

From Objects

to Components

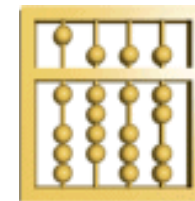
to Services

3.Lecture: Services

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Components

(I ► O) *syntactic interface* with set of
input channels I and of output channels O

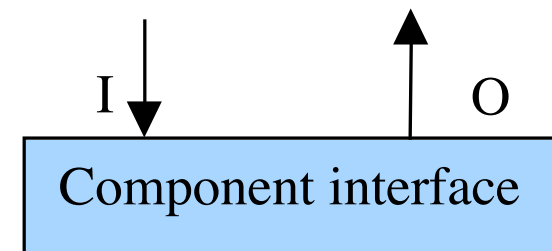
$F : \vec{I} \rightarrow \wp(\vec{O})$ component interface for (I ► O)
with *timing property*
(let $x, z \in \vec{I}, y \in \vec{O}, t \in \mathbb{N}$).

Causality

$$x \downarrow t = z \downarrow t \Rightarrow \{y \downarrow t+1 : y \in F(x)\} = \{y \downarrow t+1 : y \in F(z)\}$$

$x \downarrow t$ prefix of x with t finite sequences

A component is a **total** behaviour



Causality

Consider the identity function:

$$F : \vec{C} \rightarrow \wp(\vec{C})$$

where $y \in F.x \Rightarrow \bar{x} = \bar{y}$

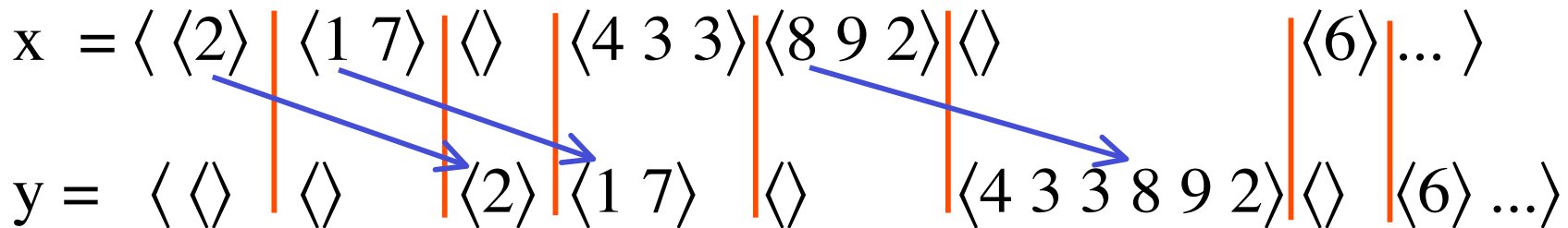
causality enforces

The golden rule of communication:

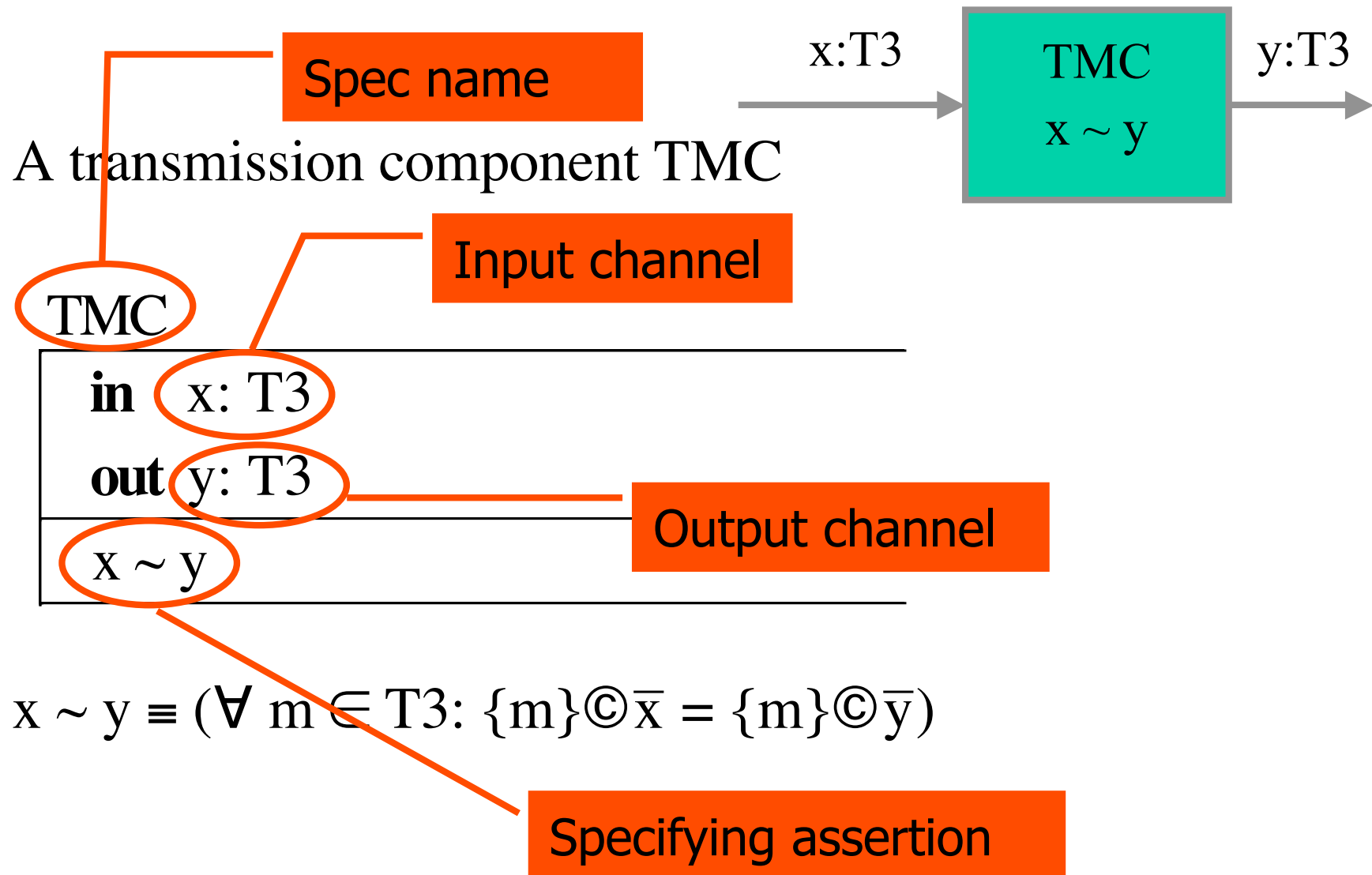
An information can never occur as output before it was received as input

$$F.x = \{y : \bar{x} = \bar{y} \wedge \forall t \in \mathbb{N} : \overline{y \downarrow t+1} \subseteq \overline{x \downarrow t}\}$$

Example:



Interface specification: How to specify services



Interface specification: How to specify services

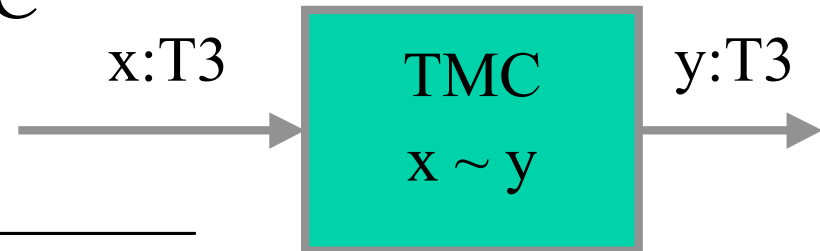
A transmission component TMC

TMC

in $x: T3$

out $y: T3$

$x \sim y$



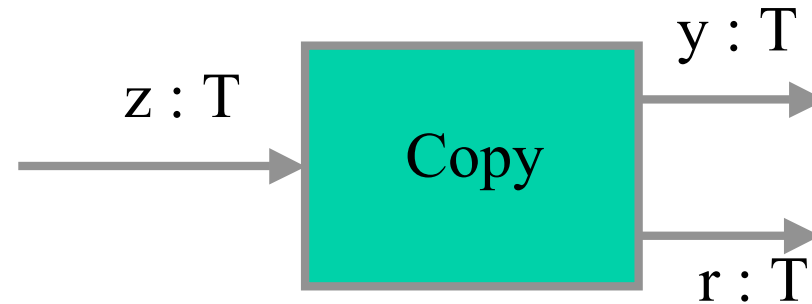
$$x \sim y \equiv (\forall m \in T3: \{m\} \odot \bar{x} = \{m\} \odot \bar{y})$$

Causality allows to conclude in addition

$$\forall t \in \mathbb{N}: \forall m \in T: \{m\} \# \bar{y} \downarrow t+1 \leq \{m\} \# \bar{x} \downarrow t$$

Example: Interface specification

Component Copy



Copy

in $z: T$

out $y, r: T$

$\bar{z} = \bar{y} \wedge \bar{z} = \bar{r}$

Causality allows to conclude in addition

$\forall t \in \mathbb{N}: \forall m \in T:$

$\{m\}\#r \downarrow t+1 \leq \{m\}\#z \downarrow t \wedge \{m\}\#y \downarrow t+1 \leq \{m\}\#z \downarrow t$

Example: Interface specification Repeater



Repeater

in $x: T$

out $r: T$

$$\forall m \in T: \{m\} \# \bar{x} > 0 \Rightarrow \{m\} \# \bar{r} = \infty$$
$$\wedge \{m\} \# \bar{x} = 0 \Rightarrow \{m\} \# \bar{r} = 0$$

Causality allows to conclude in addition

$\forall t \in \mathbb{N}$:

$$\forall m \in T: \{m\} \# \bar{x} \downarrow t = 0 \Rightarrow \{m\} \# \bar{r} \downarrow t+1 = 0$$

Remarks

- We get a very powerful, expressive model and a specification technique
- Nondeterminism/underspecification included
- The specification is done in pure higher order logic
- Verification calculus is therefore straightforward
- It is an attempt to eat the cake and keep it
 - ◇ **Time** is at your disposal whenever you need it but can be ignored whenever you like
 - ◇ **Causality** can be kept implicit, if not needed, but can be made explicit if more sophisticated reasoning is needed
 - ◇ Causality guarantees fixpoints in the case of networks of (deterministic) components

Consequences of causality

Consider the function: $F : \vec{I} \rightarrow \wp(\vec{O})$
where $F.z = \emptyset$ for some $z \in \vec{I}$
then $F.x = \emptyset$ for all $x \in \vec{I}$

Proof: Causality

$$x \downarrow t = z \downarrow t \Rightarrow \{y \downarrow t+1 : y \in F(x)\} = \{y \downarrow t+1 : y \in F(z)\}$$

implies, since $x \downarrow 0 = z \downarrow 0$

$$\{y \downarrow 1 : y \in F(x)\} = \{y \downarrow 1 : y \in F(z)\} = \emptyset$$

Thus $F(x) = \emptyset$

Services

$F : \vec{I} \rightarrow \wp(\vec{O})$ service interface with the syntactic interface $(I \blacktriangleright O)$ with

$$F.x \neq \emptyset \neq F.z \wedge x \downarrow t = z \downarrow t \Rightarrow$$

$$\{y \downarrow t+1 : y \in F(x)\} = \{y \downarrow t+1 : y \in F(z)\}$$

$$\text{Dom}(F) = \{x : F.x \neq \emptyset\}$$

the *service domain*

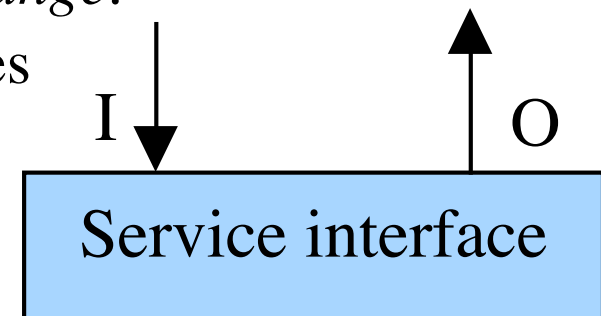
$$\text{Ran}(F) = \{y \in F.x : x \in \text{Dom}(F)\}$$

the *service range*.

$$\text{IF}[I \blacktriangleright O]$$

set of services

A service is a **partial** behaviour



Consequences of these definitions

- A component is a **total** behaviour!
- In contrast, a service is, in general, a **partial** behaviour!
- A component is the special case of a total service.
- Services can be related to components
 - ◇ A component may offer several services
- A multi-functional system/component can be defined in terms of a family of services that it offers

Example of a service: Queue

We specify the simple service of a queue

type QIn = {req} \cup Data

type QOut = Data

Queue

in x: QIn

out y: QOut

$\{\text{req}\}\#x = \text{Data}\#y \wedge \bar{y} \sqsubseteq \text{Data}\odot\bar{x}$

The input assumption here is:

$$\forall x': x' \sqsubseteq \bar{x} \Rightarrow \{\text{req}\}\#x \leq \text{Data}\#x$$

Example of a service: Box

We specify the simple service of a queue

type QIn = {req} \cup Data

type QOut = Data

Box

in x: QIn

out y: QOut

{req}#x = Data#y

$\forall d \in \text{Data}: \{d\}\#y \leq \{d\}\#x$

The input assumption here is:

$\forall x': x' \sqsubseteq \bar{x} \Rightarrow \{req\}\#x' \leq \text{Data}\#x'$

Example of a service: Unreliable Queue

We specify the simple service of an unreliable queue

type QIn = {req} \cup Data

type QOut = {fail} \cup Data

Here we have to refer explicitly to the output history in the input assumption

Queue

in x: QIn

out y: QOut

$\{\text{req}\}\#x = (\{\text{fail}\}\#y) + (\text{Data}\#y)$

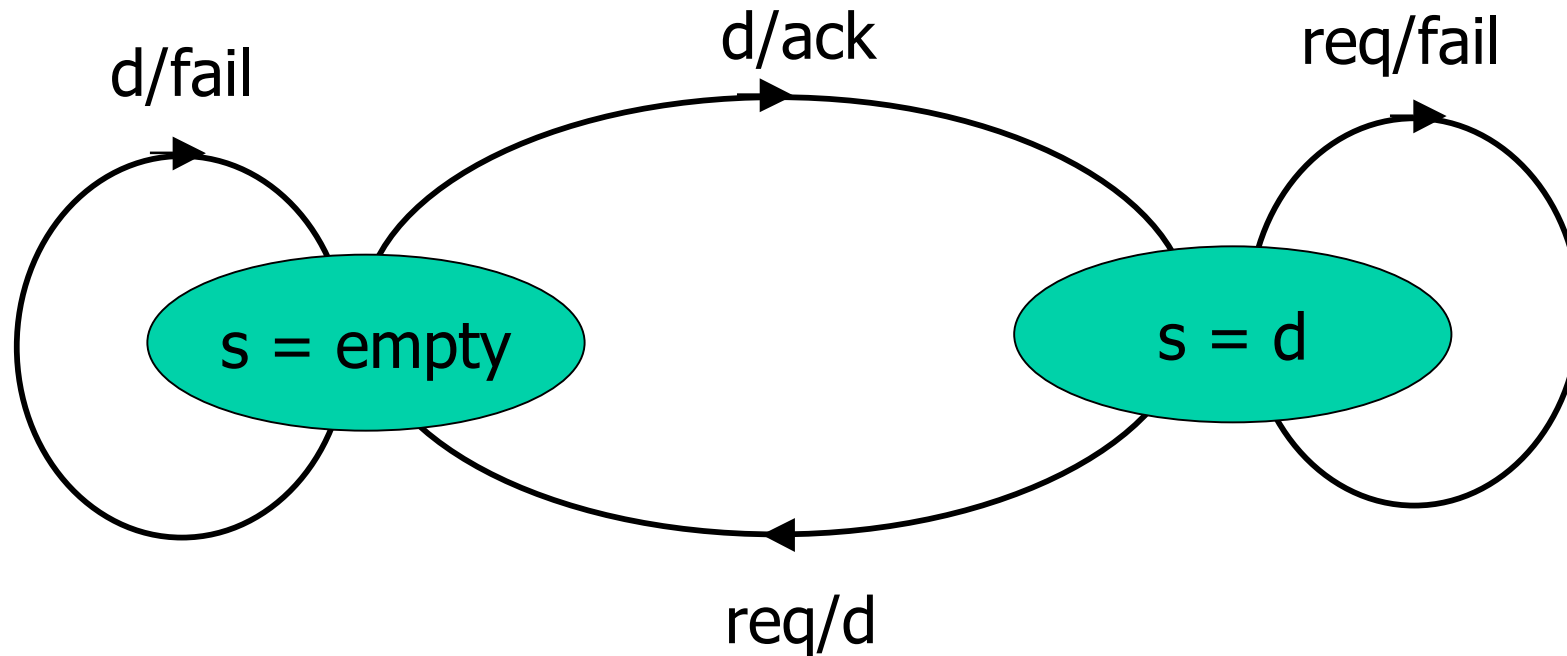
$\wedge \bar{y} \sqsubseteq \text{Data} \odot \bar{x}$

$\wedge \forall t: \{\text{req}\}\#(x \downarrow t) \leq \text{Data}\#(x \downarrow t) + (\{\text{fail}\}\#(y \downarrow t))$

The input assumption here is:

$\forall t: \{\text{req}\}\#(x \downarrow t) \leq \text{Data}\#(x \downarrow t) + (\{\text{fail}\}\#(y \downarrow t))$

A state machine for the Box



Assumption/Commitment Specifications

$F \in \text{IF}[I \blacktriangleright O]$

$A_t: \vec{I} \times \vec{O} \rightarrow \text{IB}$ input assumption at time t .

$A_t(x, y) = \exists x' \in \text{IH}[I], y' \in F.x': x \downarrow t = x' \downarrow t \wedge y \downarrow t = y' \downarrow t$

$A_{t+1}(x, y) \Rightarrow A_t(x, y)$ for all $t \in \text{IN}$

$A: \vec{I} \rightarrow \text{IB}$ input assumption

$A(x) = \exists y \in \text{IH}[O]: y \in F.x \wedge \forall t \in \text{IN}: A_t(x, y)$

$A(x) \Rightarrow \exists y \in \text{IH}[O]: A_t(x, y)$ for all $t \in \text{IN}$

Assumption/Commitment Specifications

$G_t: \vec{I} \times \vec{O} \rightarrow \mathbb{B}$ output commitment at time t

$$G_t(x, y) = \exists x' \in \text{IH}[I], y' \in F.x': x \downarrow t = x' \downarrow t \wedge y \downarrow t+1 = y' \downarrow t+1$$

$G_{t+1}(x, y) \Rightarrow G_t(x, y)$ for all $t \in \mathbb{IN}$

$$G_t(x, y) \Rightarrow A_t(x, y)$$

$G: \vec{I} \times \vec{O} \rightarrow \mathbb{B}$ commitment

$$G(x, y) = y \in F.x$$

$G(x, y) \Rightarrow G_t(x, y)$ for all $t \in \mathbb{IN}$

$$G(x, y) \Rightarrow A(x)$$

Assumption/Commitment Specification of Services

Service specification:

$$F.x = \{y: A(x) \wedge G(x, y)\}$$

$$F.x = \emptyset \quad \text{iff} \quad \neg A(x)$$

Component specification:

$$F.x = \{y: A(x) \Rightarrow G(x, y) \\ \wedge \forall t \in \mathbb{N}: A_t(x, y) \Rightarrow G_t(x, y)\}$$

$$F.x = (F.x) \downarrow t^{\vec{0}} \quad \text{“chaos”} \quad \text{iff} \quad \forall y: \neg A_t(x, y)$$

Composition of Components and Services

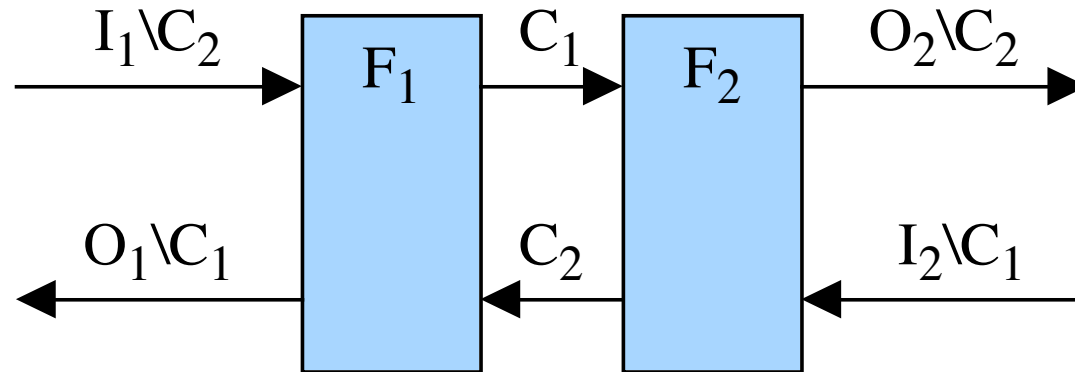
Given

$$F_1 \in \text{IF}[I_1 \blacktriangleright O_1]$$

$$F_2 \in \text{IF}[I_2 \blacktriangleright O_2]$$

$$C_1 \subseteq O_1 \cap I_2$$

$$C_2 \subseteq O_2 \cap I_1$$



$$F_1[C_1 \leftrightarrow C_2]F_2 \in \text{IF}[I_1 \setminus C_2 \cup I_2 \setminus C_1 \blacktriangleright O_1 \setminus C_1 \cup O_2 \setminus C_2],$$

$$(F_1[C_1 \leftrightarrow C_2]F_2).x = \{z \mid (O_1 \setminus C_1) \cup (O_2 \setminus C_2) : x = z \mid I_1 \setminus C_2 \cup I_2 \setminus C_1 \\ \wedge z \mid O_1 \in F_1(z \mid I_1) \wedge z \mid O_2 \in F_1(z \mid I_2)\}$$

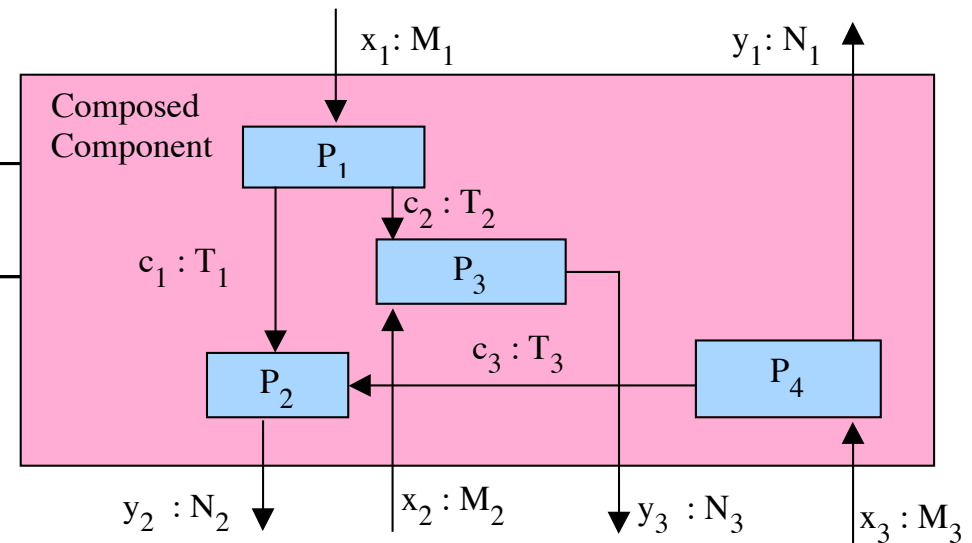
Architectures: Specification of composed components

Composed components spec

in $x_1: T_1, x_2: T_2, \dots$

out $y_1: S_1, y_2: S_2, \dots$

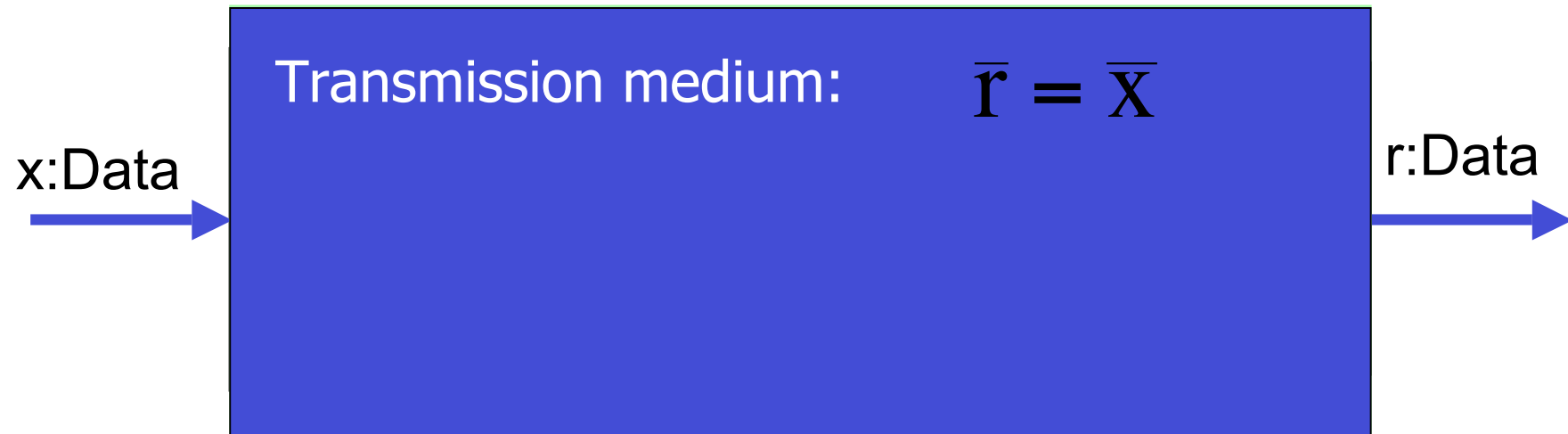
$\exists c_1, c_2, \dots : P_1 \wedge \dots \wedge P_n$



Parallel composition = logical and

Channel hiding = existential quantification

Example: Alternating Bit Protokoll



Leads in the case of deterministic components to the equations :

$$c1 = \text{sender}(x, c4)$$

$$c2 = \text{medium}(c1)$$

$$(r, c3) = \text{receiver}(c2)$$

$$c4 = \text{medium}(c3)$$

In case of nondeterminism

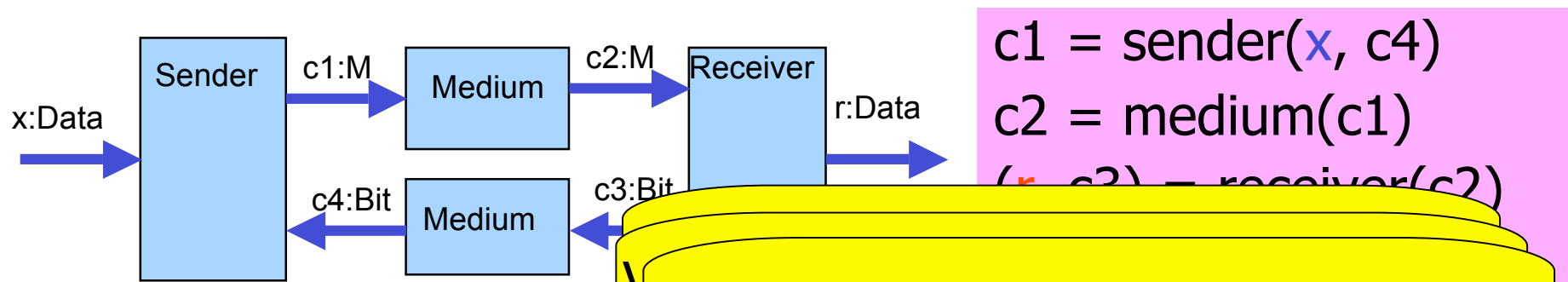
$$c1 \in \text{sender}(x, c4)$$

$$c2 \in \text{medium}(c1)$$

$$(r, c3) \in \text{receiver}(c2)$$

$$c4 \in \text{medium}(c3)$$

Fixpoints



Given stream x how

Value does only depend on values in time intervals $t < 2$

time	1	2	3	4	5	6	7	8	9	10	11
x	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >	< ... >
c1	< ... >	< ... >	< ... >								
c2	< ... >	< ... >	< ... >								
c3	< ... >	< ... >	< ... >								
c4	< ... >	< ... >	< ... >								
r	< ... >	< ... >	< ... >								

Property Refinement

Given two service interfaces

$$F1, F2 \in IF[I \blacktriangleright O]$$

we call service $F2$ a **property refinement** of $F1$
if for all input histories $x \in IH[I]$

$$F2.x \subseteq F1.x$$

Then we write

$$F1 \Rightarrow F2$$

we get **compositionality**:

$$F1 \Rightarrow F2 \quad G1 \Rightarrow G2$$

$$F1[C1 \leftrightarrow C2]G1 \Rightarrow F2[C1 \leftrightarrow C2]G2$$

Using Queues for Boxes

- We can use the component Queue wherever we need a component Box since:

Box \equiv > Queue

since:

$$\begin{aligned} \text{req}\#x = \text{Data}\#y \wedge \bar{y} \sqsubseteq \text{Data}\odot\bar{x} &\Rightarrow \\ \text{req}\#x = \text{Data}\#y \wedge \forall d \in \text{Data}: \{d\}\#y \leq \{d\}\#x & \end{aligned}$$

To prove this implication proves the refinement relation.

Remarks

- Property refinement

$$F_1 \equiv \triangleright F_2$$

is easily proved by proving

$$P_2 \Rightarrow P_1$$

for the specifying assertion P_1 and P_2 of F_1 and F_2 resp.

- Property refinement is not appropriate for relating services with components (in property refinement the domain is always decreased)
 - ◇ One solution: chaos completion
 - ◇ Another solution: a more refined refinement relation