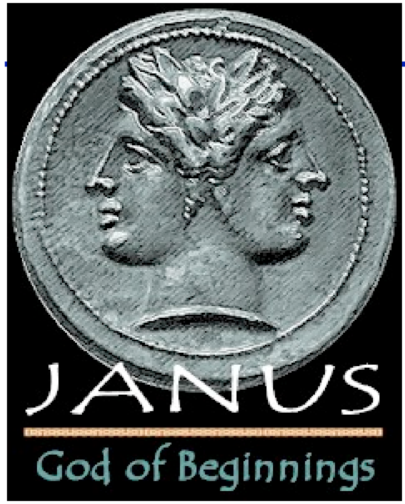

From Objects

to Components

to Services

Formal models for service-oriented interfaces and layered architectures

Manfred Broy

Technische Universität München
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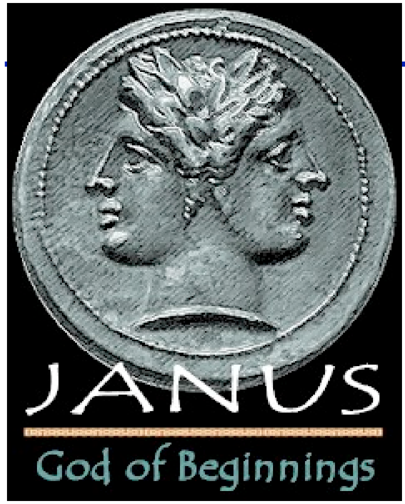
D-80290 München, Germany



From Objects

to Components

to Services

2.Lecture: Components

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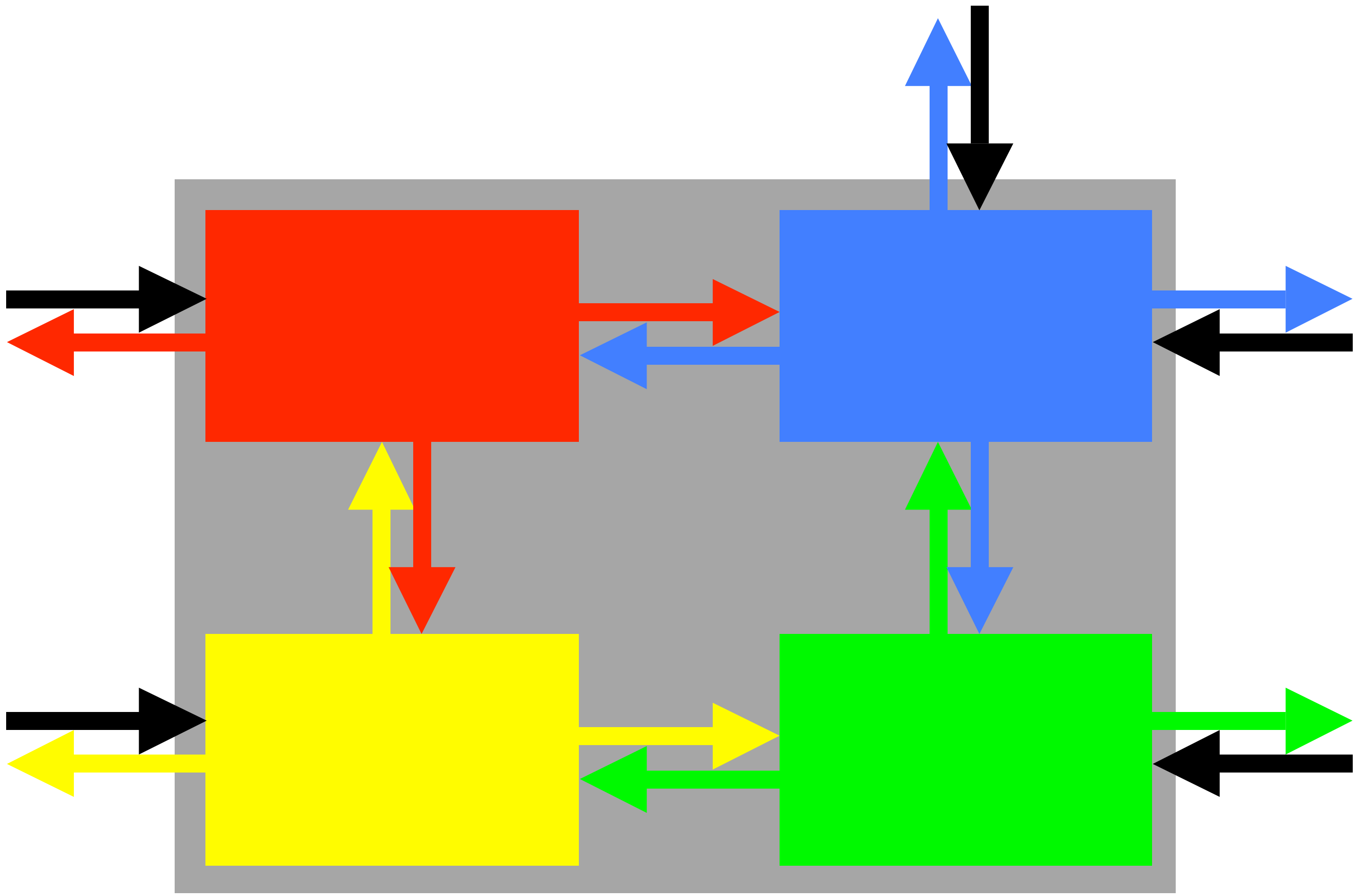
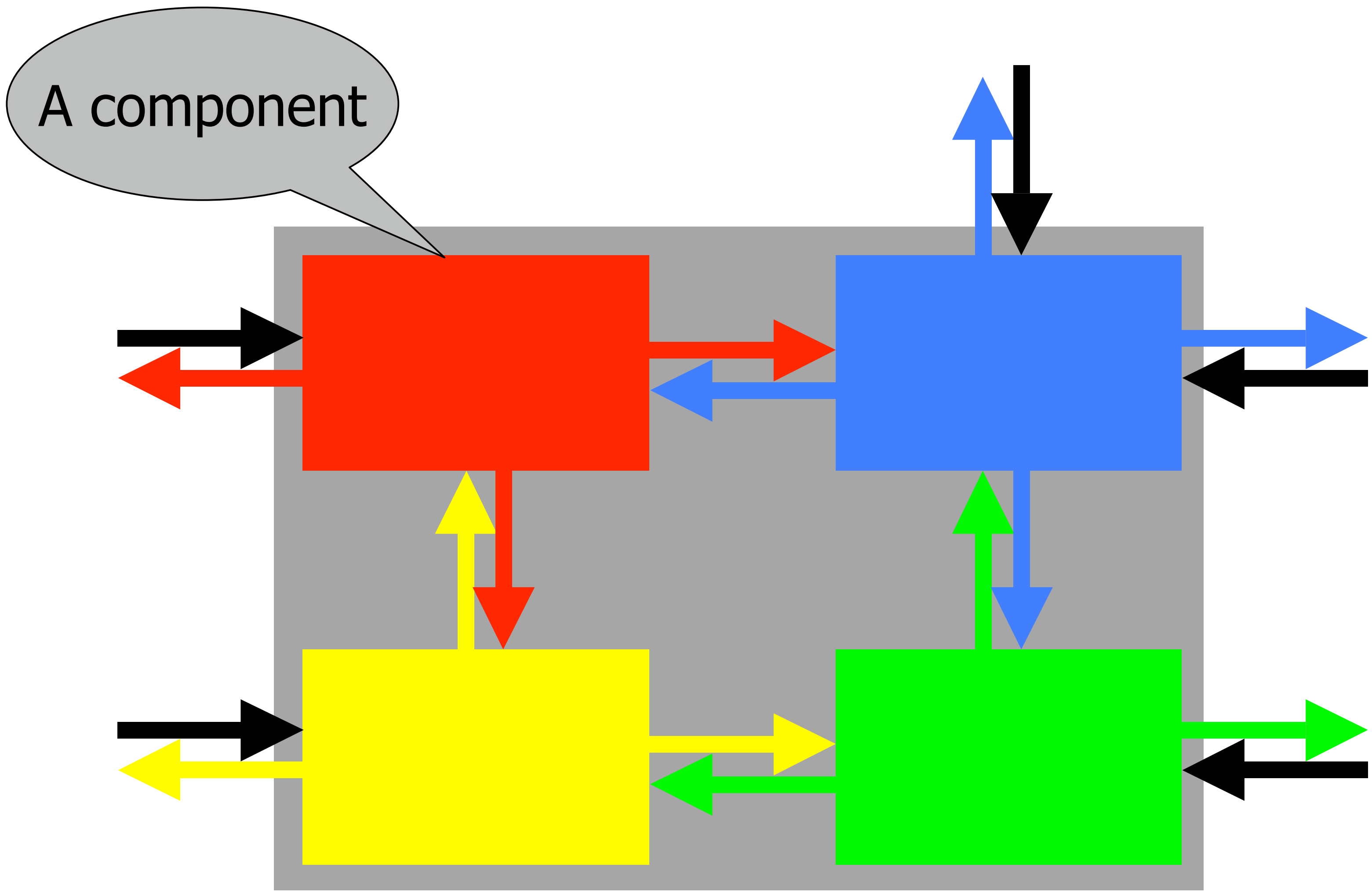
System structures and architectures



System structures and architectures



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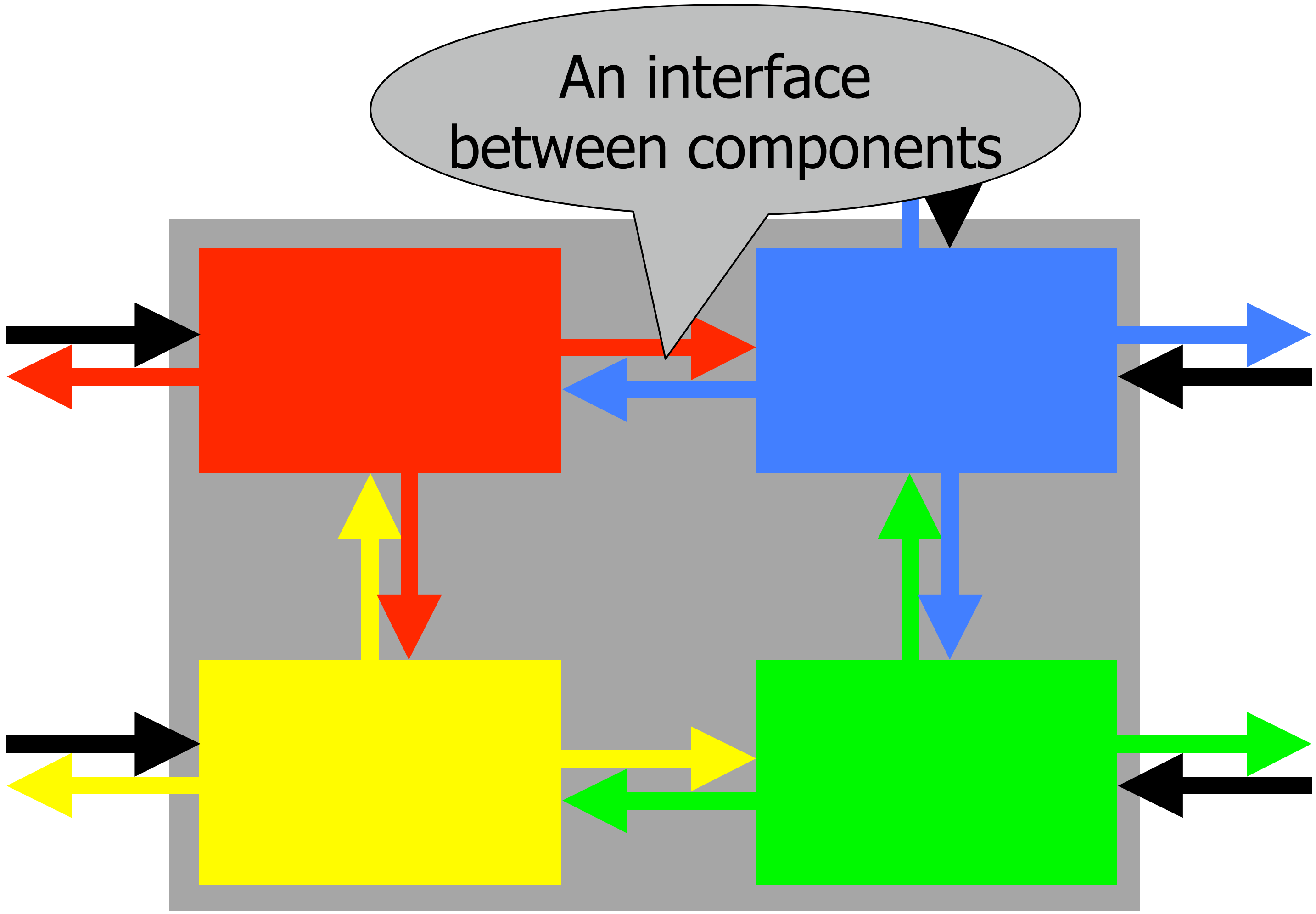

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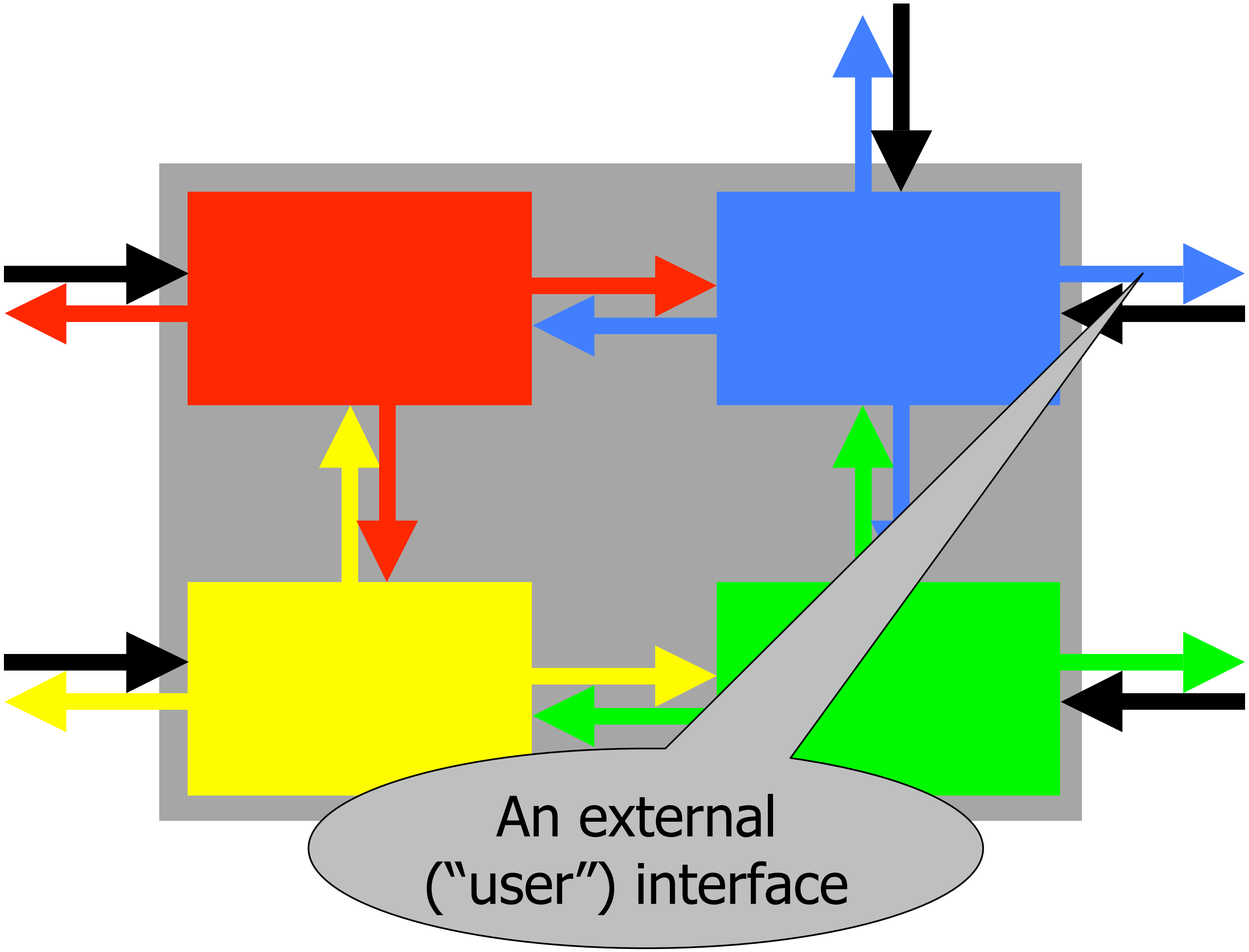

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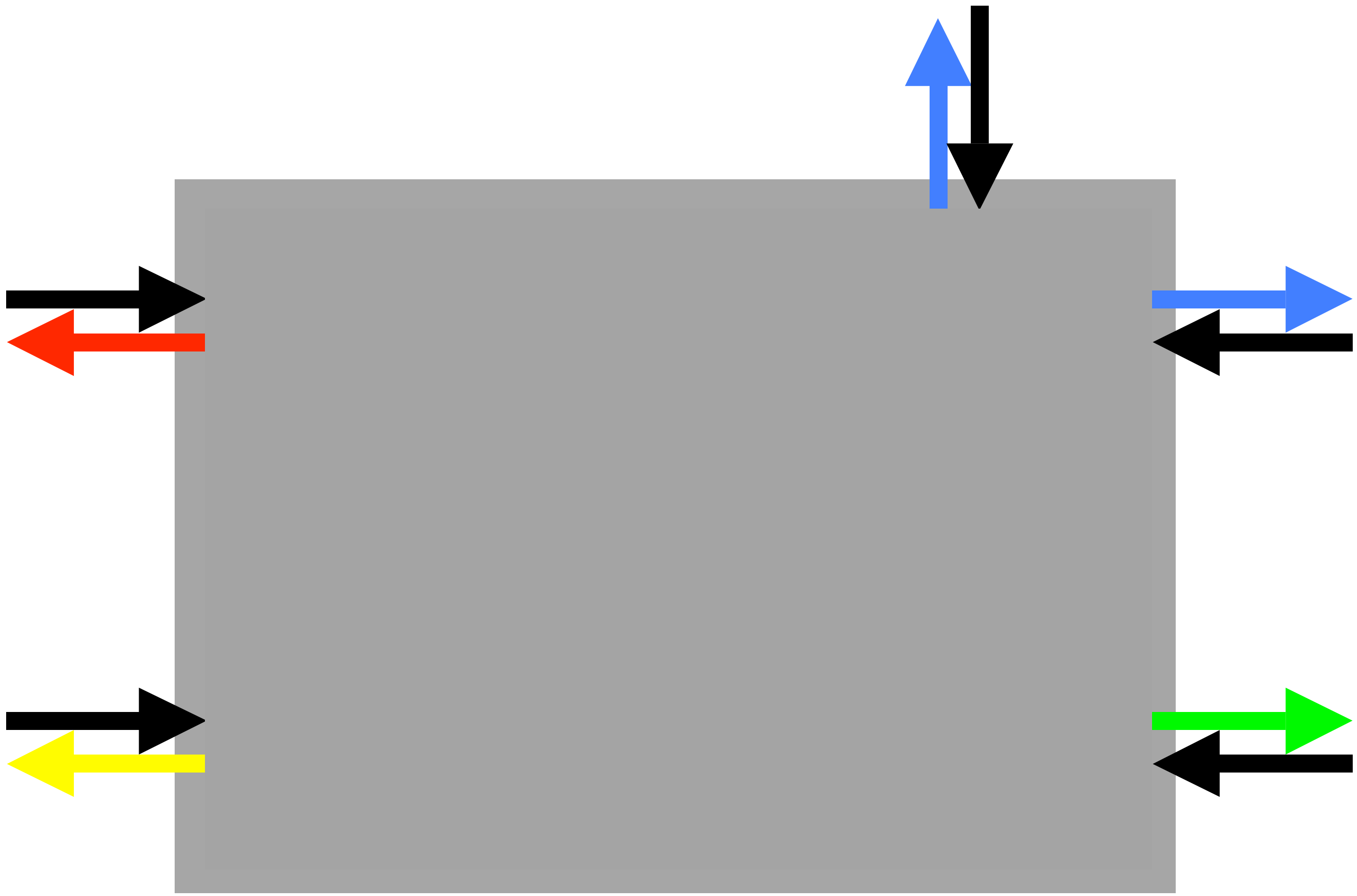
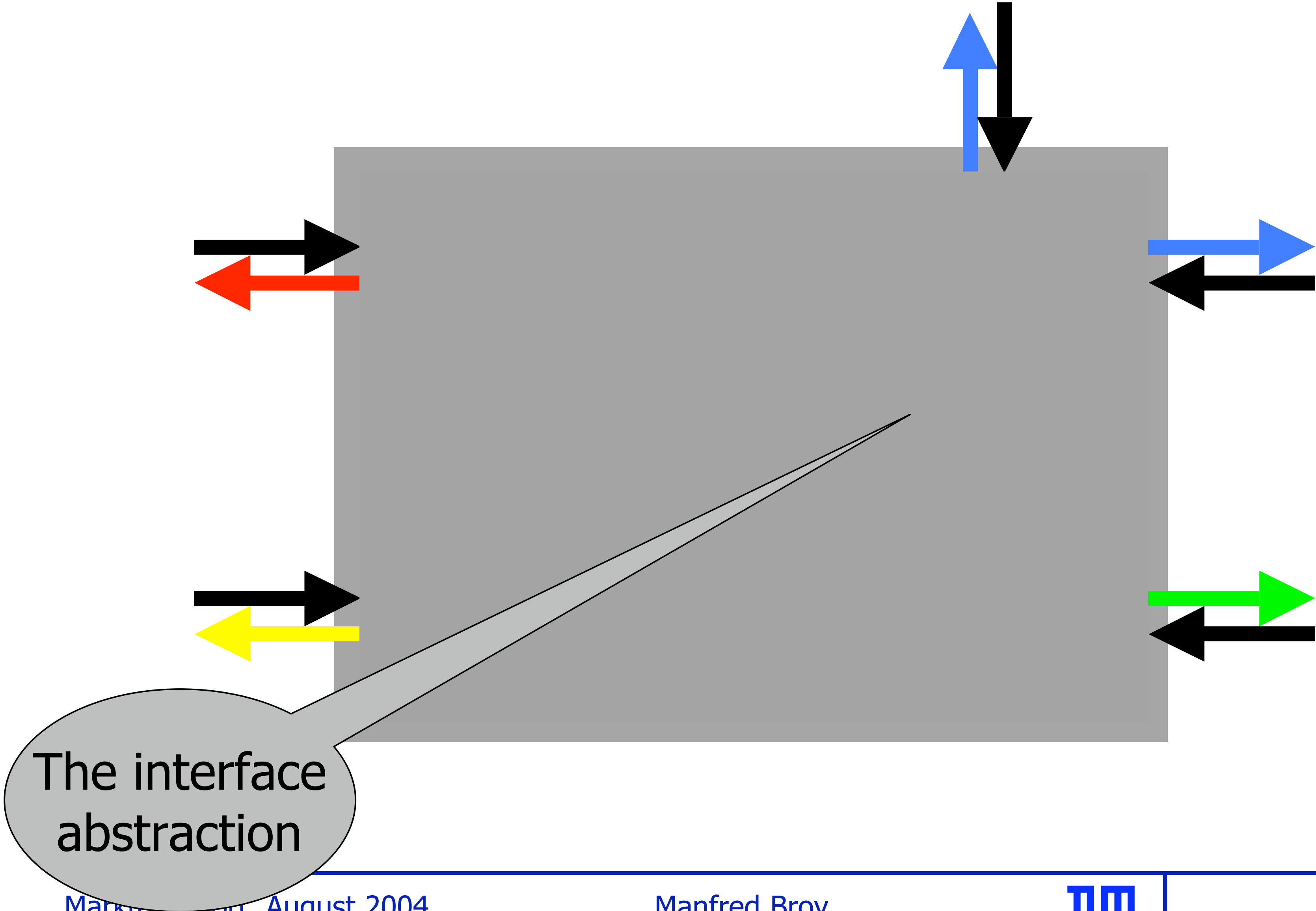


System structures and architectures



Component and systems

A
system
is
a component
is
a system

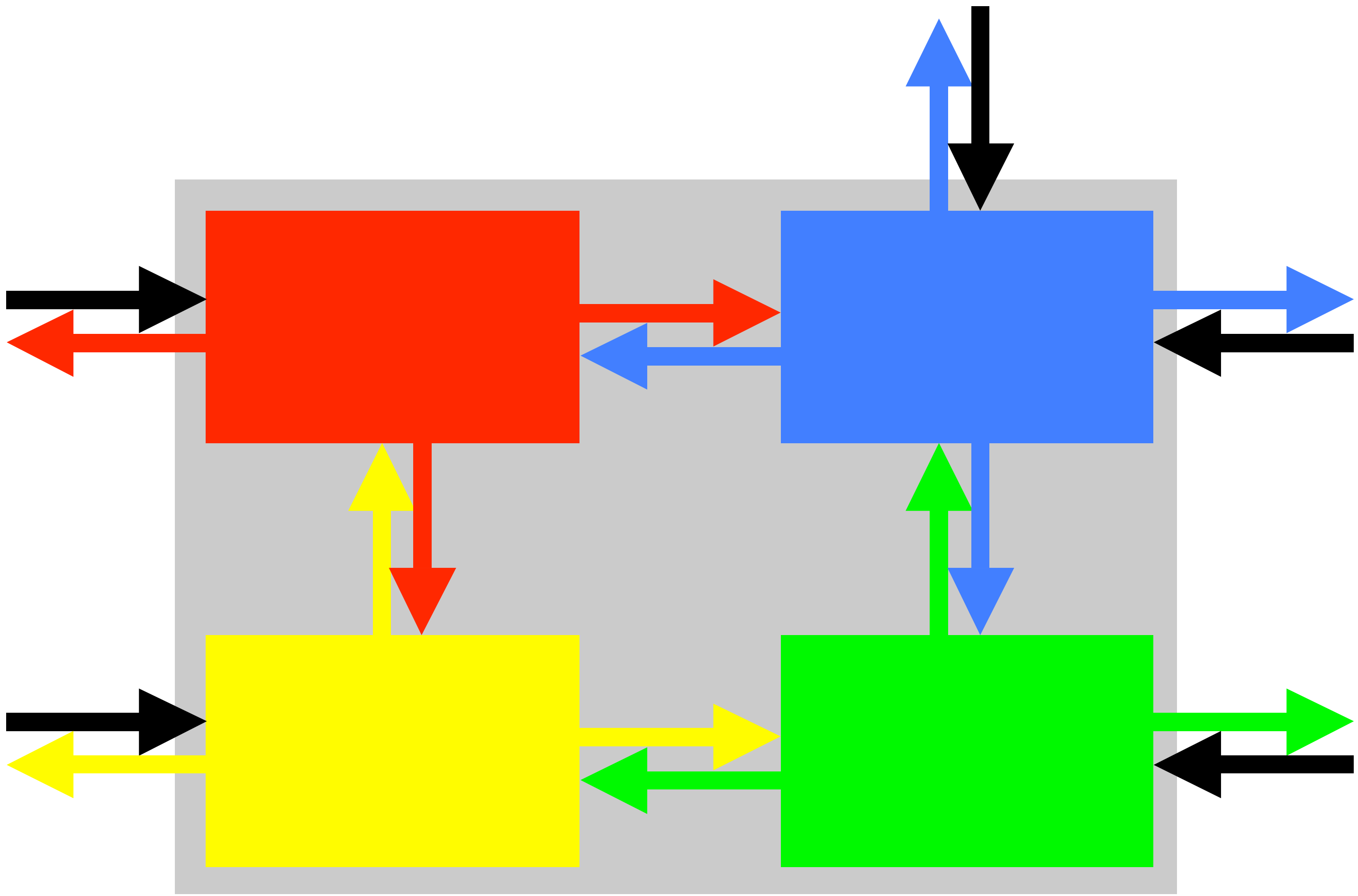
What is an observable/black box/interface behaviour

Component C1 is observable/behavioural compatible (has compatible interface behaviour) to component C2, if we can replace C2 in every (syntactically) correct system by C1 without violating the correctness.

If C1 is compatible to C2 and vice versa we call C1 and C2 observable/behavioural equivalent(having the same interface behaviours).

Note: Classes with quite different state spaces may nevertheless be compatible/equivalent

System structures and architectures



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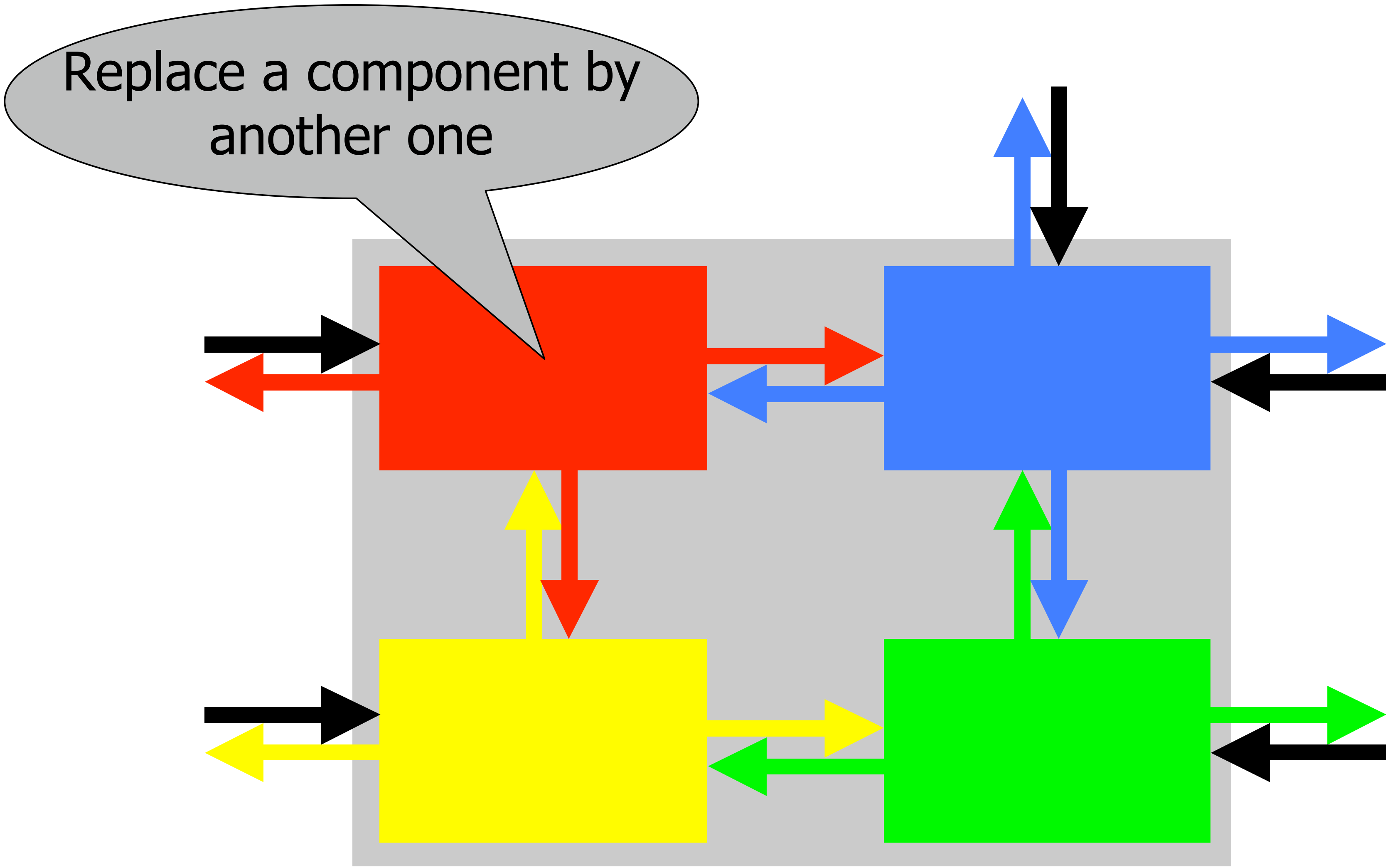

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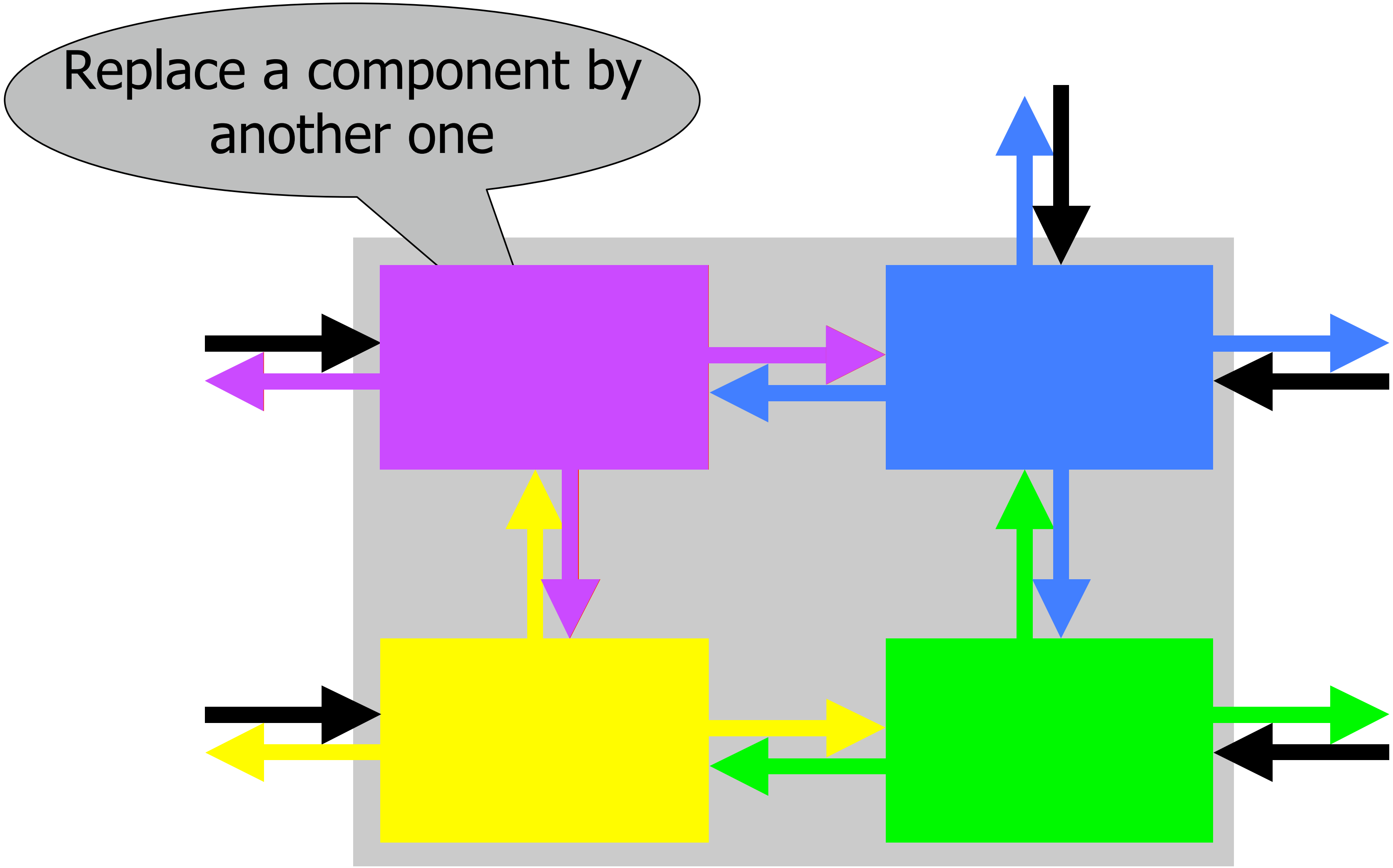
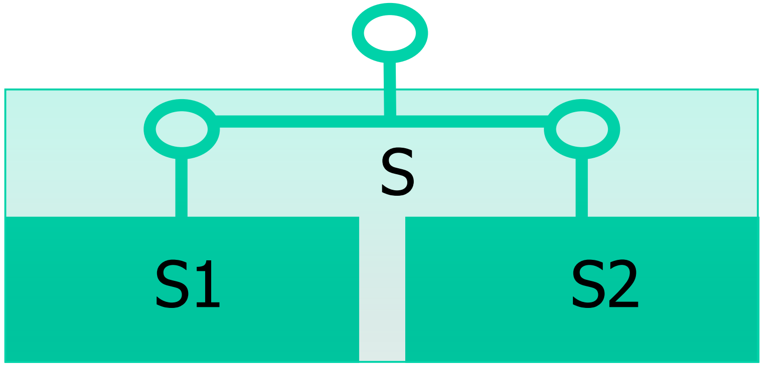


Composition / Decomposition of Systems



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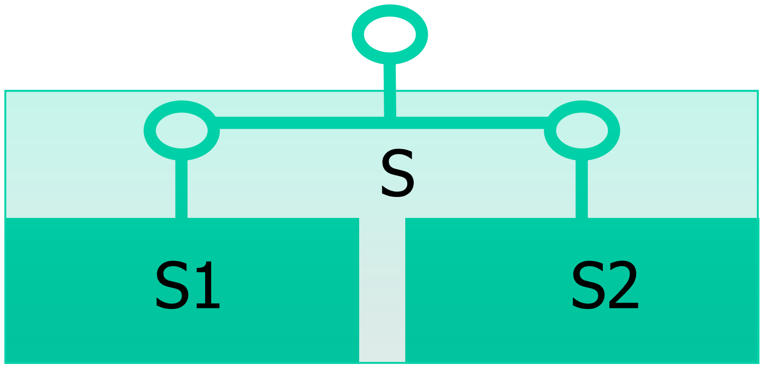
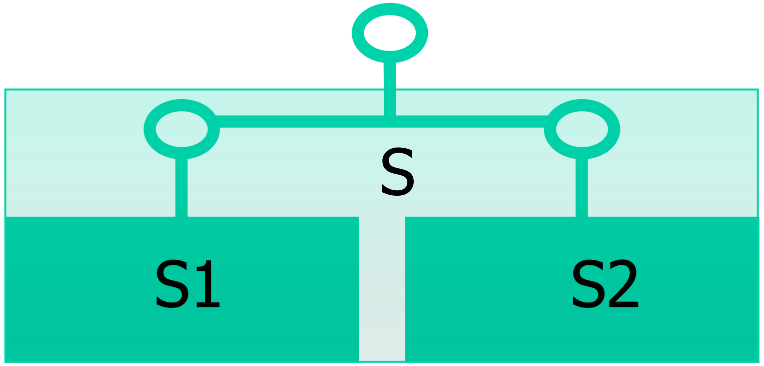


Composition $\otimes$: System $\times$ System $\rightarrow$ System *"syntactic" construction*

Composition / Decomposition of Systems



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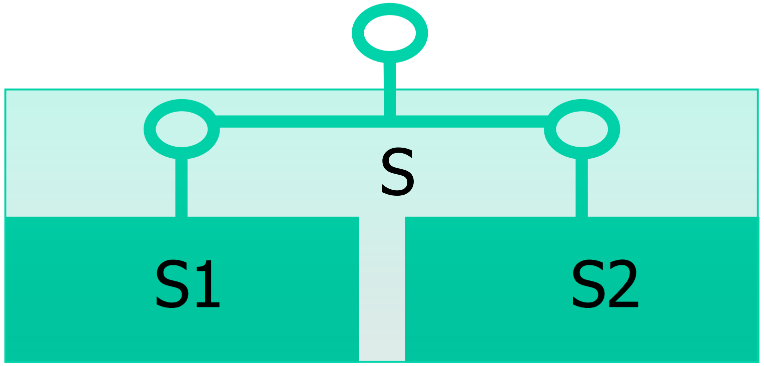
$S = S1 \otimes S2$

Composition / Decomposition of Systems

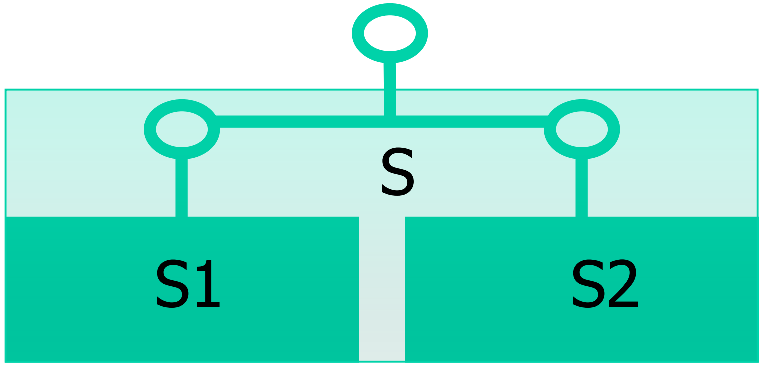


Composition $\otimes$: System $\times$ System $\to$ System *"syntactic" construction*

$S = S1 \otimes S2$

Observation: β : System $\to$ Observation *provides abstraction*

Composition / Decomposition of Systems



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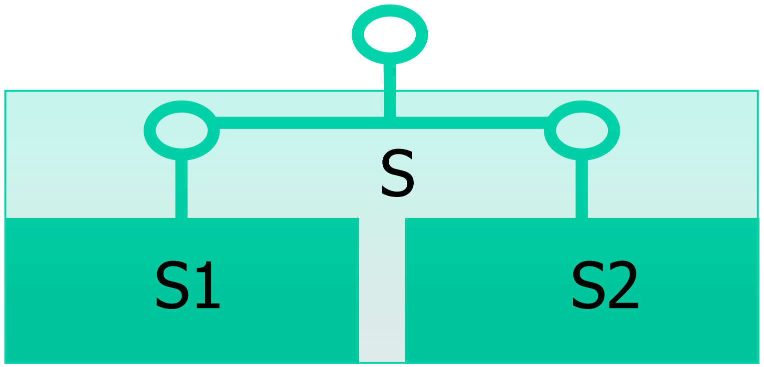
Behaviour model α : System → Behaviour

Composition / Decomposition of Systems



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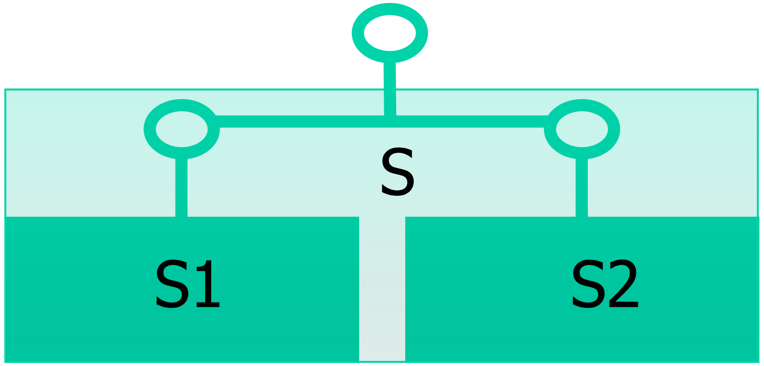

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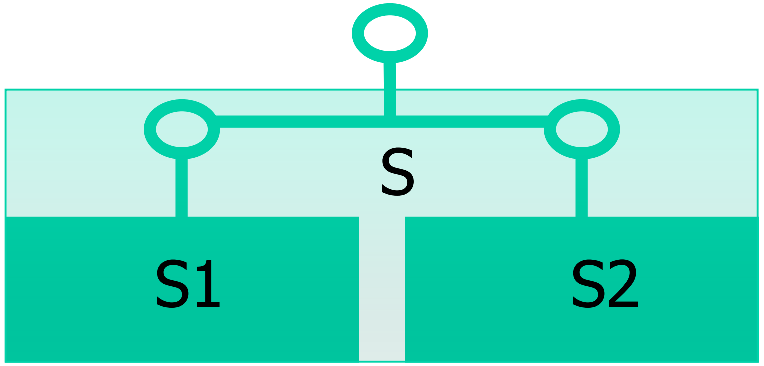
$\alpha(S) = \alpha(S1) \otimes' \alpha(S2)$

Composition / Decomposition of Systems



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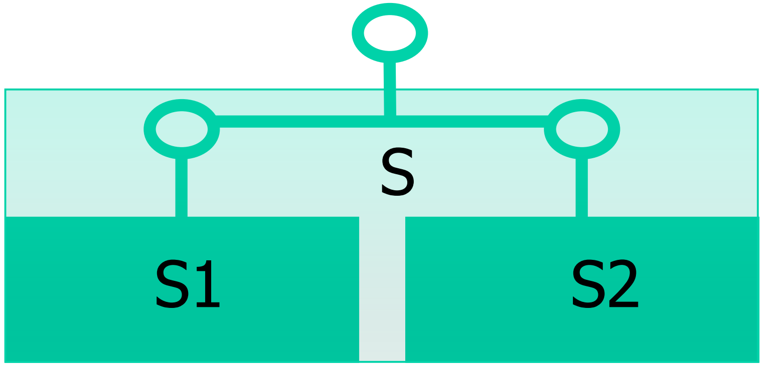
Expressivity: γ : Behaviour → Observation

Composition / Decomposition of Systems



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Observation: β : System → Observation *provides abstraction*

Behaviour model α : System → Behaviour

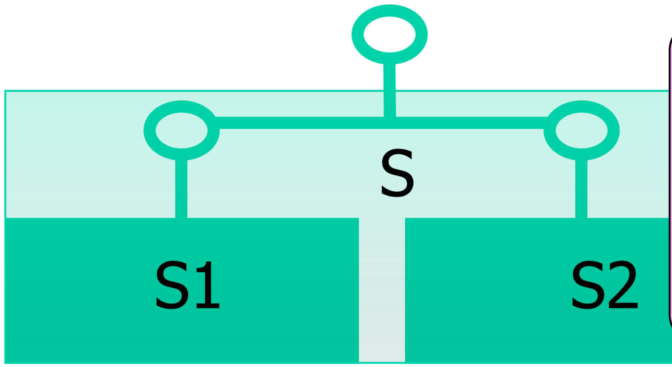
Compositionality: $\otimes$': Behaviour × Behaviour → Behaviour

$\alpha(S) = \alpha(S1) \otimes' \alpha(S2)$

Expressivity: γ : Behaviour → Observation

$\gamma(\alpha(S)) = \beta(s)$

Composition / Decomposition of Systems



In general, there does not exist an operator

$\otimes''$: Observation × Observation → Observation

such that

$\beta(S) = \beta(S1) \otimes'' \beta(S2)$

Composition $\otimes$: System × S… → System *"syntactic" construction*

$S = S1 \otimes S2$

Observation: β : System → Observation *provides abstraction*

Behaviour model α : System → Behaviour

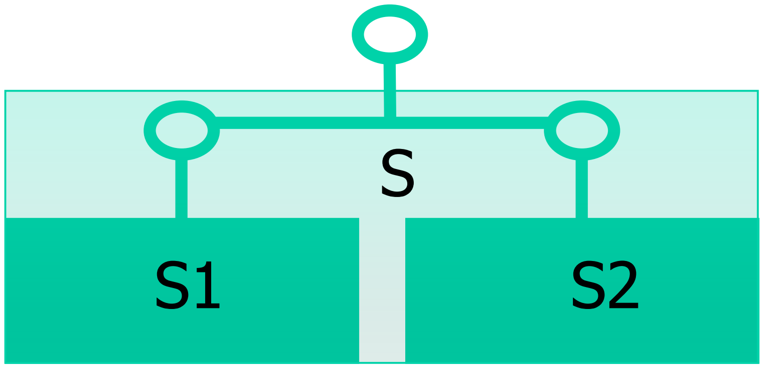
Compositionality: $\otimes'$: Behaviour × Behaviour → Behaviour

$\alpha(S) = \alpha(S1) \otimes' \alpha(S2)$

Expressivity: γ : Behaviour → Observation

$\gamma(\alpha(S)) = \beta(s)$

Composition / Decomposition of Systems



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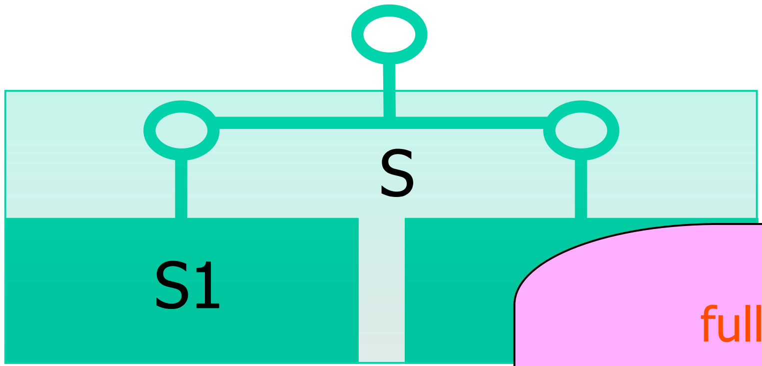


α is called
fully abstract (w.r.t. the observation β),
if there does not exist
a behaviour Behaviour’ and an abstraction function

α‛ : Behaviour → Behaviour’

where for α°α‛ compositionality and expressivity holds

Composition ⊗: Sy

S = S

Observation: β : Sy

Behaviour model α : System our

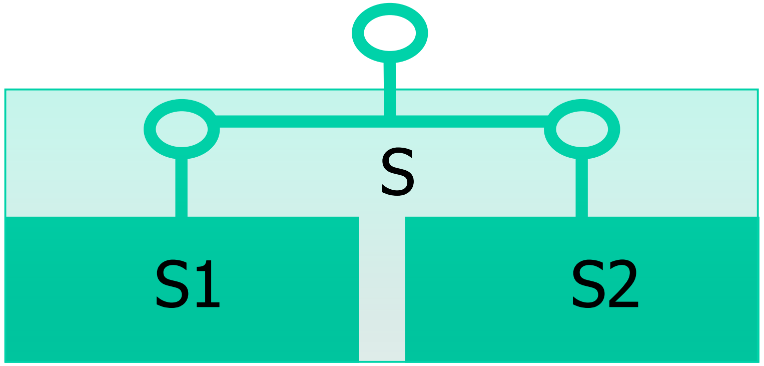
Compositionality: ⊗‛: Be our × Behaviour → Behaviour

α(S) = α(S1) ⊗‛ α(S2)

Expressivity: γ : Behaviour → Observation

γ(α(S)) = β(s)

Composition / Decomposition of Systems



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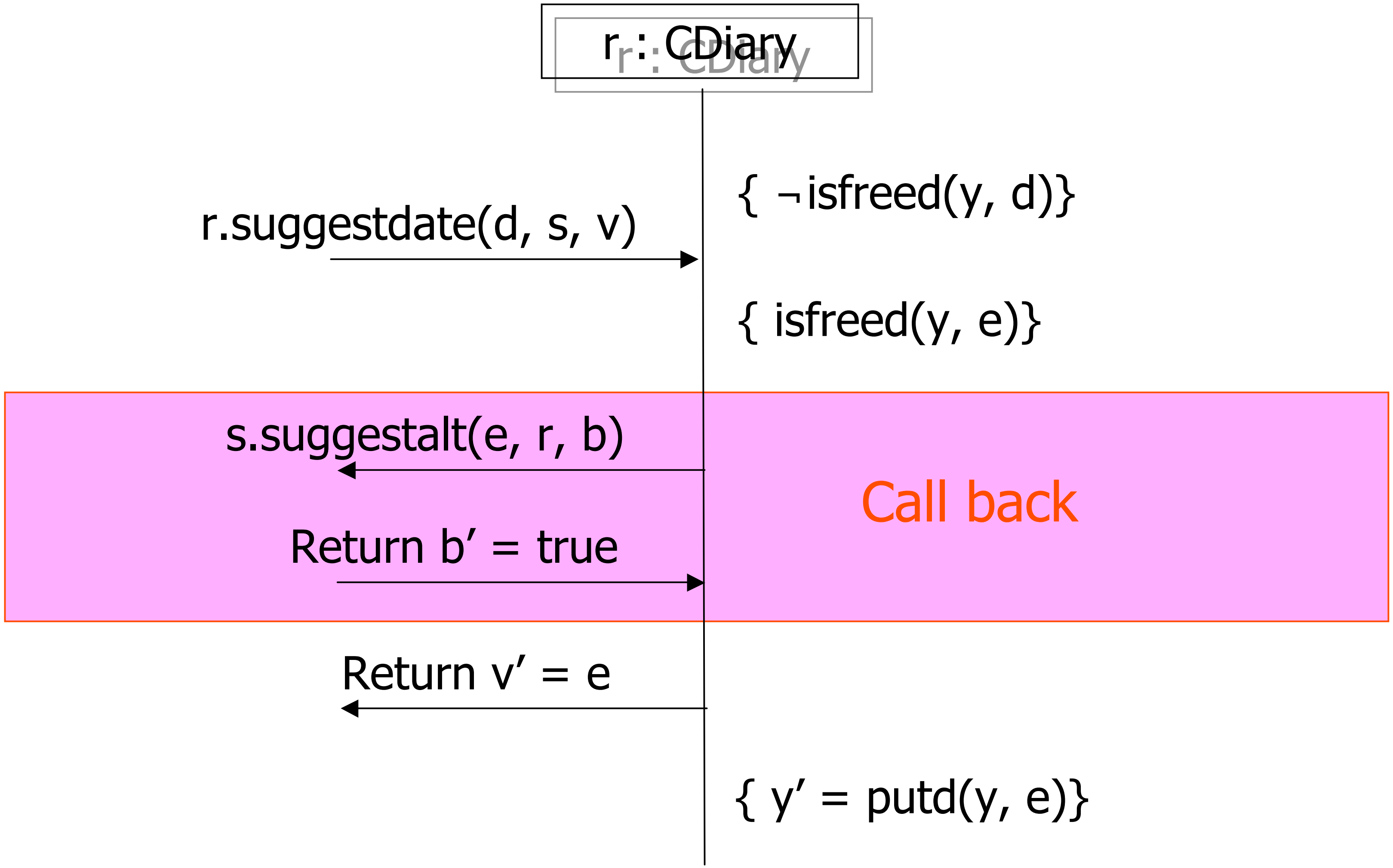
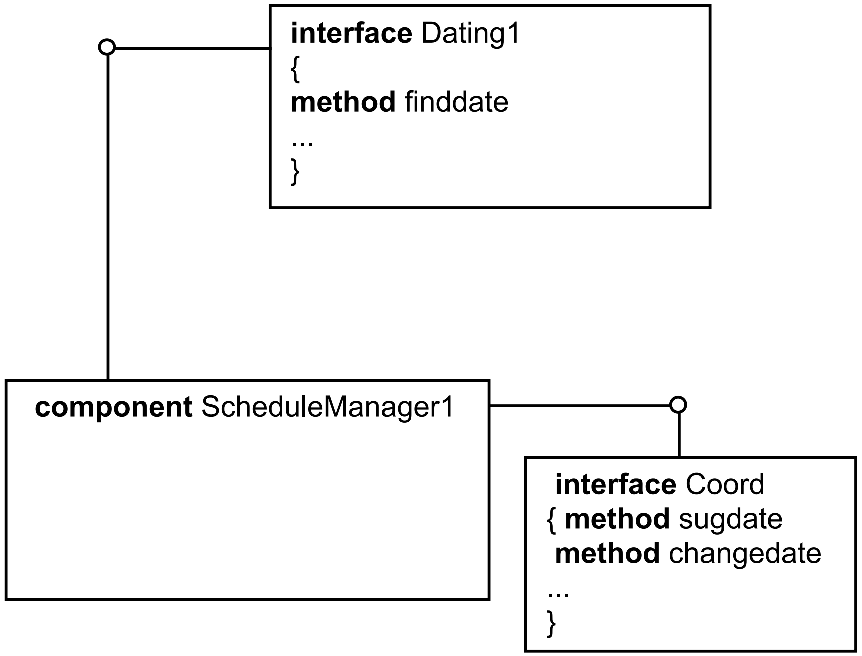
Complications in OO: call backs/forwarded calls



Forming architectures in OO: Interface specifications

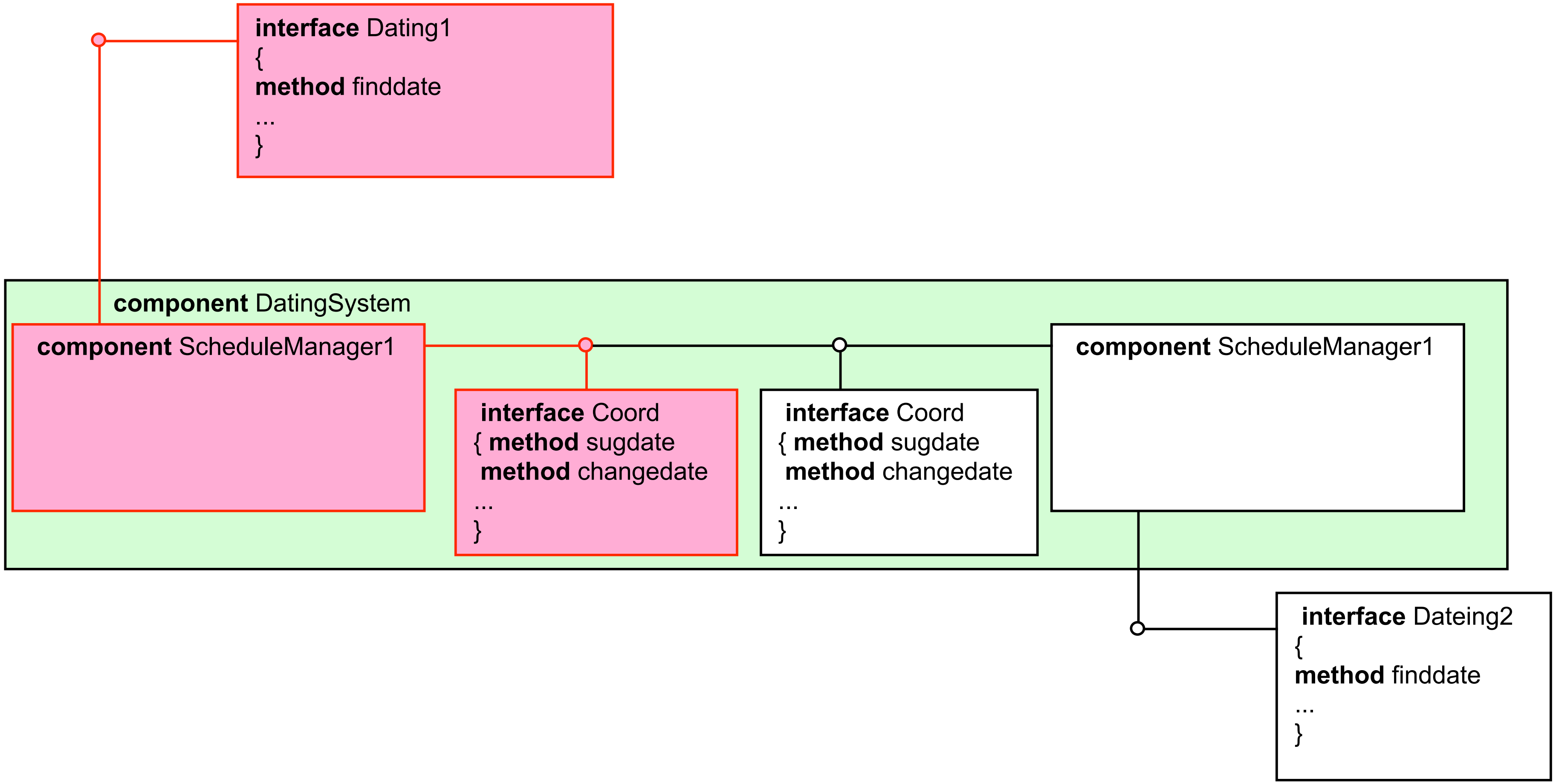


What is a component in OO?

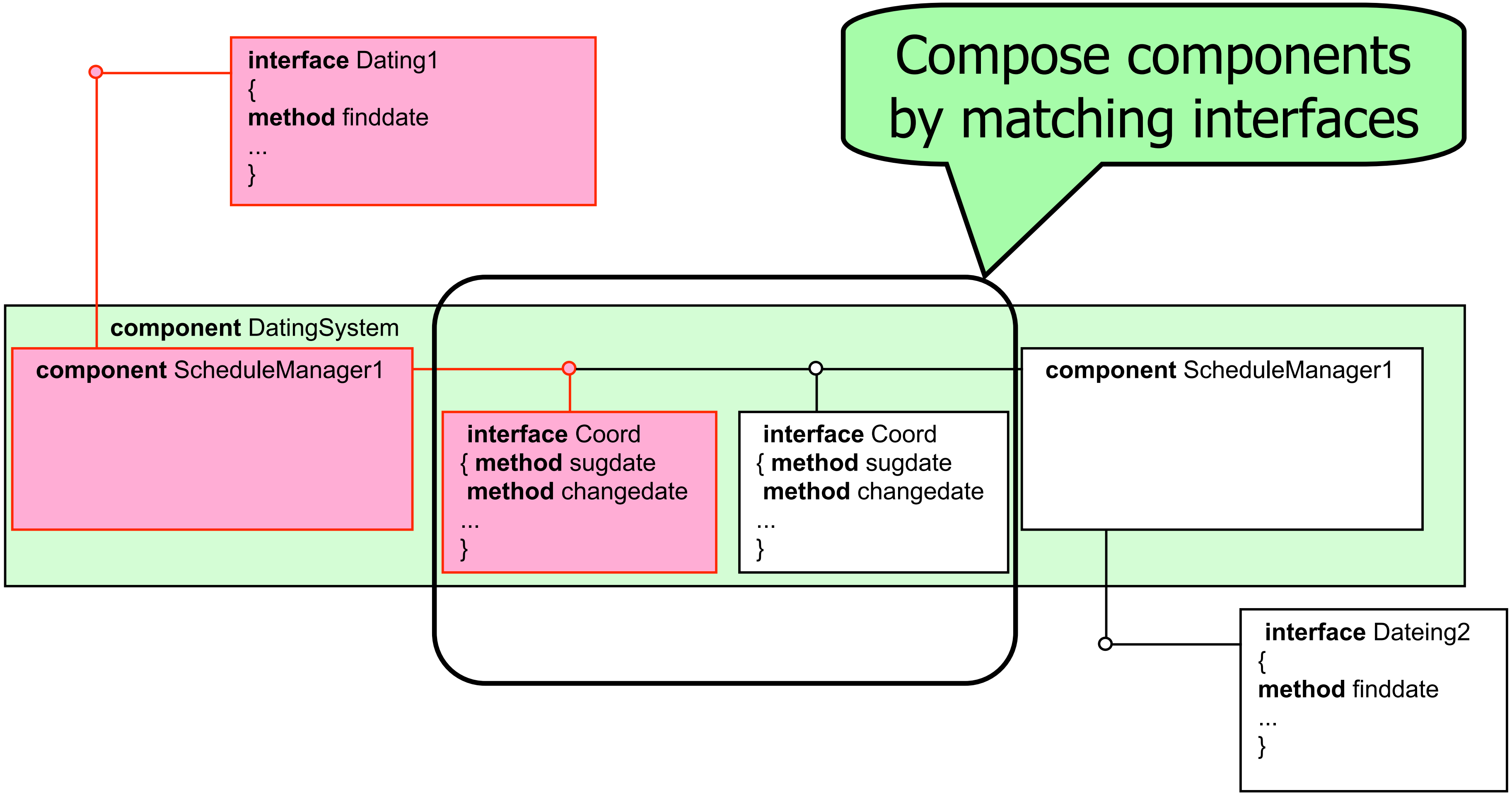
- A class?

- A unit with several sub-interfaces all with export and import methods?

Forming architectures: Composition



Forming architectures: Composition

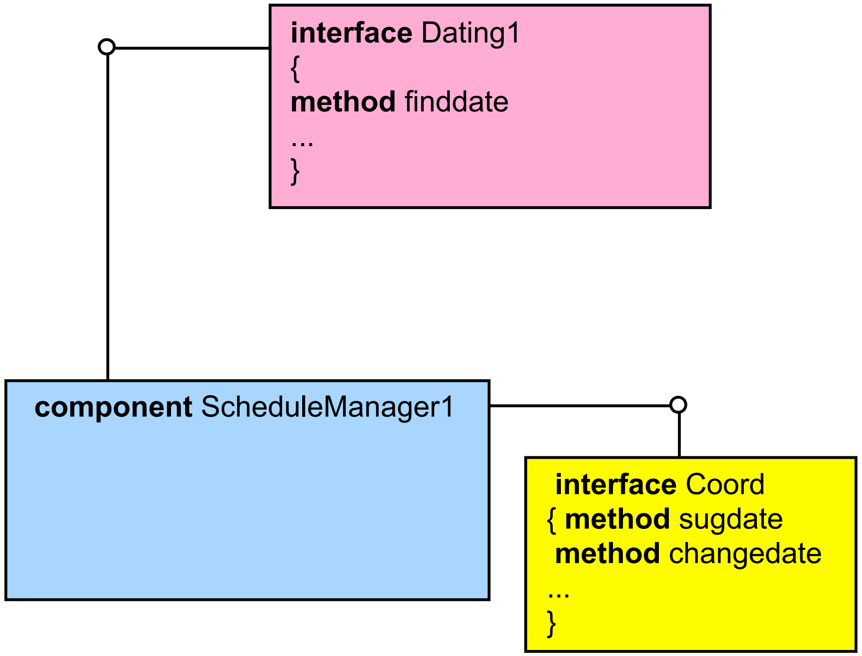


OO: why classes/objects are not enough

Conventional OO has the following deficiencies:

- Synchronous method invocation inadequate concept
  - for system with varying availability and QoS
  - inherently sequential
- Interface specifications in OO insufficient
  - Design by contract breaks principle of encapsulation
  - In the presence of forwarded calls atomicity of method invocation does not work - design by contract fails
  - Export/import specs needed
- Appropriate notion of component missing
- Concept of composition missing/unclear/too complicated
- No support of hierarchical composition/decomposition
- No build-in concept of real time/concurrency

Forming architectures: Decomposition into layers



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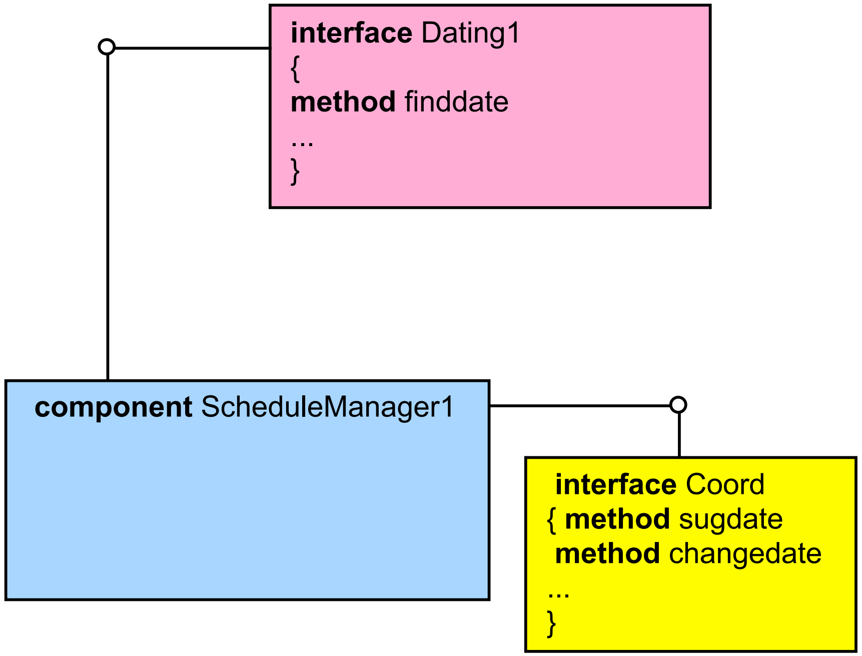

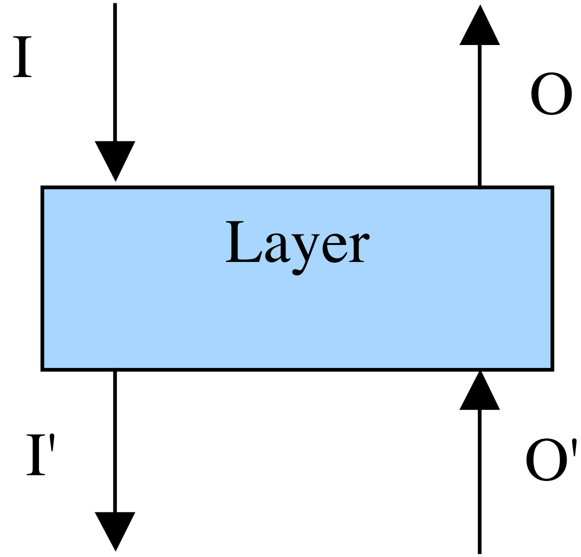

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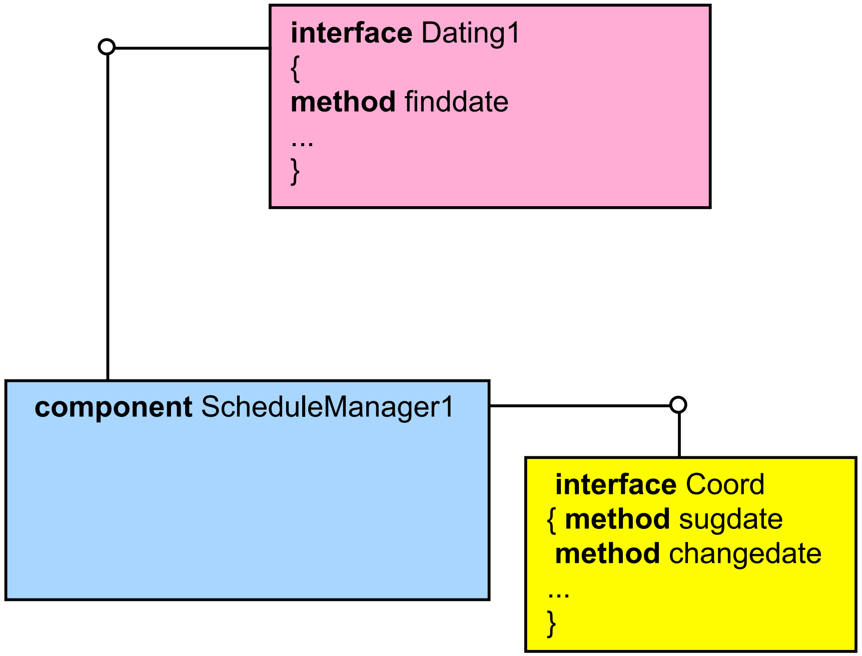
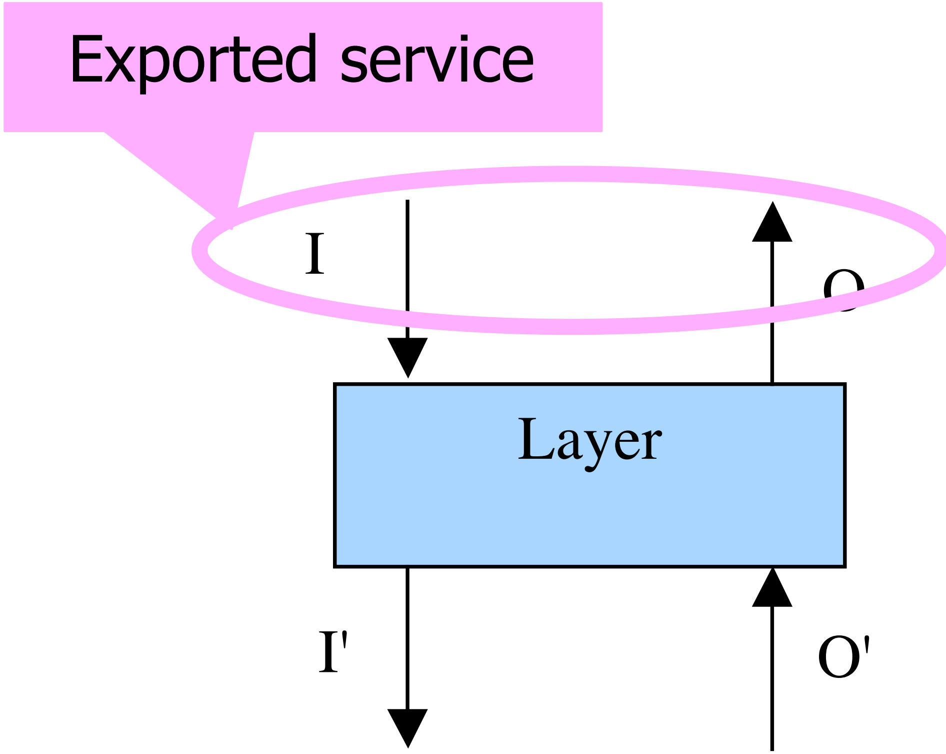
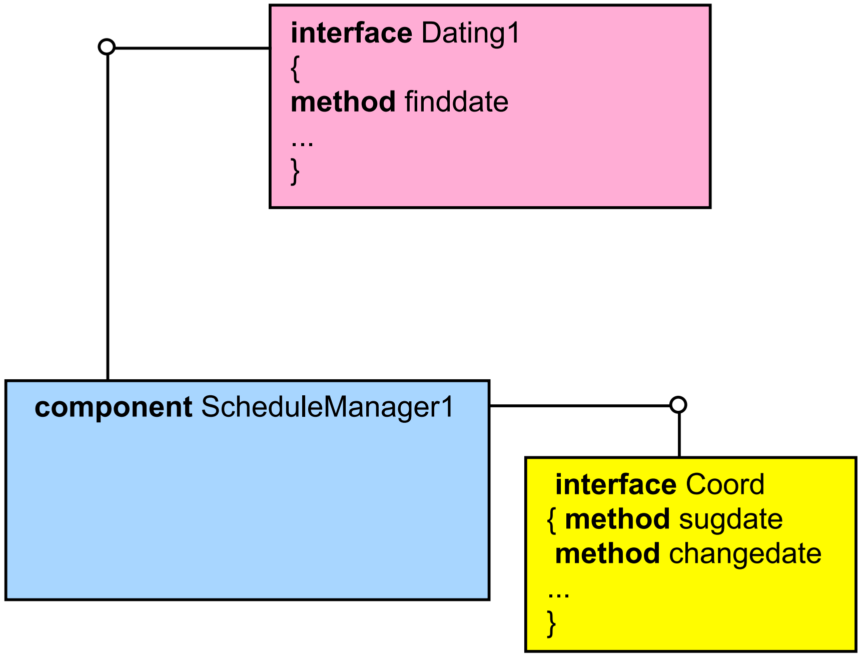
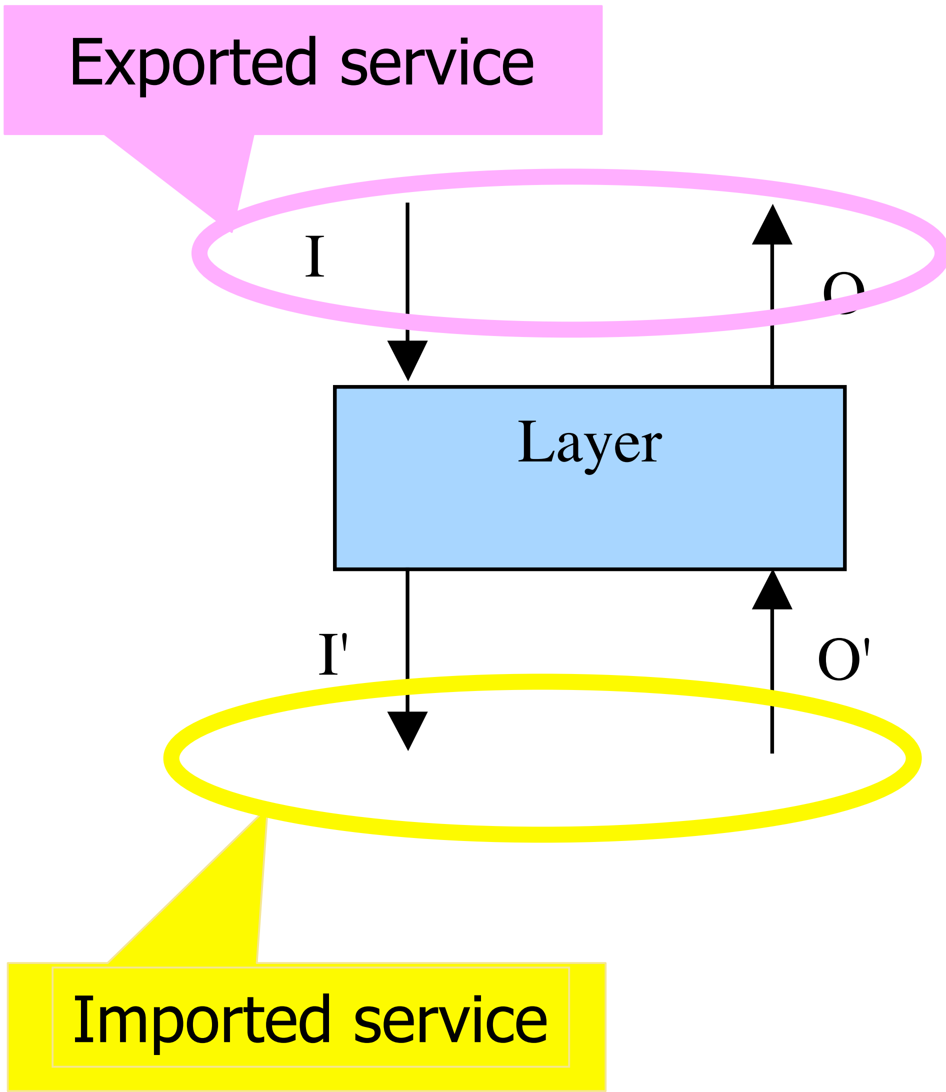




Forming architectures: Decomposition into layers





What is a service?

The term service is use extensively in IT!
Often without a precisely defined meaning!

- Service:
  - ◇ A set of interaction patterns!
- Typical service structures:
  - ◇ Service access protocol
  - ◇ Service provision protocol
- Essential concepts:
  - ◇ Service specification
  - ◇ Service composition
- Service composition
  - ◇ Service import/export
- Service refinement

Goals of the work

- A formal model for services, layers and layered architectures
- A theory for relating, composing, and refining services, layers, and layered architectures
- Techniques for specifying services, layers, and layered architectures
- Techniques for verifying services, layers, and layered architectures
- A methodology for designing services, layers, and layered architectures
- Design patterns for services, layers, and layered architectures

Streams

Streams are communication histories for sequential communication devices called channels.

A stream of digits:

$x = \langle\ 2\quad 1\ 7\quad 4\ 3\ 3\quad 8\ 9\ 2\quad 6\ \ldots\ \rangle$

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time

Streams

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A stream of digits:

$$x = \langle\, | \; 2 \; | \; 1\;7 \; | \; | \; 4\;3\;3 \; | \; 8\;9\;2 \; | \; | \; 6 \; \ldots \,\rangle$$

time

Timed Streams

Timed Streams are communication histories for sequential communication devices called channels.

A timed stream of digits:

$$x = \langle \; | \; \langle 2 \rangle \; | \; \langle 1\ 7 \rangle \; | \; \langle \rangle \; | \; \langle 4\ 3\ 3 \rangle \; | \; \langle 8\ 9\ 2 \rangle \; | \; \langle \rangle \; | \; \langle 6 \rangle \; \dots \; \rangle$$

time

Timed Streams

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A timed stream of digits:

$$x = \langle\ \langle 2\rangle\quad \langle 1\ 7\rangle\quad \langle\rangle\qquad \langle 4\ 3\ 3\rangle\langle 8\ 9\ 2\rangle\ \ \langle\rangle\quad \langle 6\rangle\ \ \dots\quad \rangle$$

time

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$x : \mathrm{IN} \to \{0, ..., 9\}^*$ timed stream of digits

Basics of our System Model: Streams and Behaviours

M	Universe of messages/data elements
$TYPE$	Set of all types
C	set of typed channels a channel is an identifier with a type
$\mathbb{N} \to M^*$	timed stream
$x : C \to (\mathbb{N} \to M^*)$	channel history for the channel set x(c) is a timed stream with messages of the type of channel c
$\mathbb{H}(C)$ and $\vec{C}$	set of channel histories for channel set C
$(z \oplus z') \in \mathbb{H}(C \cup C')$	union of histories

Notation

Let s, r be streams

$z\hat{\ }s$	concatenation of a sequence or stream z to a stream s,
$s \sqsubseteq r$	s is a prefix of r $\quad s \sqsubseteq r \equiv \exists\, u: s\hat{\ }u = r$
S©s	substream of s with only the elements in the set S,
S#s	number of elements in s that are elements in the set S,
s.k	k-th sequence in the stream s,
$s\downarrow k$	prefix of the first k sequences in the timed stream s,
$s\uparrow k$	stream s without the first k sequences,
$\bar{s}$	finite or infinite (nontimed) stream that is the result of concatenating all sequences in s

All these notions apply also for channel histories

Notation

Notation

let s $= \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle\ \ldots\ \rangle$

—

Notation

$$\text{let } s = \langle \langle 2 \rangle \; \langle 1\ 7 \rangle \; \langle \rangle \; \langle 4\ 3\ 3 \rangle \; \langle 8\ 9\ 2 \rangle \; \langle \rangle \; \langle 6 \rangle \; \ldots \rangle$$

$$\{2, 3\} \text{©} s = \langle \langle 2 \rangle \; \langle \rangle \quad \langle \rangle \; \langle 3\ 3 \rangle \quad \langle 2 \rangle \quad \langle \rangle \; \langle \rangle \; \ldots \rangle$$

—

Notation

$$\text{let } s = \langle \langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle\ \ldots\ \rangle$$

$$\{2, 3\}©s = \langle \langle 2\rangle\ \langle\rangle\quad\ \langle\rangle\ \langle 3\ 3\rangle\quad \langle 2\rangle\qquad \langle\rangle\ \ \langle\rangle\ \ldots\ \rangle$$

$$s.3 = \langle\rangle$$

—

Notation

let s $= \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle\ \ldots\ \rangle$

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s.3 $= \langle\rangle$

s↓3 $= \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\rangle$

—

Notation

$\text{let } s = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$

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$s.3 = \langle\rangle$

$s\downarrow 3 = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\rangle$

$s\uparrow 3 = \langle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$

Notation

$$\text{let } s = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$$

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$$s\downarrow 3 = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\rangle$$

$$s\uparrow 3 = \langle \quad \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$$

$$\overline{s} = \langle 2 \quad 1\ 7 \quad 4\ 3\ 3 \quad 8\ 9\ 2 \quad 6 \ldots \rangle$$

Notation

let s	$= \langle \langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$
$\{2, 3\}$©s	$= \langle \langle 2\rangle\ \langle\rangle\ \langle\rangle\ \langle 3\ 3\rangle\ \langle 2\rangle\ \langle\rangle\ \langle\rangle \ldots \rangle$
s.3	$= \langle\rangle$
$s\downarrow 3$	$= \langle \langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle \rangle$
$s\uparrow 3$	$= \langle \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$
$\overline{s}$	$= \langle 2\ \ 1\ 7\ \ 4\ 3\ 3\ \ 8\ 9\ 2\ \ 6 \ldots \rangle$
let z	$= \langle \langle 3\ 5\rangle\ \langle 5\ 6\ 7\rangle\ \langle\rangle \rangle$

Notation

$\text{let } s = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$

$\{2, 3\}©s = \langle\langle 2\rangle\ \langle\rangle\quad \langle\rangle\ \langle 3\ 3\rangle\quad \langle 2\rangle\quad \langle\rangle\ \langle\rangle \ldots \rangle$

$s.3 = \langle\rangle$

$s\downarrow 3 = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\rangle$

$s\uparrow 3 = \langle\quad \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$

$\overline{s} = \langle 2\quad 1\ 7\quad 4\ 3\ 3\quad 8\ 9\ 2\quad 6 \ldots \rangle$

$\text{let } z = \langle\langle 3\ 5\rangle\ \langle 5\ 6\ 7\rangle\ \langle\rangle\rangle$

$\{5, 7\}\#z = 3$

Notation

$$\text{let } s = \langle\langle 2\rangle\ \langle 1\ 7\rangle\ \langle\rangle\ \langle 4\ 3\ 3\rangle\ \langle 8\ 9\ 2\rangle\ \langle\rangle\ \langle 6\rangle \ldots \rangle$$

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Components

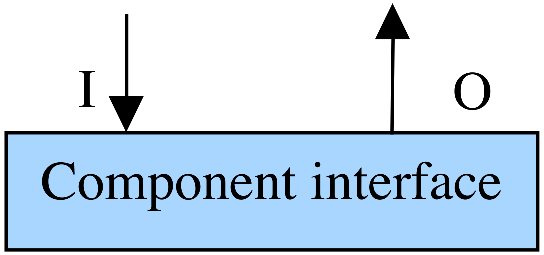
$(I \blacktriangleright O)$ — *syntactic interface* with set of input channels I and of output channels O

$F : \vec{I} \rightarrow \wp(\vec{O})$ — component interface for $(I \blacktriangleright O)$ with *timing property* (let $x, z \in \vec{I}, y \in \vec{O}, t \in \mathbb{N}$):

$$x{\downarrow}t = z{\downarrow}t \Rightarrow \{y{\downarrow}t{+}1 : y \in F(x)\} = \{y{\downarrow}t{+}1 : y \in F(z)\}$$

$x{\downarrow}t$ — prefix of x with t finite sequences

A component is a total behaviour



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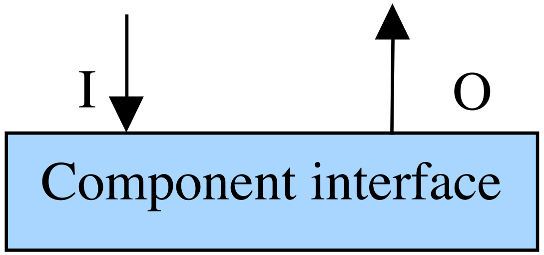

Causality

Consider the identity function:

$$F : \vec{C} \to \wp(\vec{C})$$

where $y \in F.x \Rightarrow \bar{x} = \bar{y}$

causality enforces

$$F.x = \{y: \bar{x} = \bar{y} \wedge \forall\, t \in \mathrm{IN}: \overline{y \downarrow t+1} \sqsubseteq \overline{x \downarrow t}\}$$

Example:

$x = \langle \langle 2 \rangle\ \langle 1\ 7 \rangle\ \langle \rangle\ \langle 4\ 3\ 3 \rangle \langle 8\ 9\ 2 \rangle \langle \rangle\ \langle 6 \rangle \ldots \rangle$

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$y =$	$\langle \langle \rangle$	$\langle \rangle$	$\langle 2 \rangle$	$\langle 1\ 7 \rangle$	$\langle \rangle$	$\langle 4\ 3\ 3\ 8\ 9\ 2 \rangle$	$\langle \rangle$	$\langle 6 \rangle \ldots \rangle$

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Causality

Consider the identity function:

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The golden rule of communication:

An information can never occur as output before it was received as input

where $\quad y \in F.x \Rightarrow \bar{x} = \bar{y}$

causality enforces

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