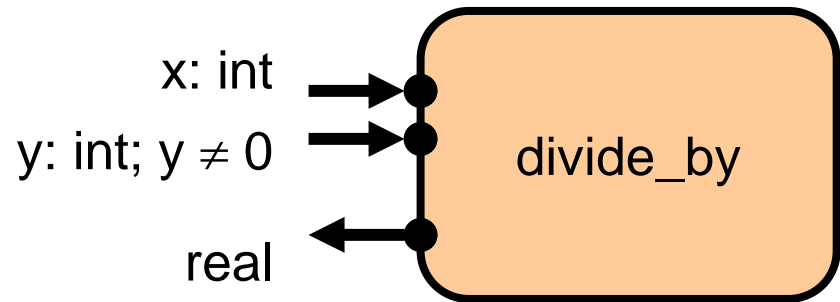


# Interface-based Design 3

Tom Henzinger  
EPFL and UC Berkeley

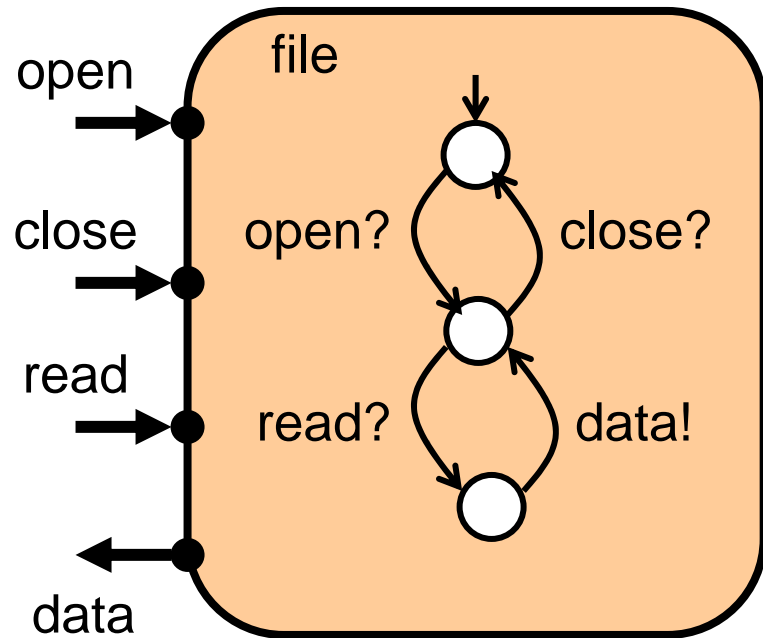
# An Assertional Interface

This interface  
constrains the  
client's **data**.

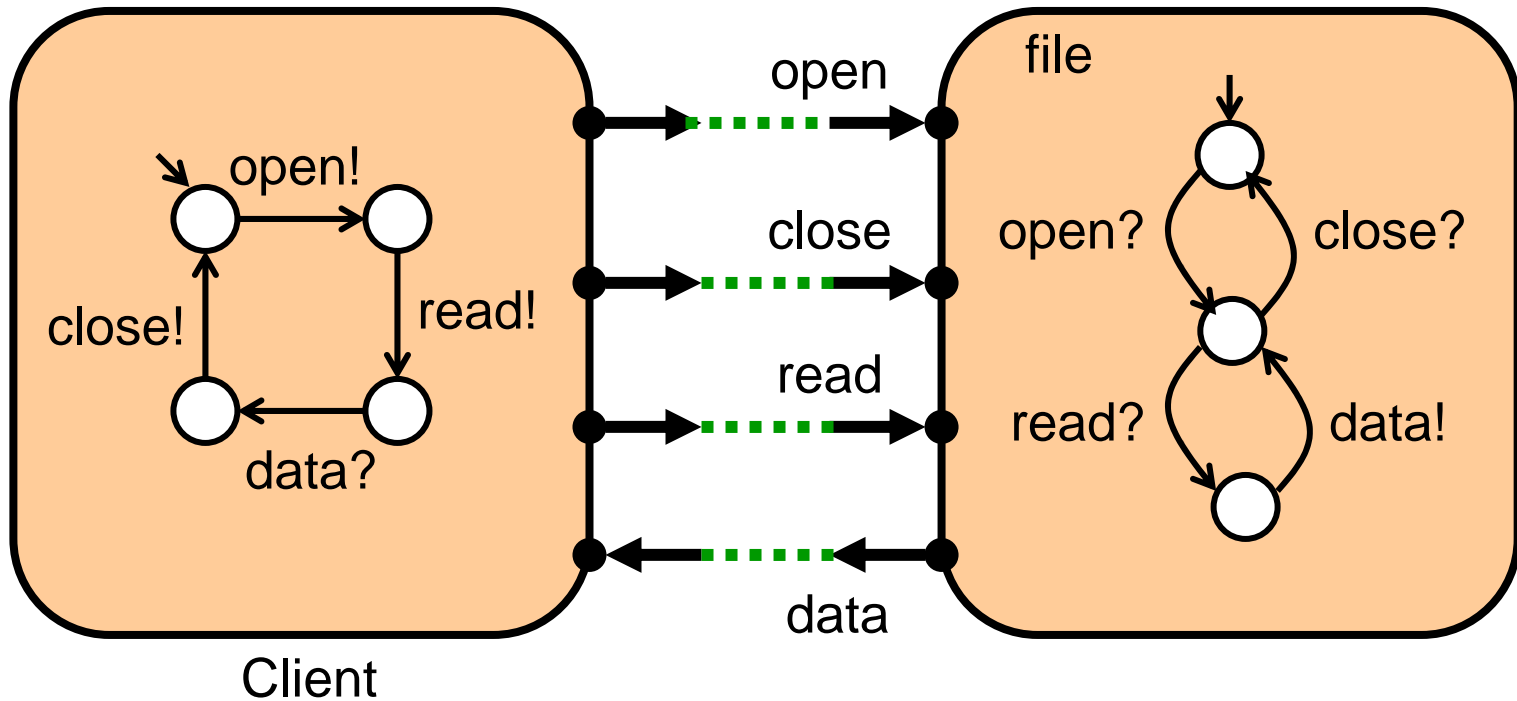


# An Automaton Interface

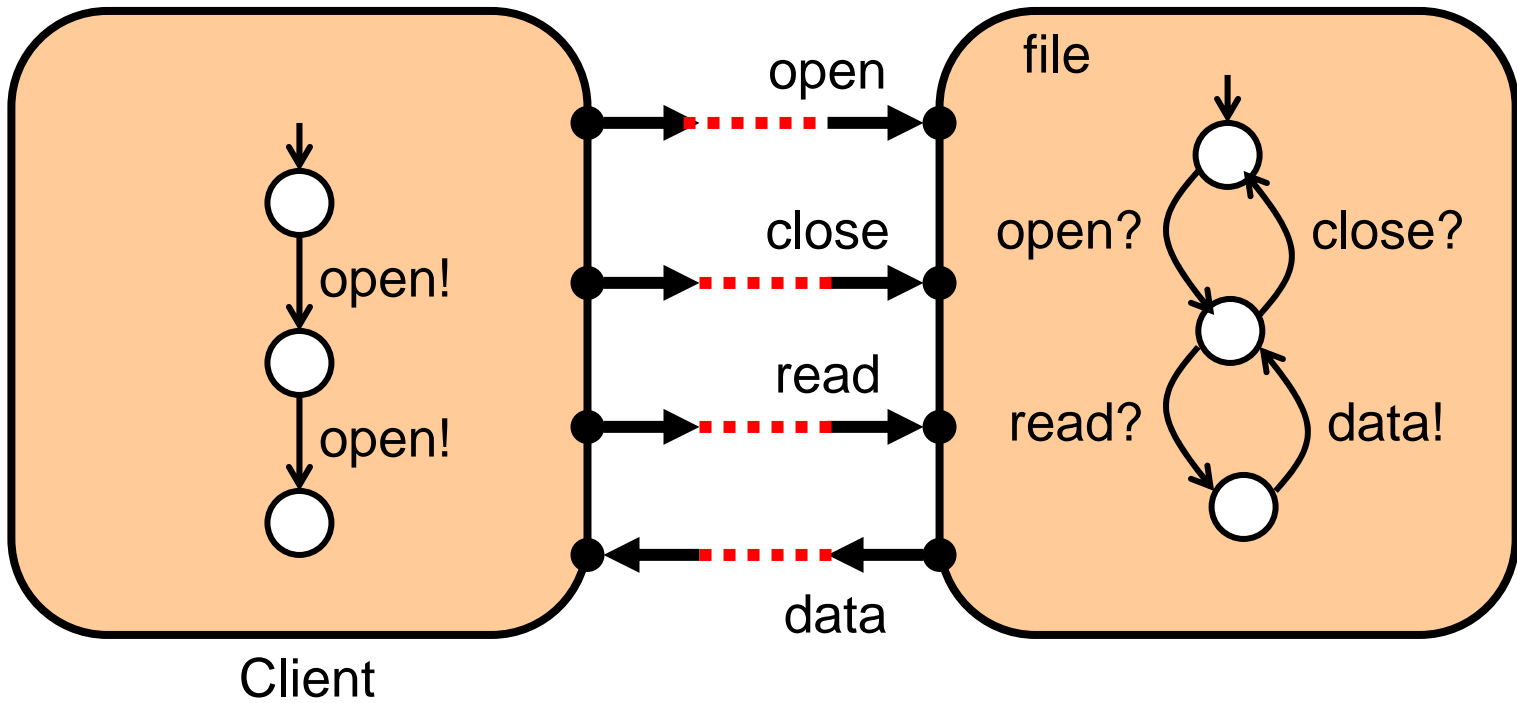
This interface  
constrains the  
client's **control**.



# Two **Compatible** Automaton Interfaces



# Two **Incompatible** Automaton Interfaces



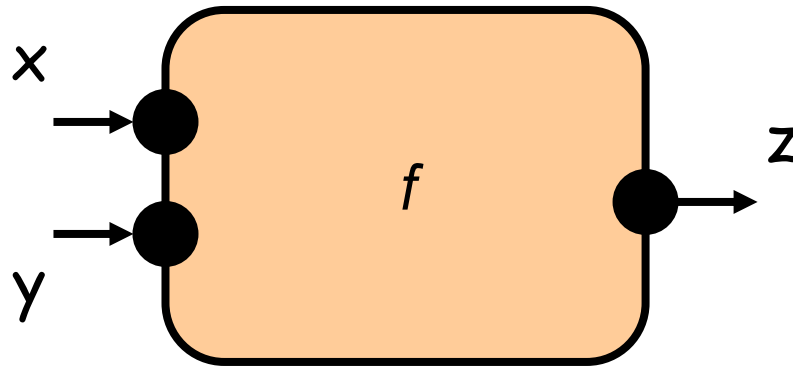
*Today's Lecture:*

How do we check the **compatibility** of automaton interfaces ?

What is the **composition** of two compatible automaton interfaces ?

## Free Inputs:

*Interface Composition propagates Environment Constraints.*

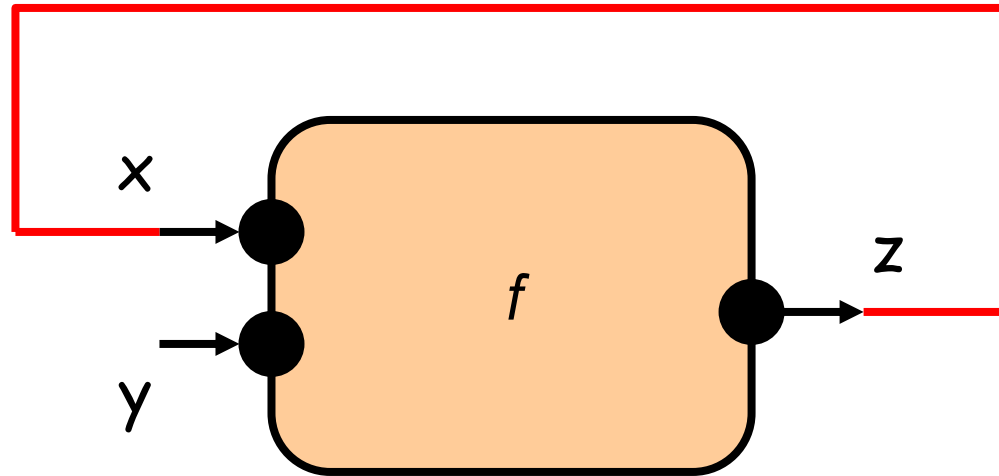


$$x=0 \Rightarrow y=0$$

true

## Free Inputs:

*Interface Composition propagates Environment Constraints.*



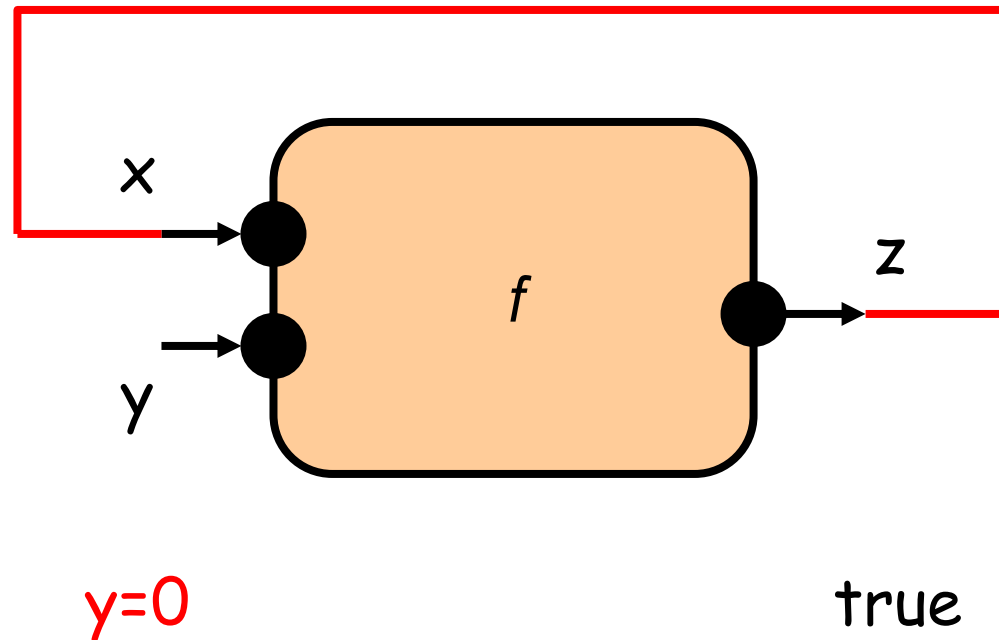
$$x=0 \Rightarrow y=0$$

true



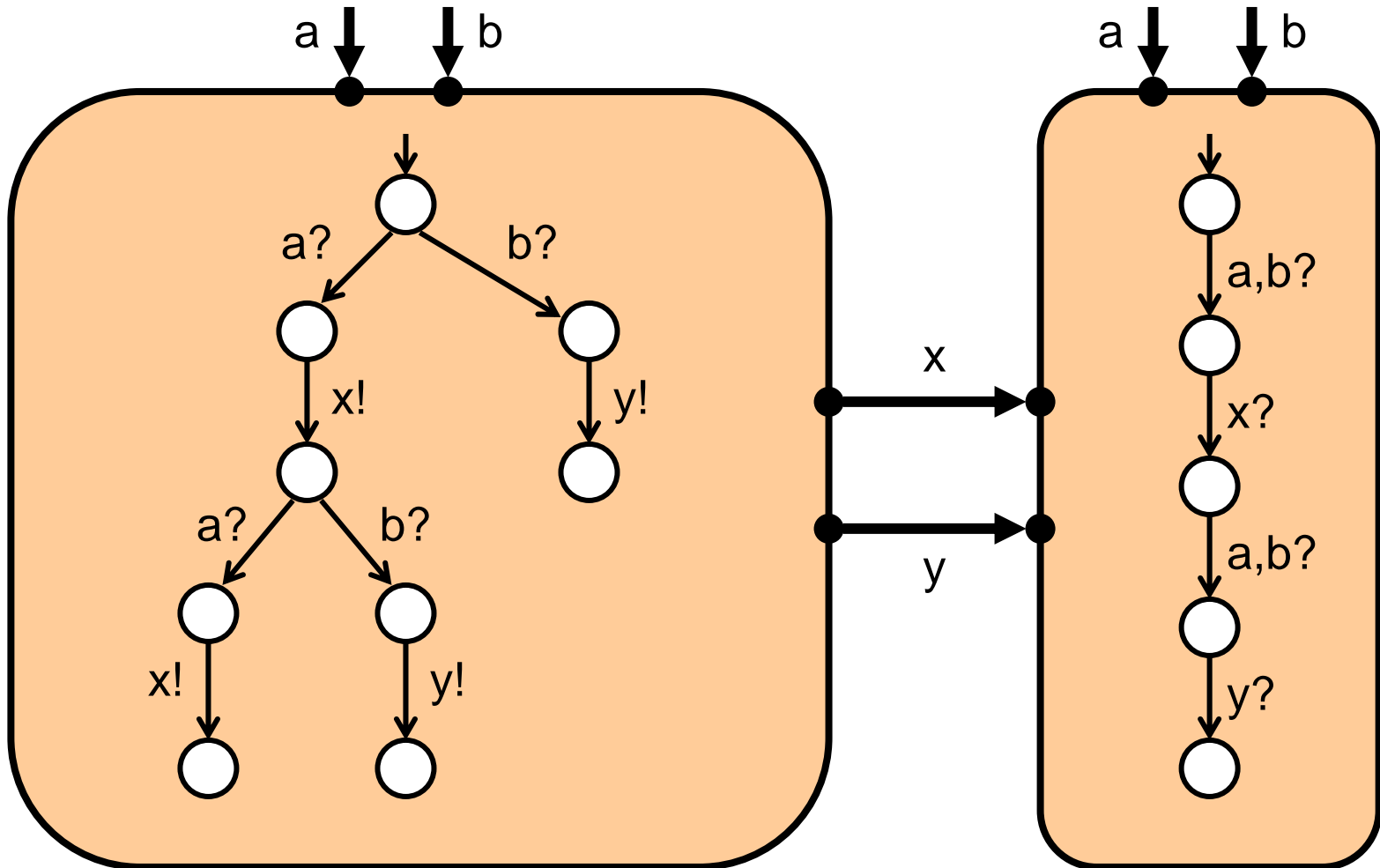
## Free Inputs:

*Interface Composition propagates Environment Constraints.*

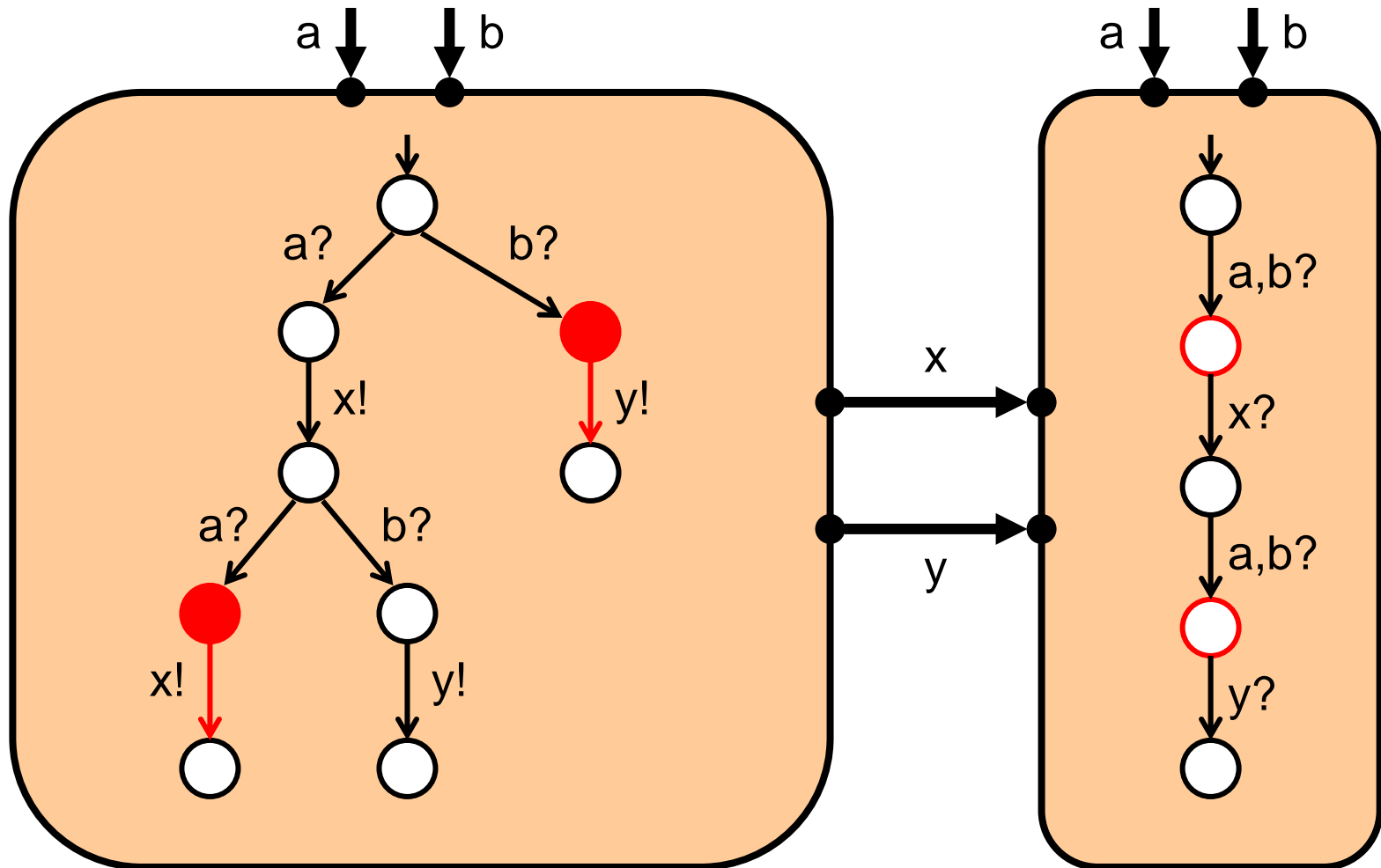


The environment is helpful !

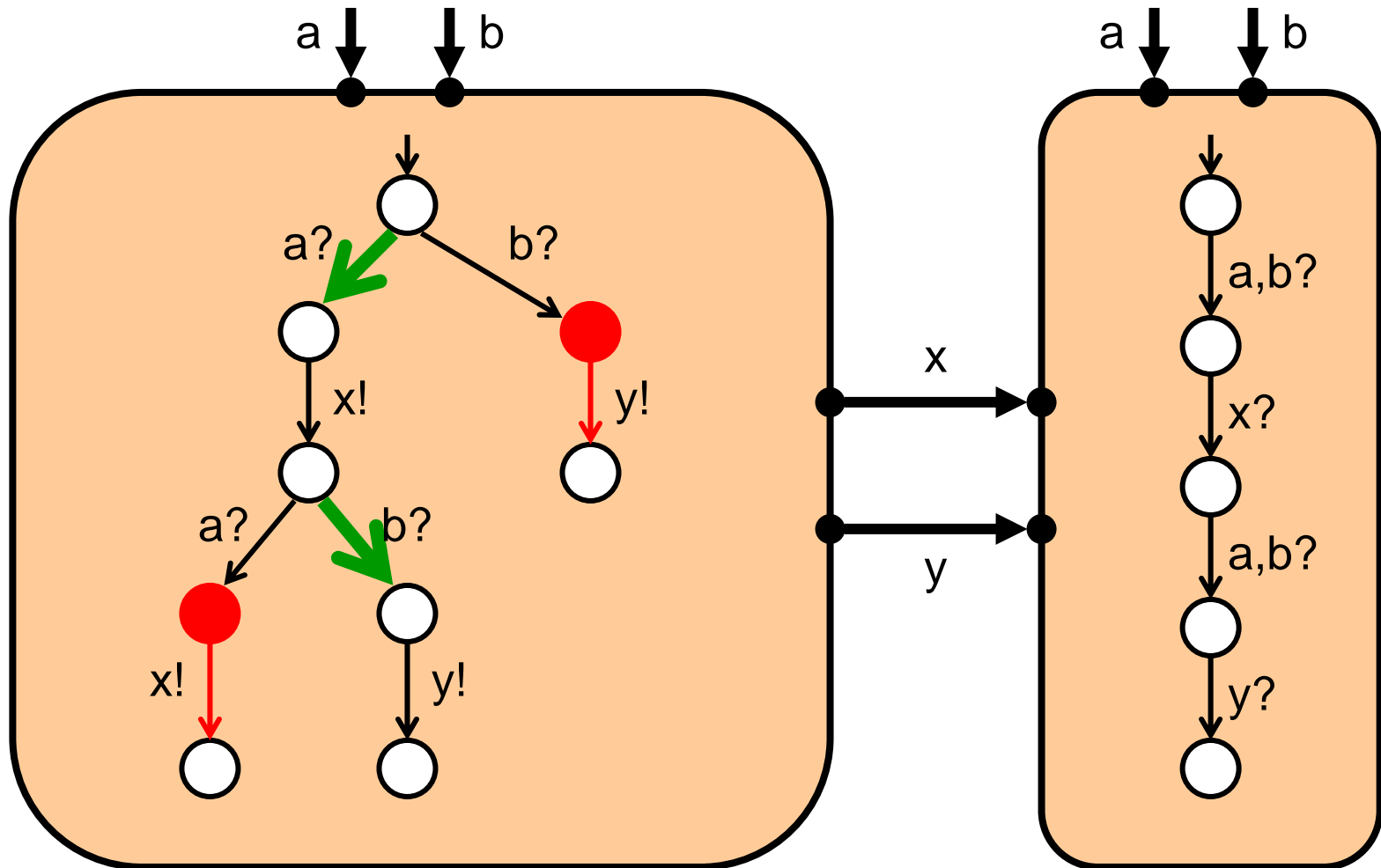
# Automaton Interface Composition



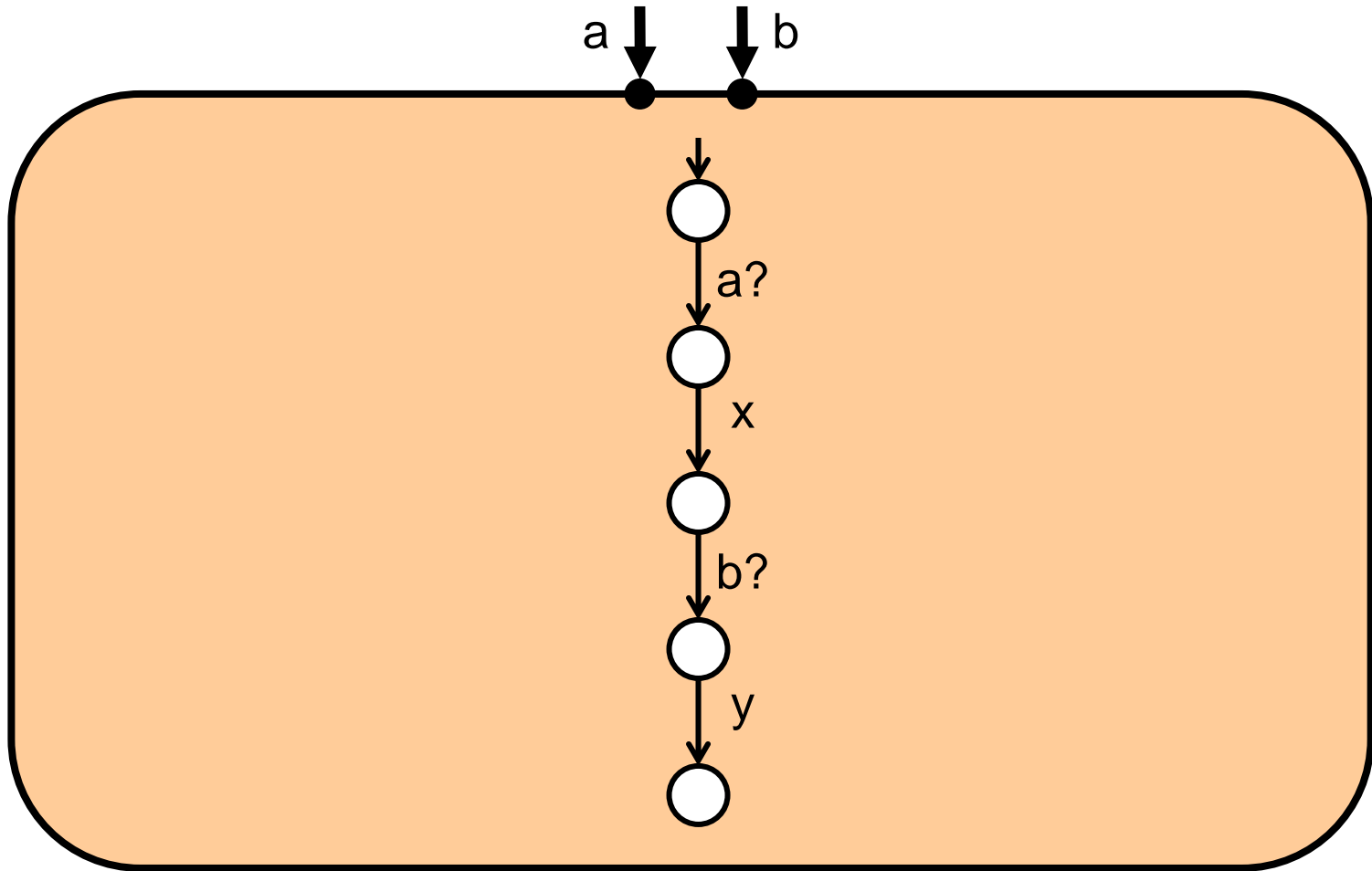
# Automaton Interface Composition



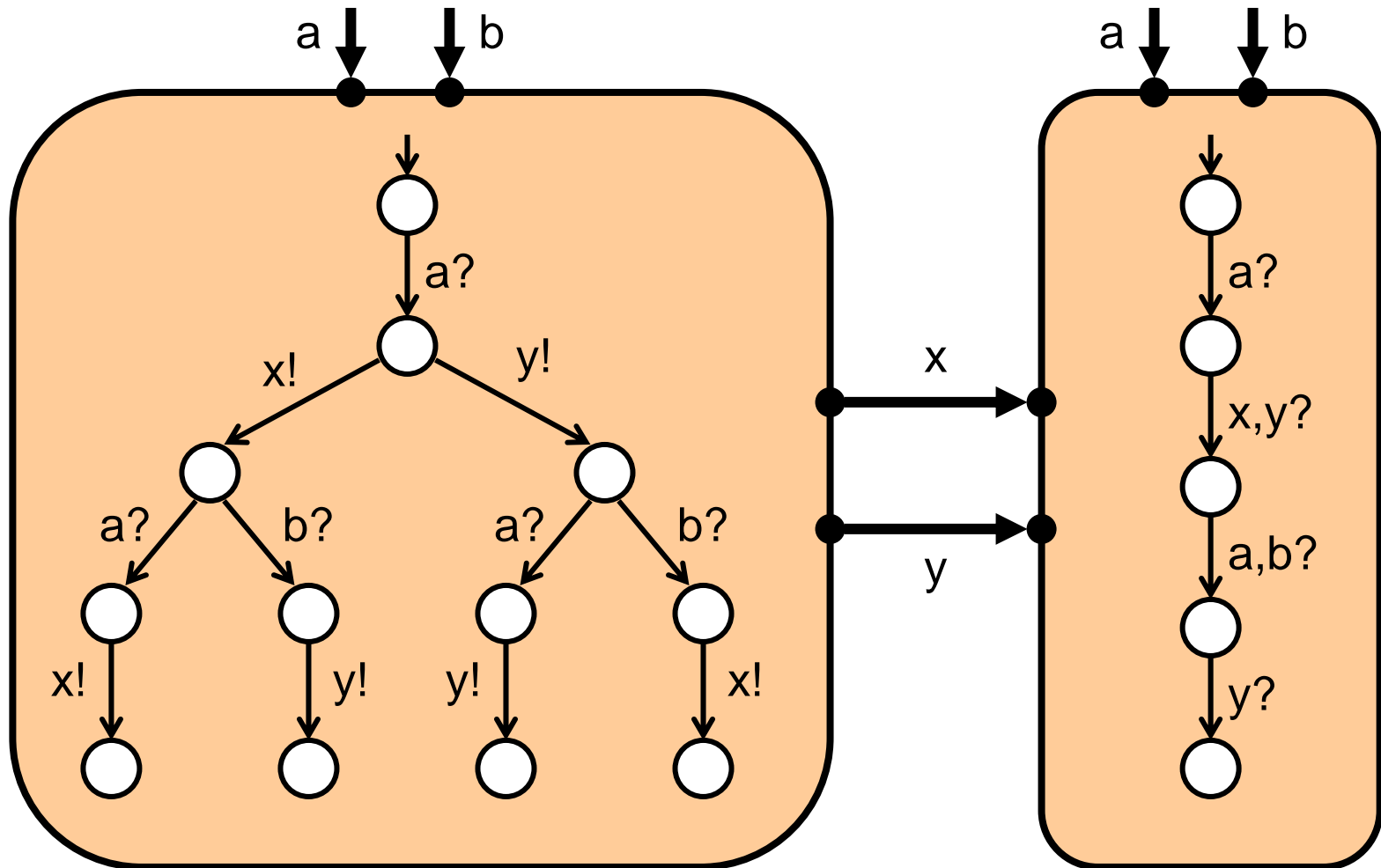
# Automaton Interface Composition



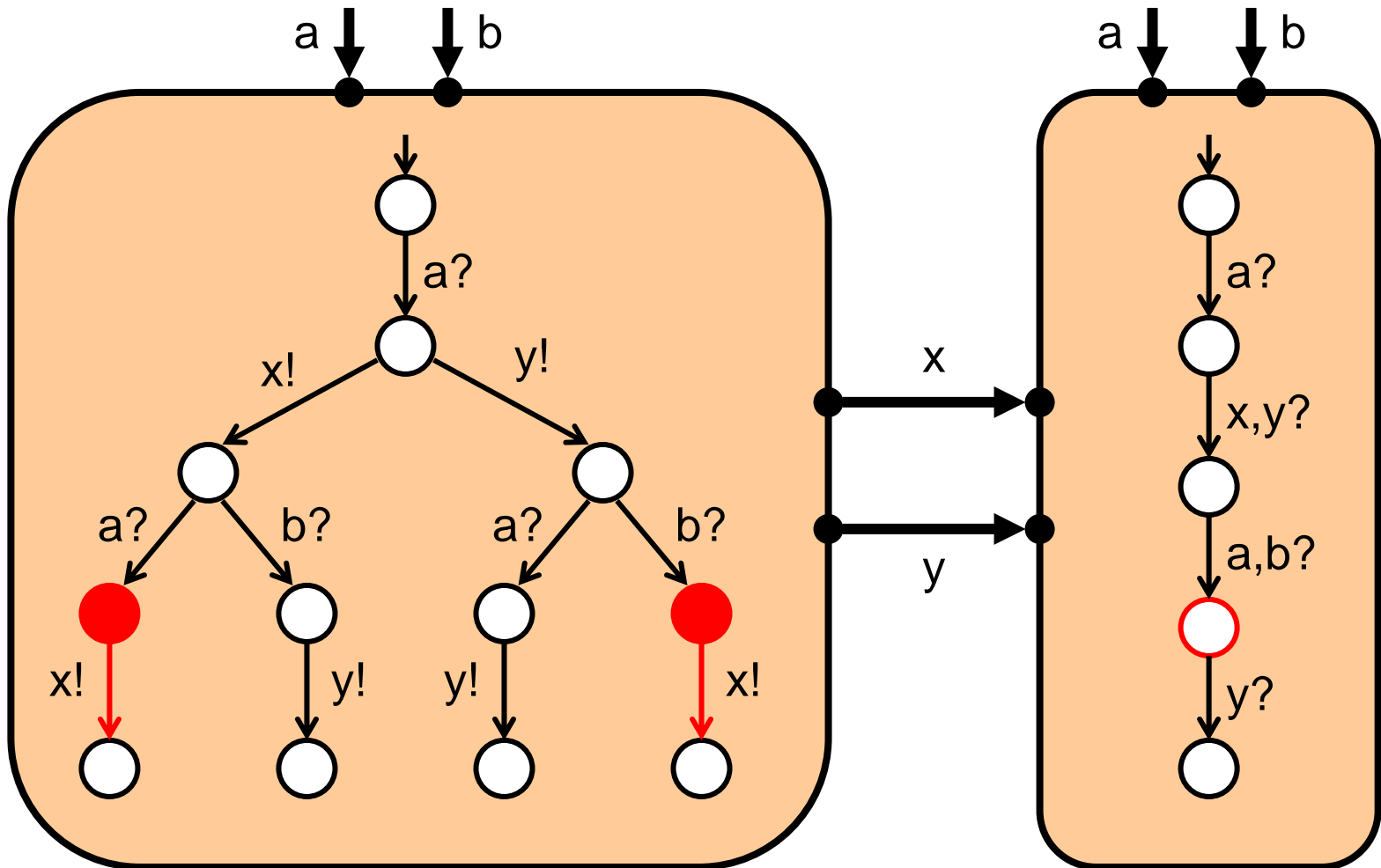
# The Composite Interface



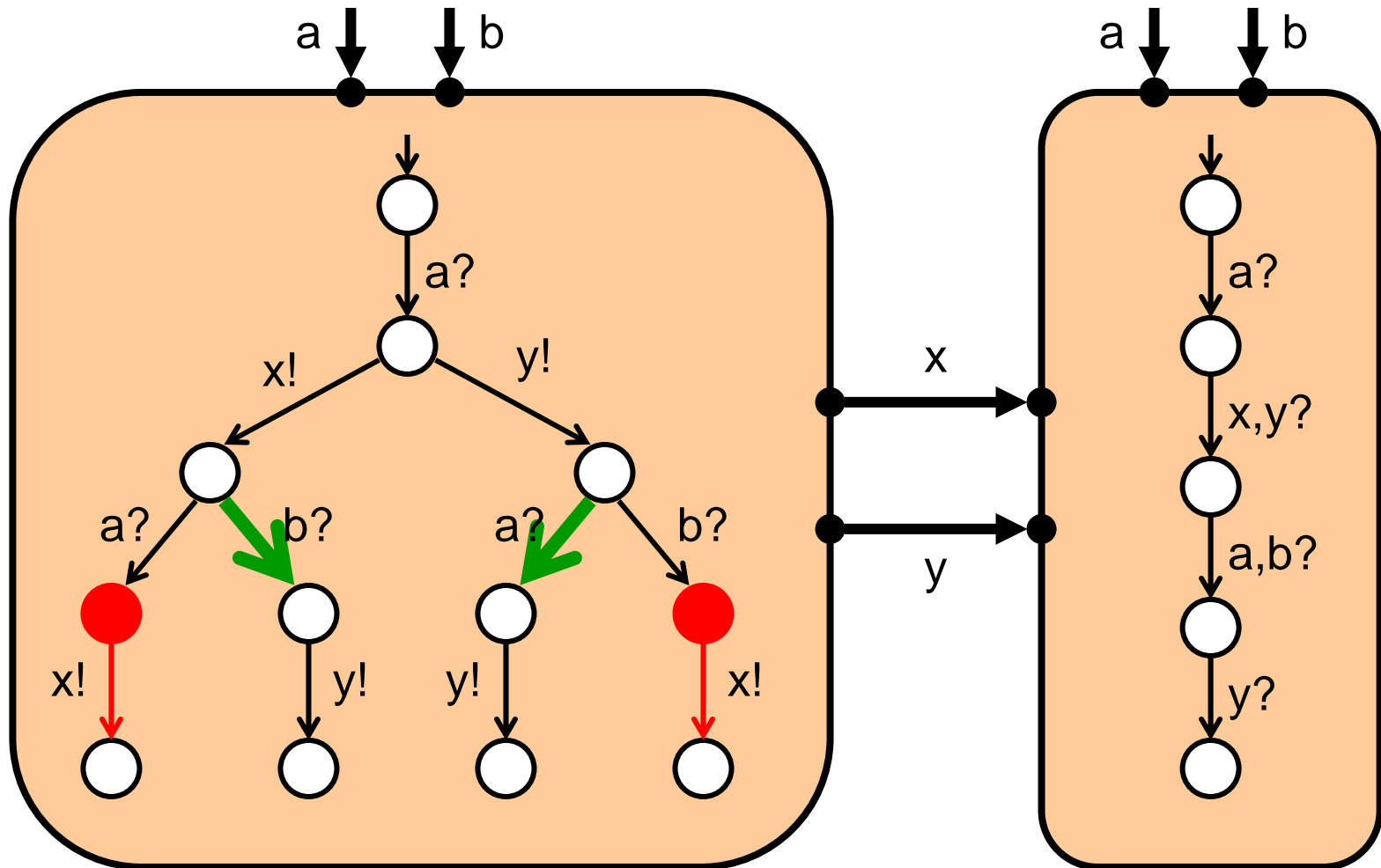
# Automaton Interface Composition



# Automaton Interface Composition

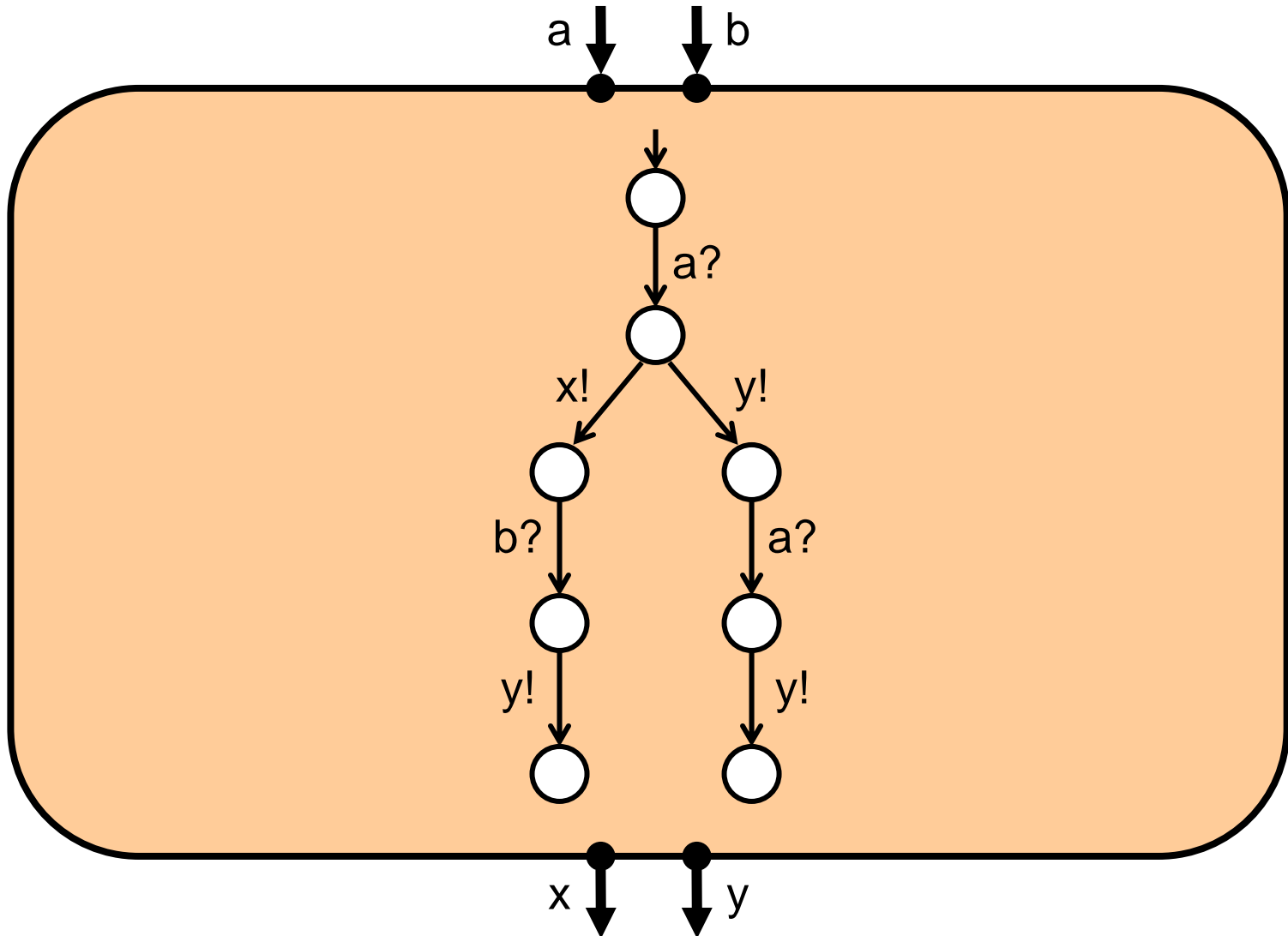


# Automaton Interface Composition





# The Composite Interface

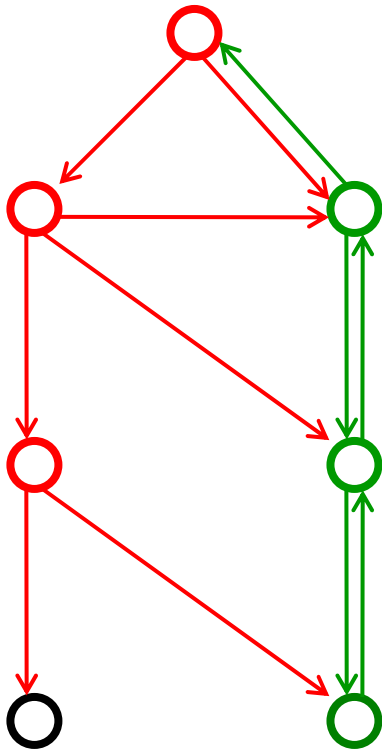


## Lesson 3:

# Stateful Interfaces are Games!

- Player Input vs. player Internal.
- The composite interface is the product restricted to those states from which player Input has a strategy to avoid incompatibilities.

# Graph Games

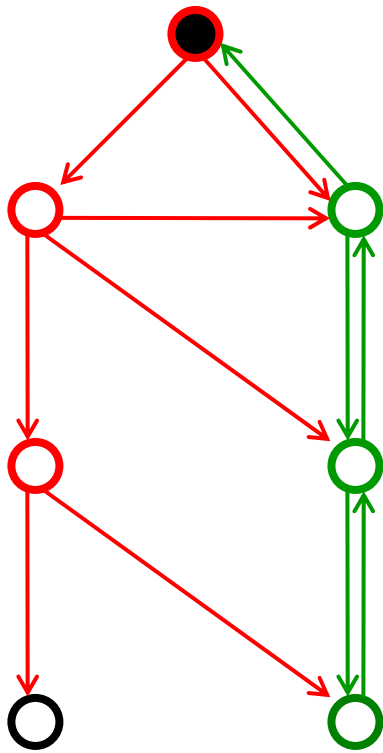


AND-OR Graph:

OR Nodes

AND Nodes

# Graph Games

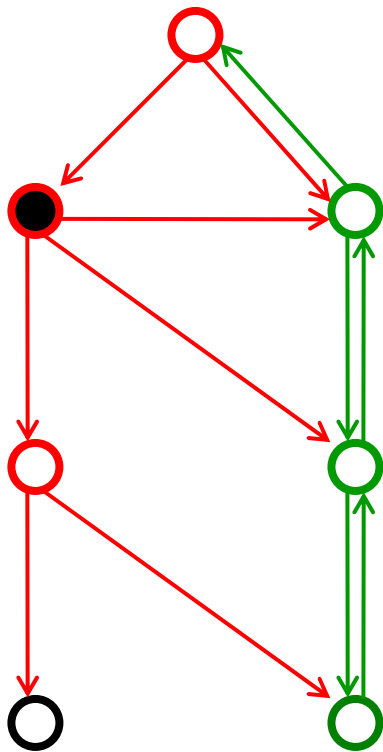


AND-OR Graph:

OR Player

AND Player

# Graph Games

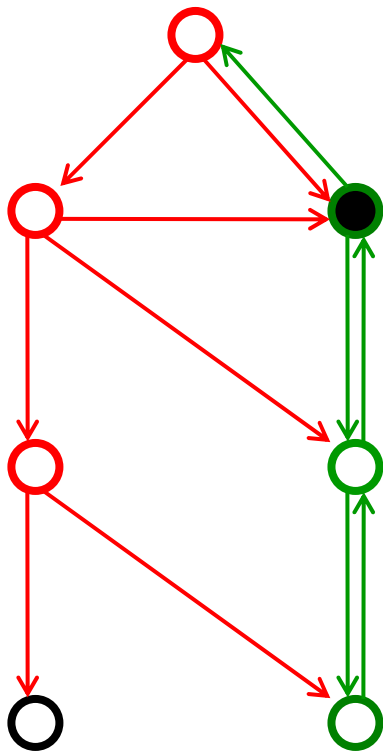


AND-OR Graph:

OR Player

AND Player

# Graph Games

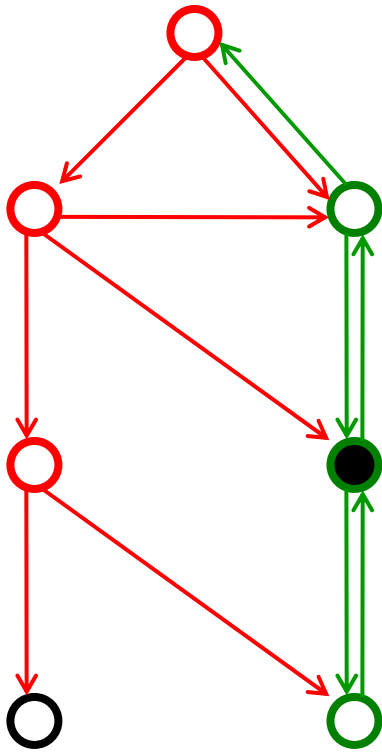


AND-OR Graph:

OR Player

AND Player

# Graph Games

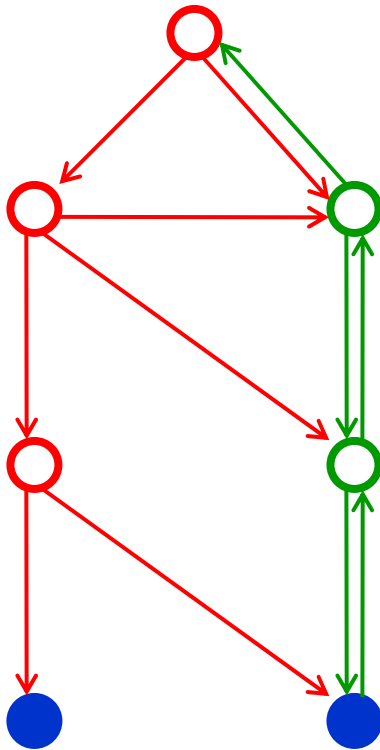


AND-OR Graph:

OR Player

AND Player

# Safety Games



AND-OR Graph:

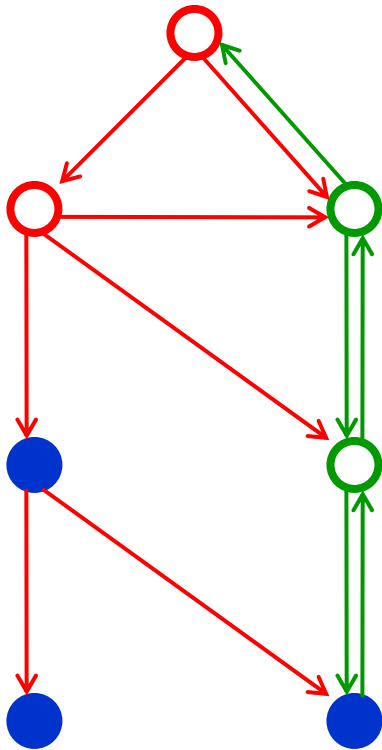
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?



# Safety Games



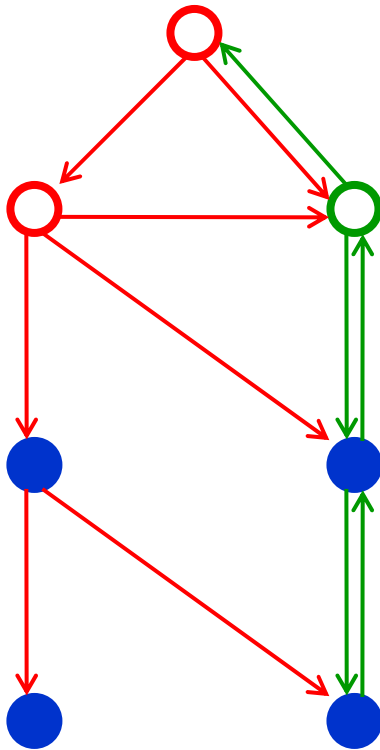
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



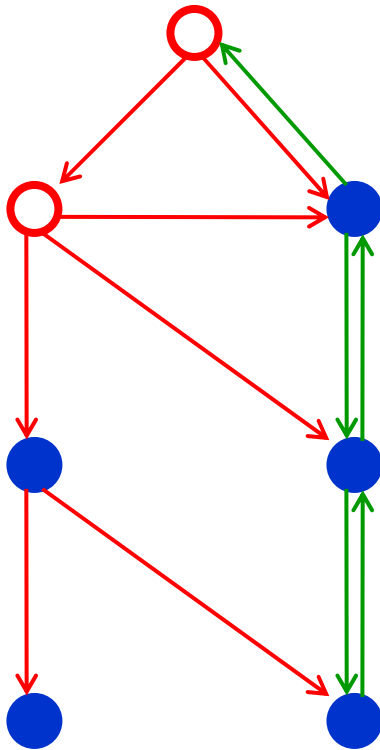
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



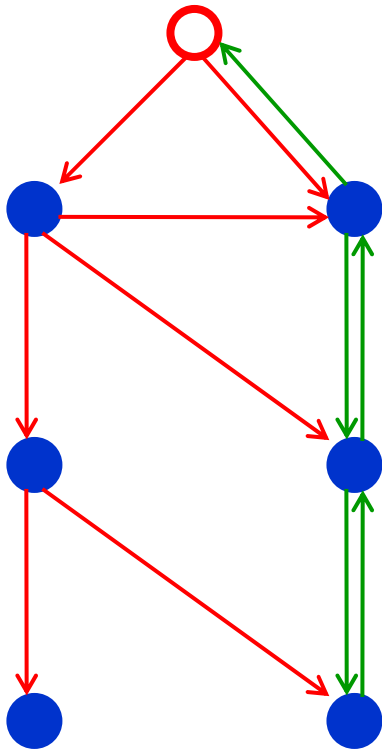
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



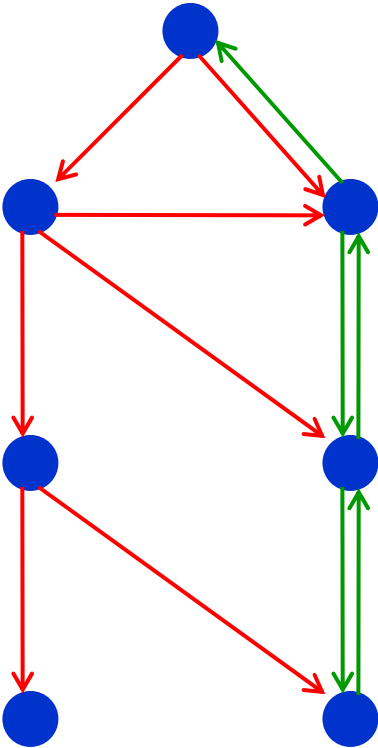
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



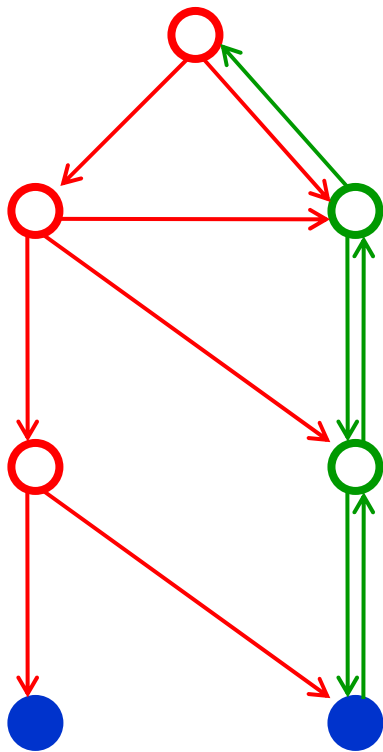
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



AND-OR Graph:

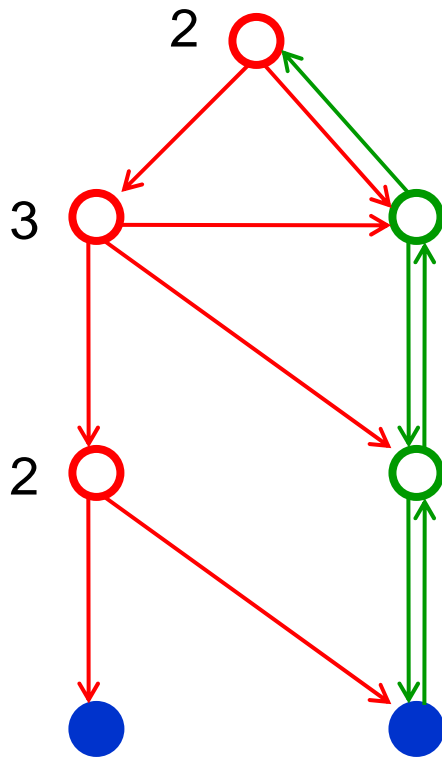
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

Complexity?

# Safety Games



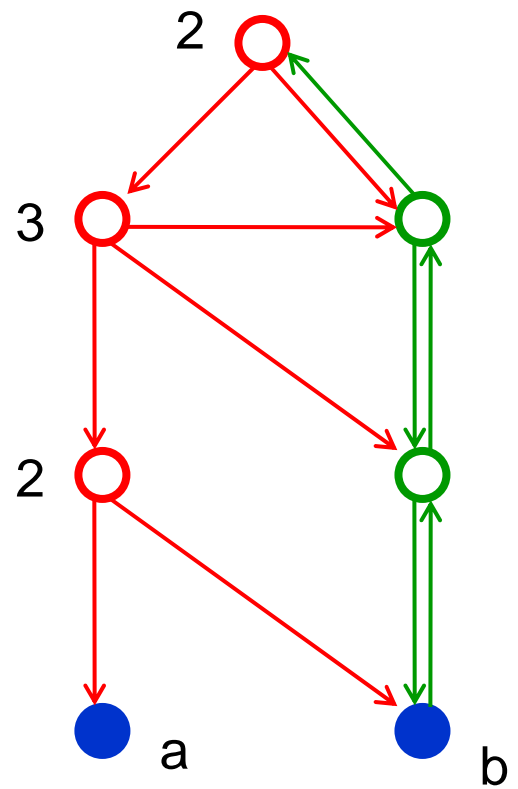
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



AND-OR Graph:

OR Player

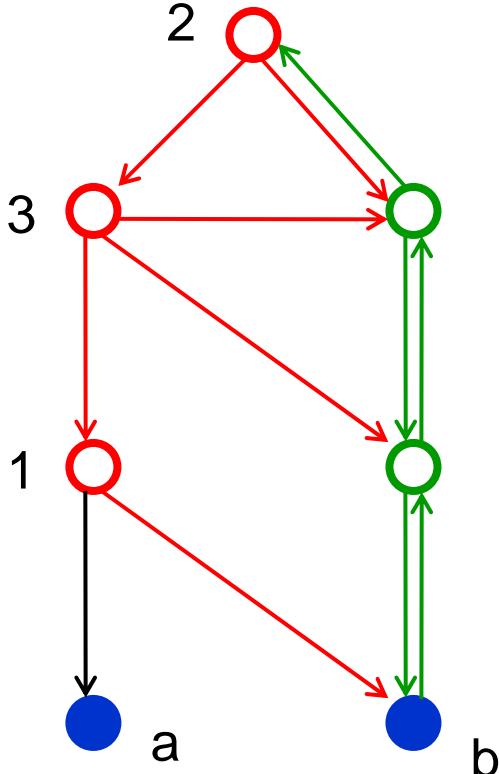
AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

a b
-----



# Safety Games



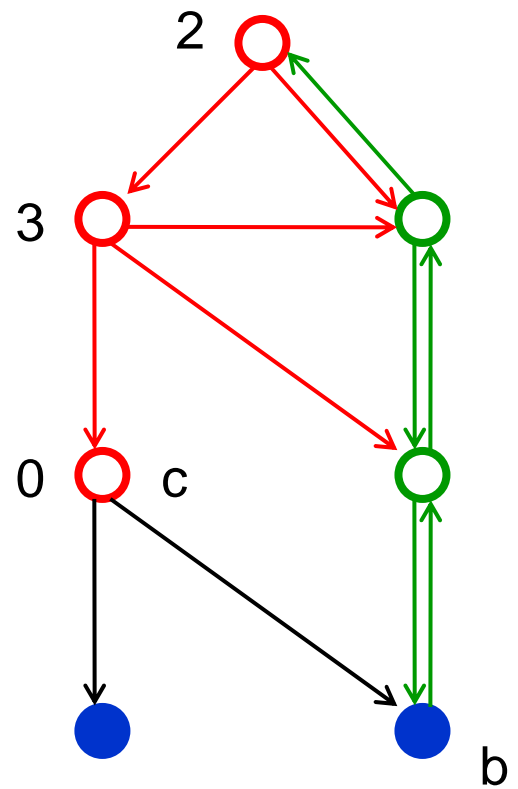
AND-OR Graph:

OR Player  
AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

b

# Safety Games



AND-OR Graph:

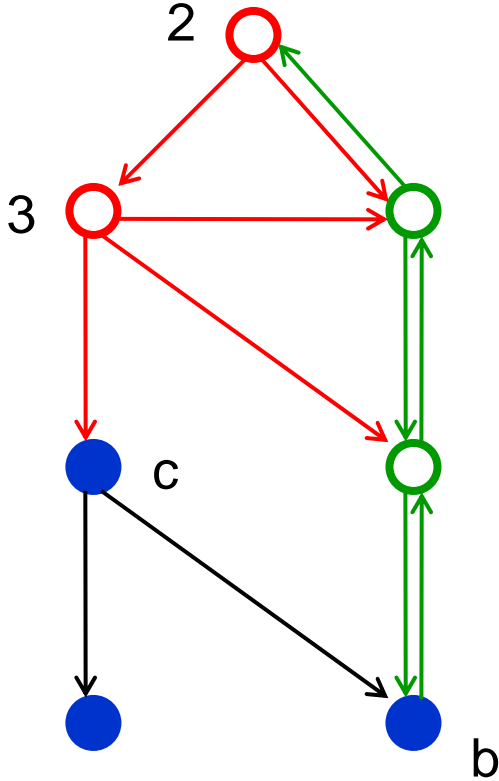
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

b c

# Safety Games



AND-OR Graph:

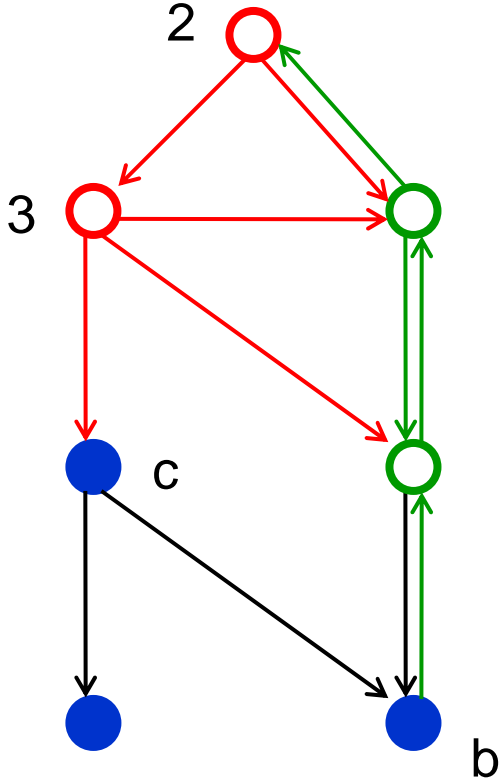
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

b c

# Safety Games



AND-OR Graph:

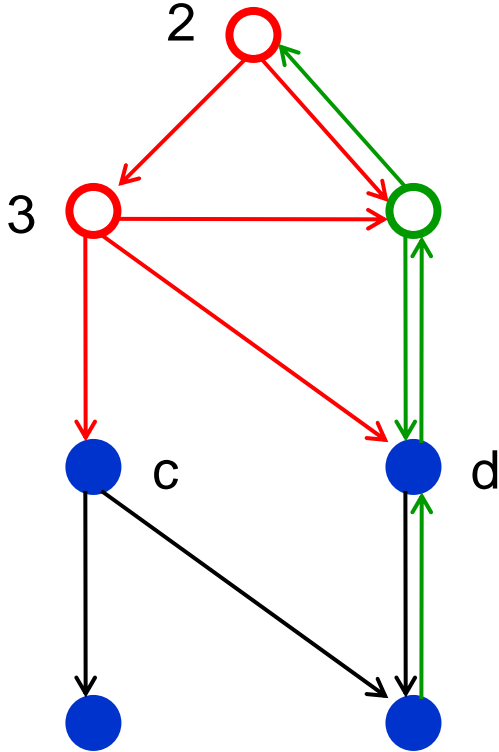
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

c

# Safety Games



AND-OR Graph:

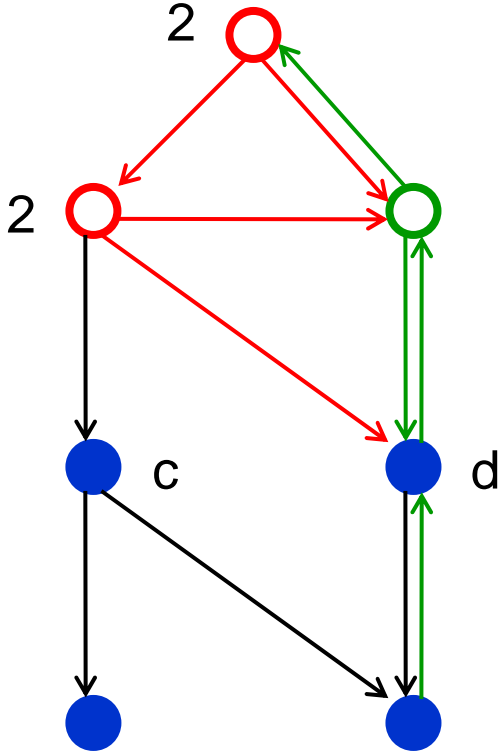
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

c d

# Safety Games



AND-OR Graph:

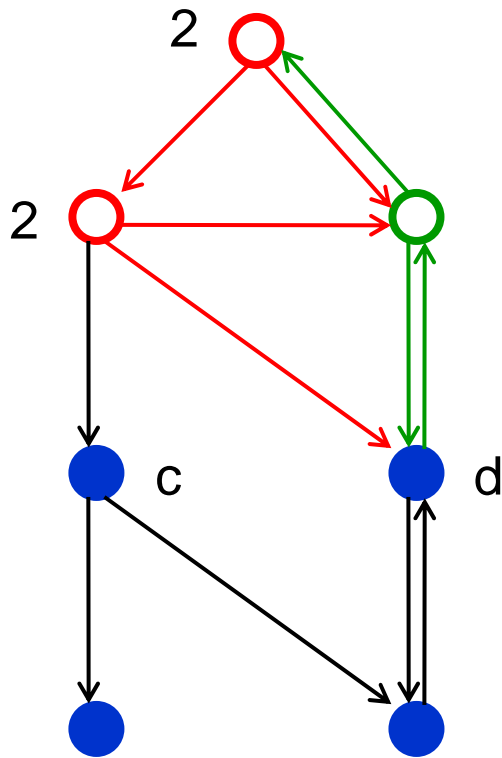
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid ERROR Nodes ?

d

# Safety Games



AND-OR Graph:

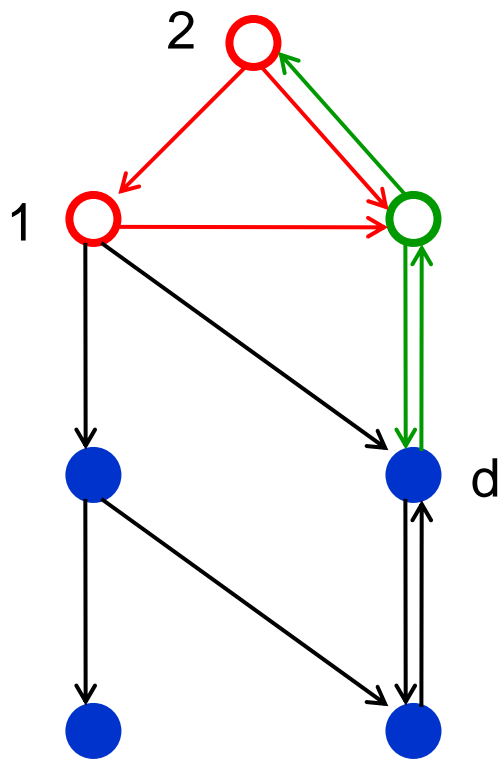
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

d

# Safety Games



AND-OR Graph:

OR Player

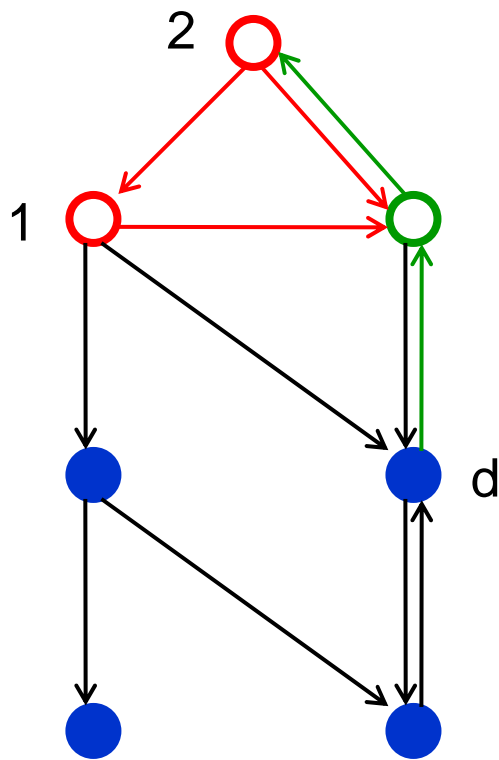
AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

d



# Safety Games



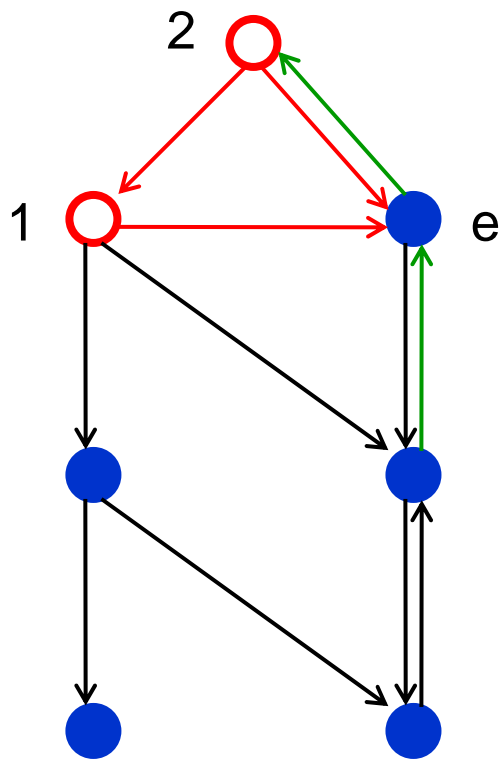
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



AND-OR Graph:

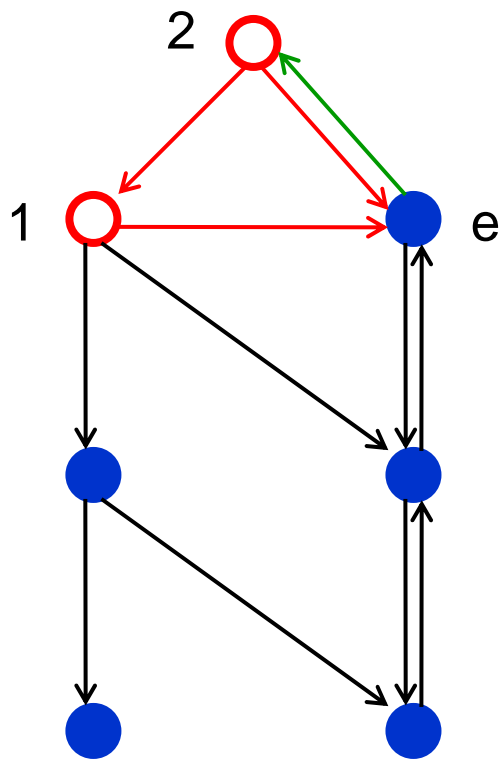
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

e

# Safety Games



AND-OR Graph:

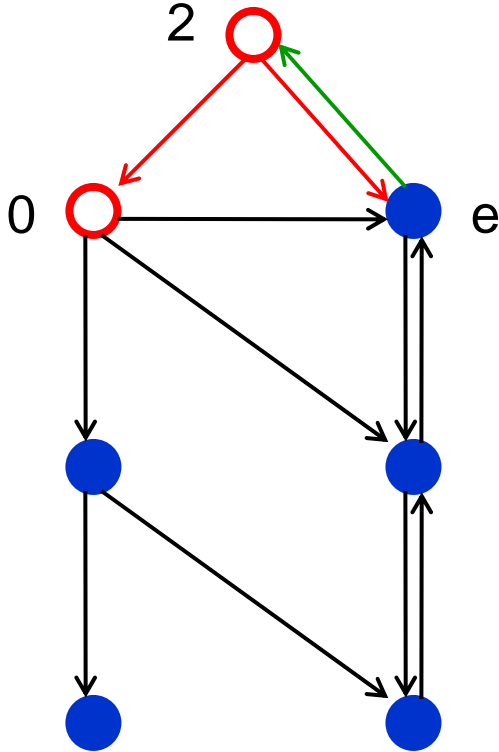
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

e

# Safety Games



AND-OR Graph:

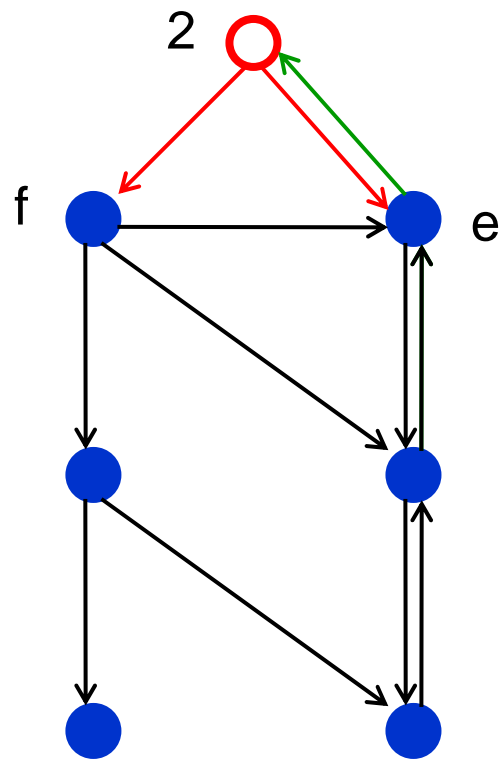
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

e

# Safety Games



AND-OR Graph:

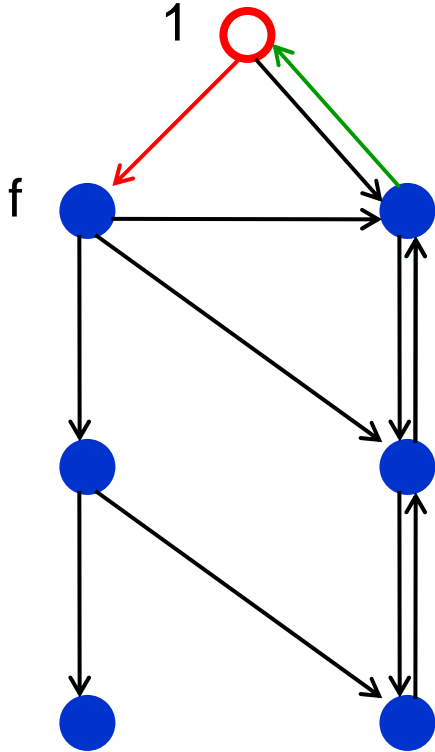
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

e f

# Safety Games



AND-OR Graph:

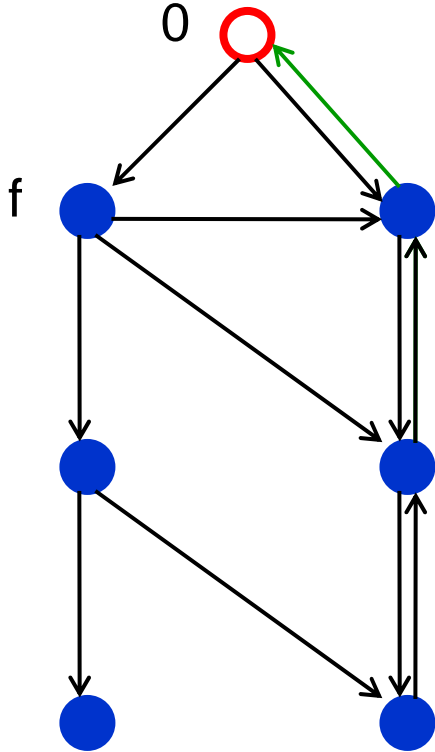
OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

f

# Safety Games



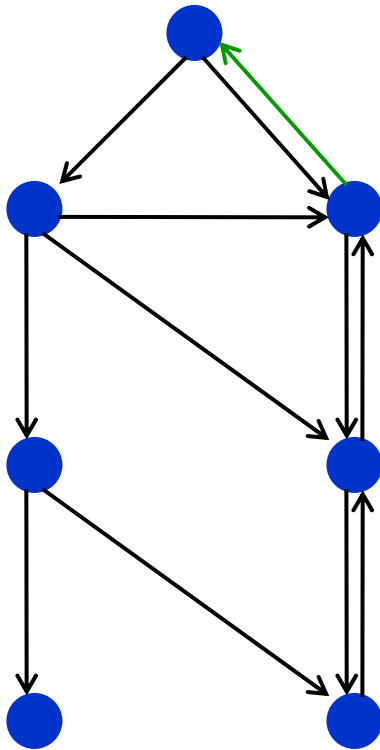
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



AND-OR Graph:

OR Player

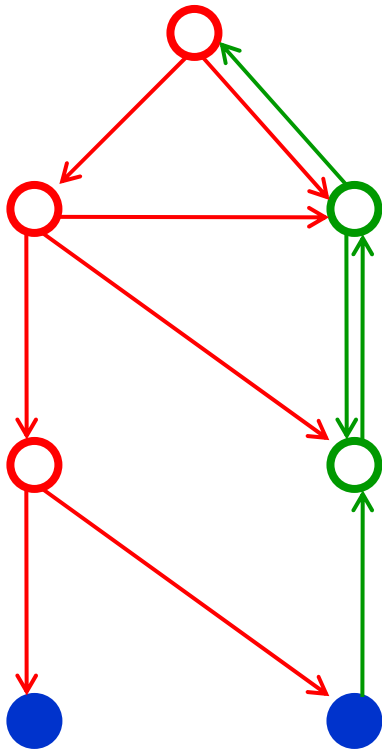
AND Player

From which nodes does the  
OR Player have a strategy  
to avoid **ERROR Nodes** ?

Linear Time (P-Complete)



# Safety Games



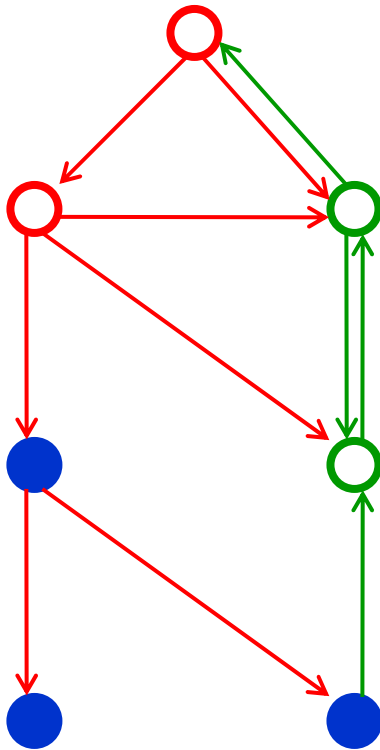
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



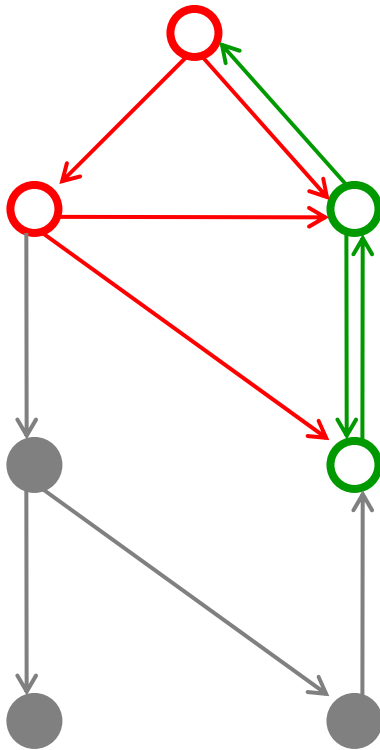
AND-OR Graph:

OR Player

AND Player

From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

# Safety Games



AND-OR Graph:

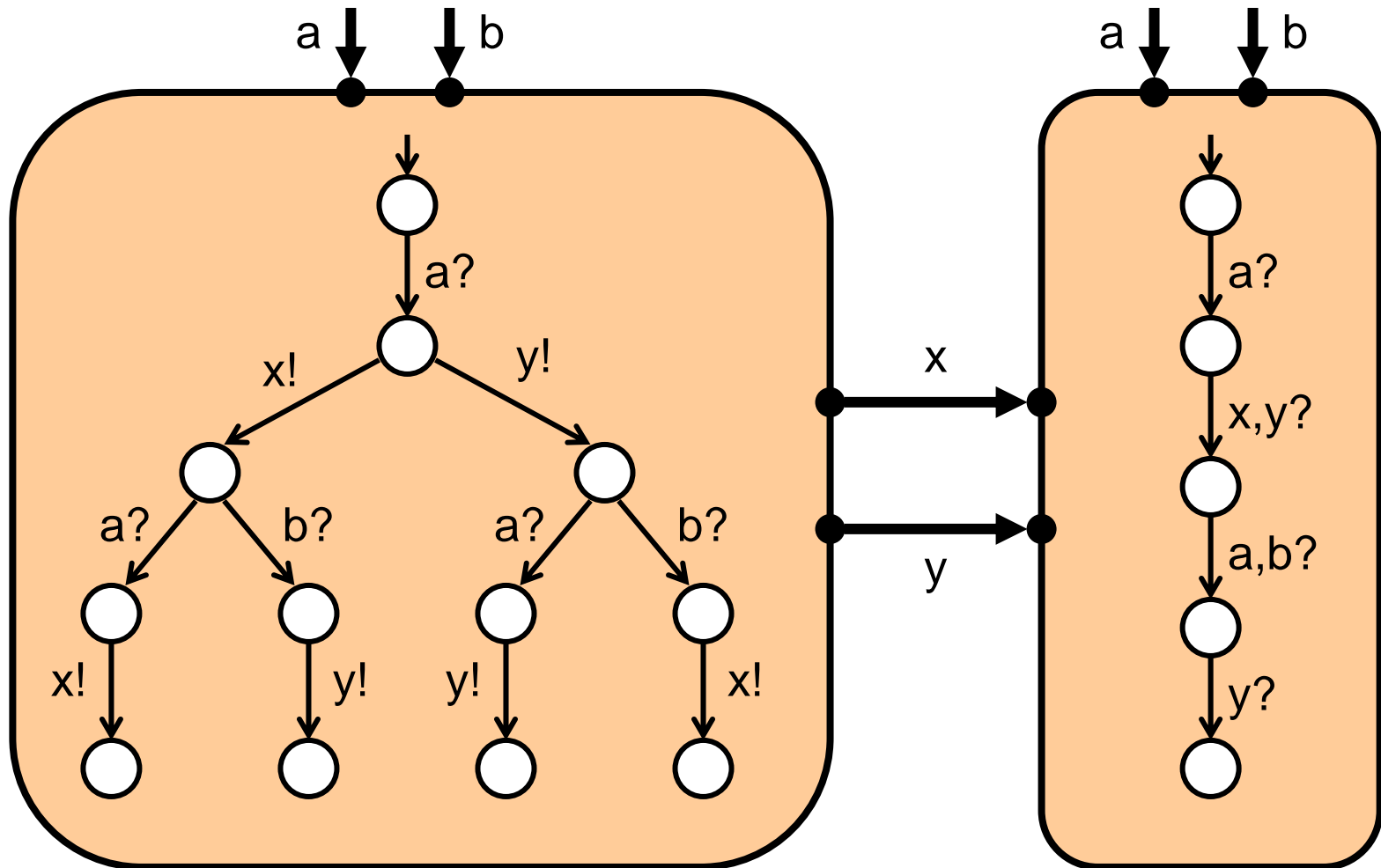
OR Player

AND Player

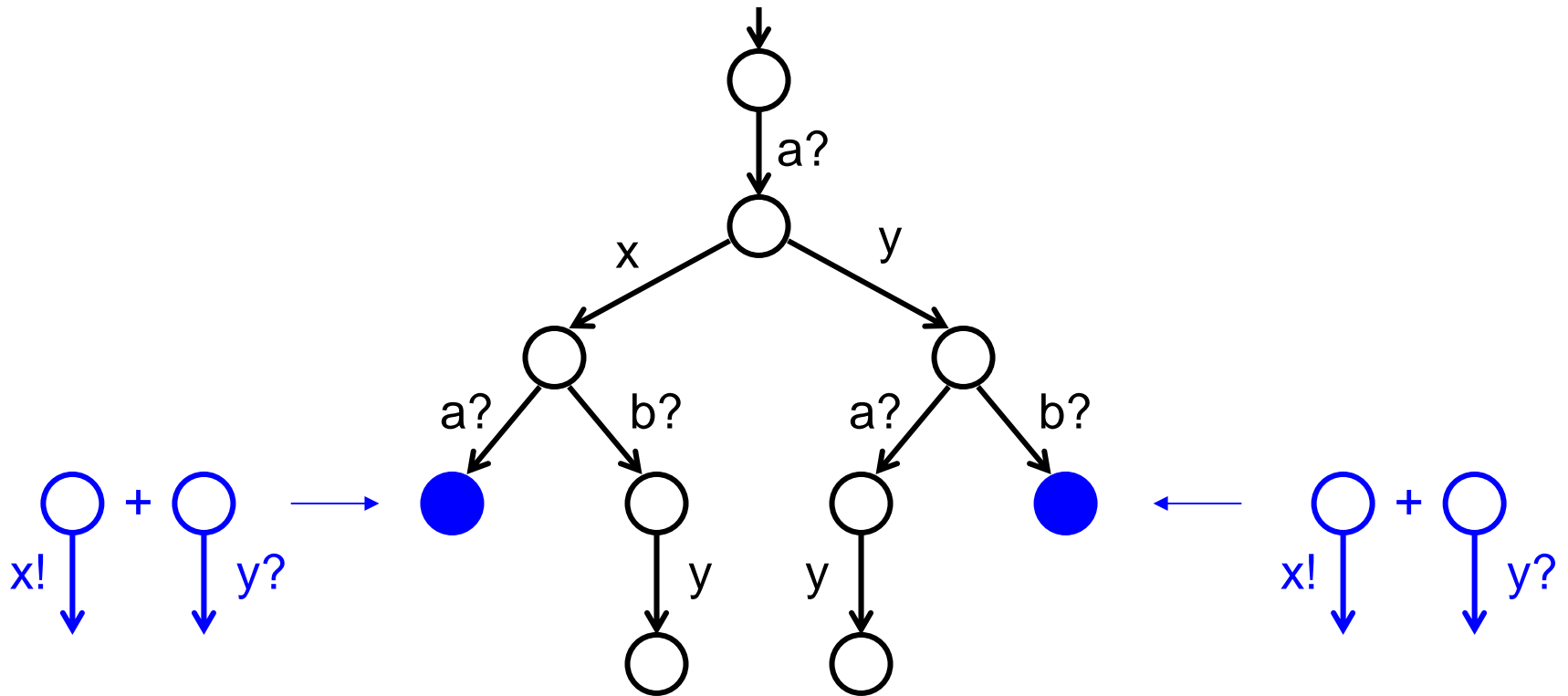
From which nodes does the OR Player have a strategy to avoid **ERROR Nodes** ?

Most general memoryless pure strategy exists.

# From Interfaces to Games

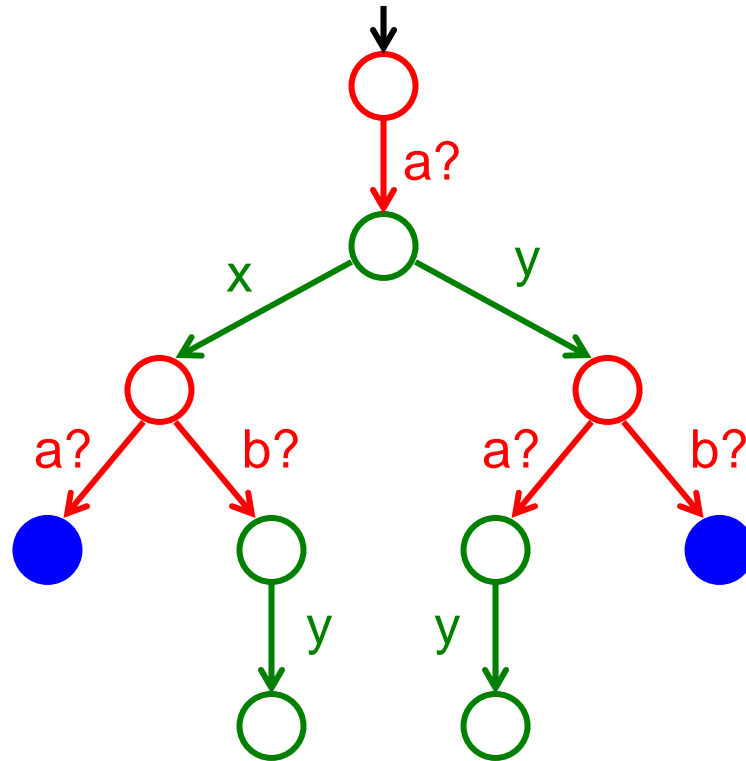


# From Interfaces to Games



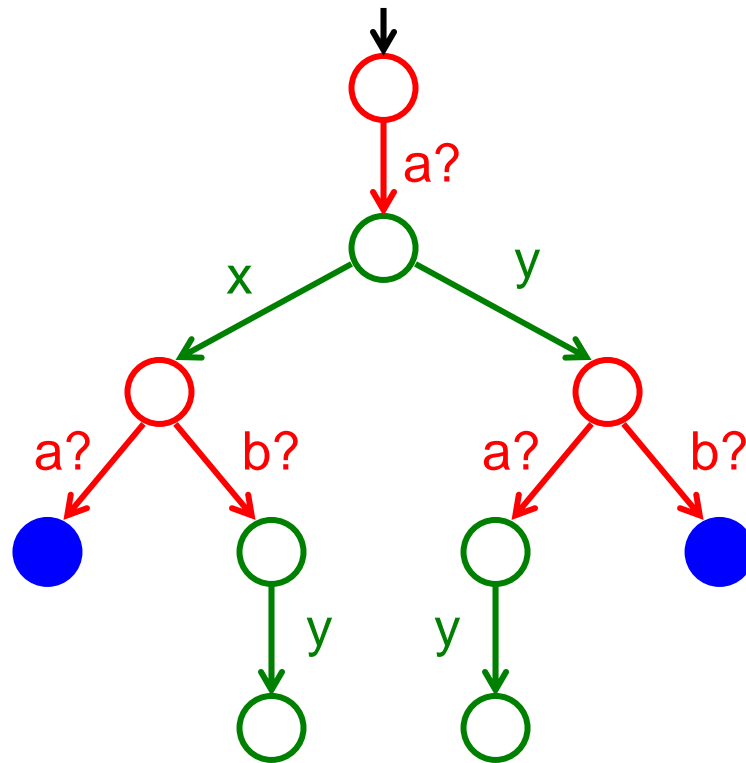
ERROR states of product automaton.

# From Interfaces to Games



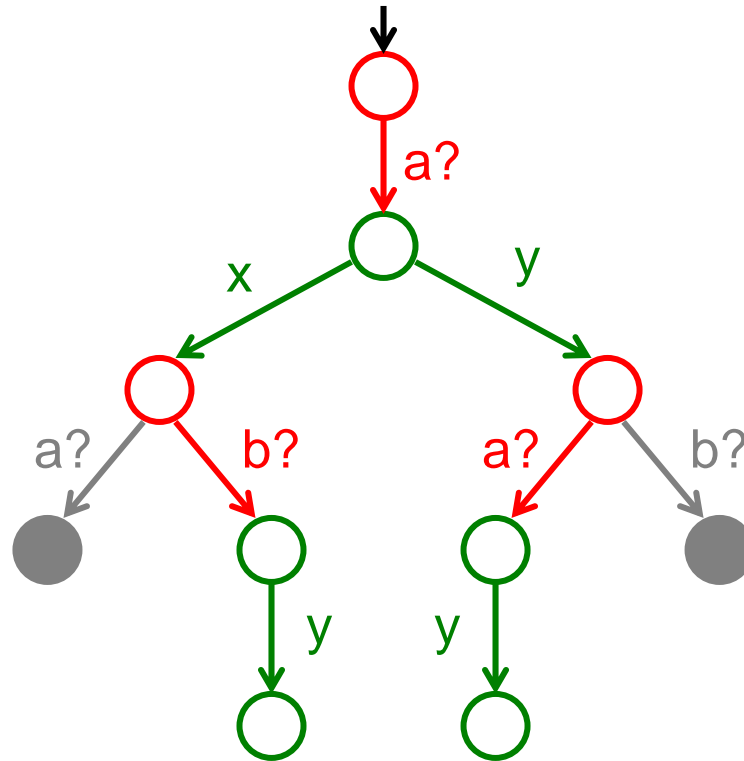
OR Player ... External choices made by the environment  
AND Player ... Internal choices made by the interface product

# From Interfaces to Games



Does the Environment have a strategy to avoid the ERROR states?

# From Interfaces to Games

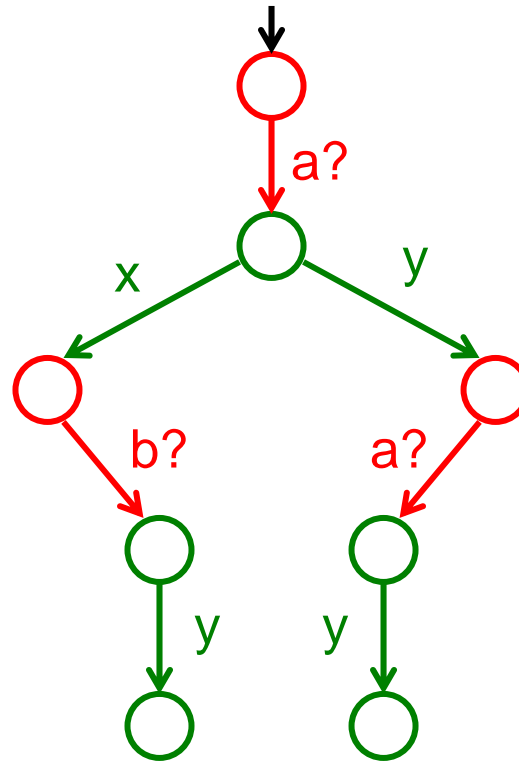


Yes

Does the Environment have a strategy to avoid the ERROR states?

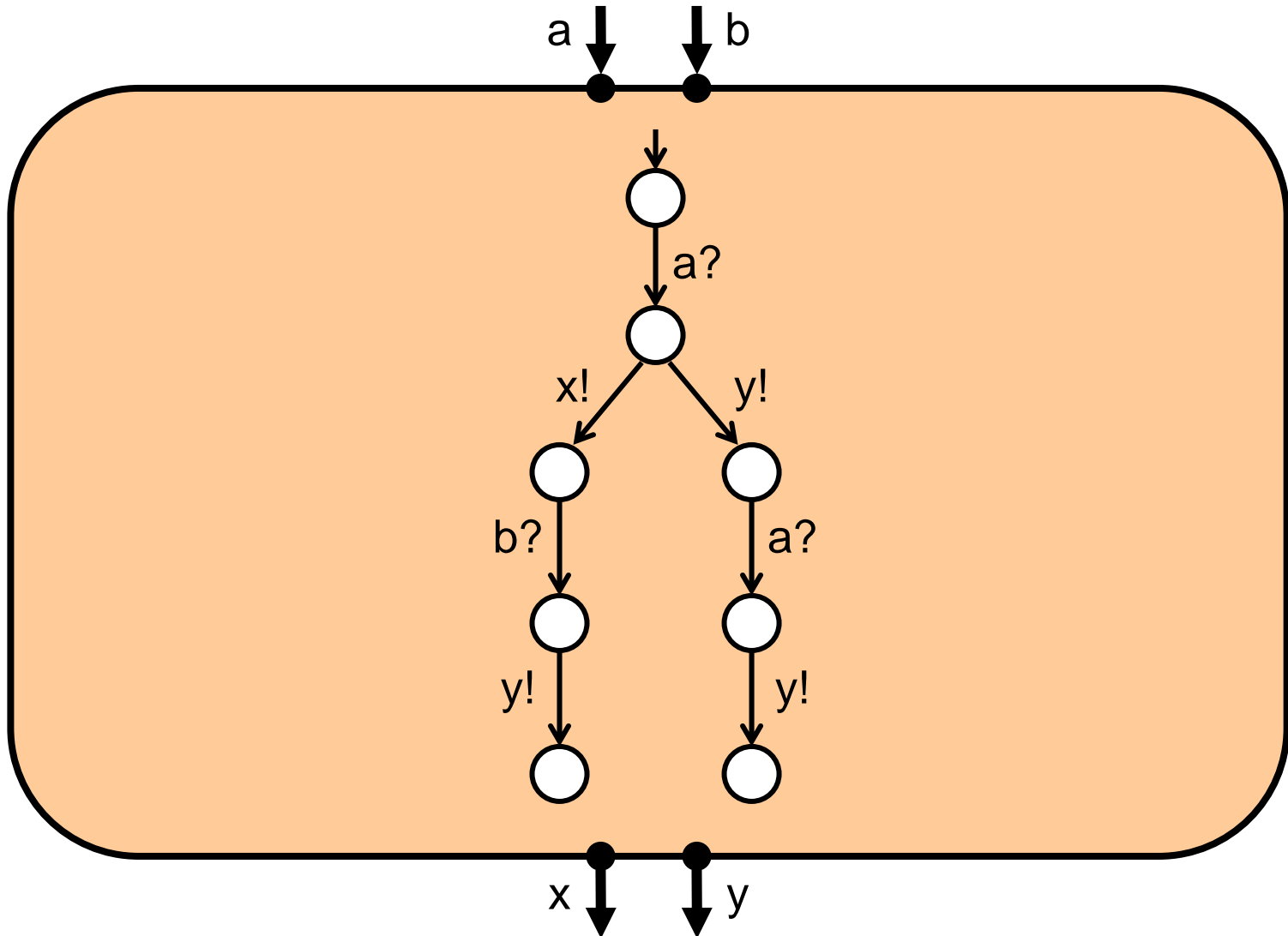


# From Interfaces to Games



The most general environment strategy.

# The Composite Interface



So far, we have used a simple lock-step model of concurrency.

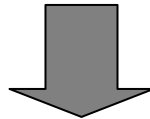
An interface formalism can be built around any model of concurrency.

So far, we have used a simple lock-step model of concurrency.

An interface formalism can be built around any model of concurrency.

Example:

I/O Automata [Lynch]



Interface Automata [deAlfaro,H]

So far, we have used a simple lock-step model of concurrency.

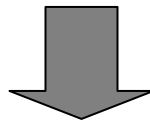
An interface formalism can be built around any model of concurrency.

Example:

I/O Automata [Lynch]

- Total composition
- A process model

*same syntax*



*different semantics*

Interface Automata [deAlfaro,H]

- Partial composition
- Compatibility check
- An interface model

# An Interface Automaton

$$F = (Q_F, Q_F^0, A_F^I, A_F^O, A_F^H, T_F)$$

$Q_F$  ... set of *states*

$Q_F^0 \subseteq Q_F$  ... set of *initial states*

$A_F^I$  ... *input actions*

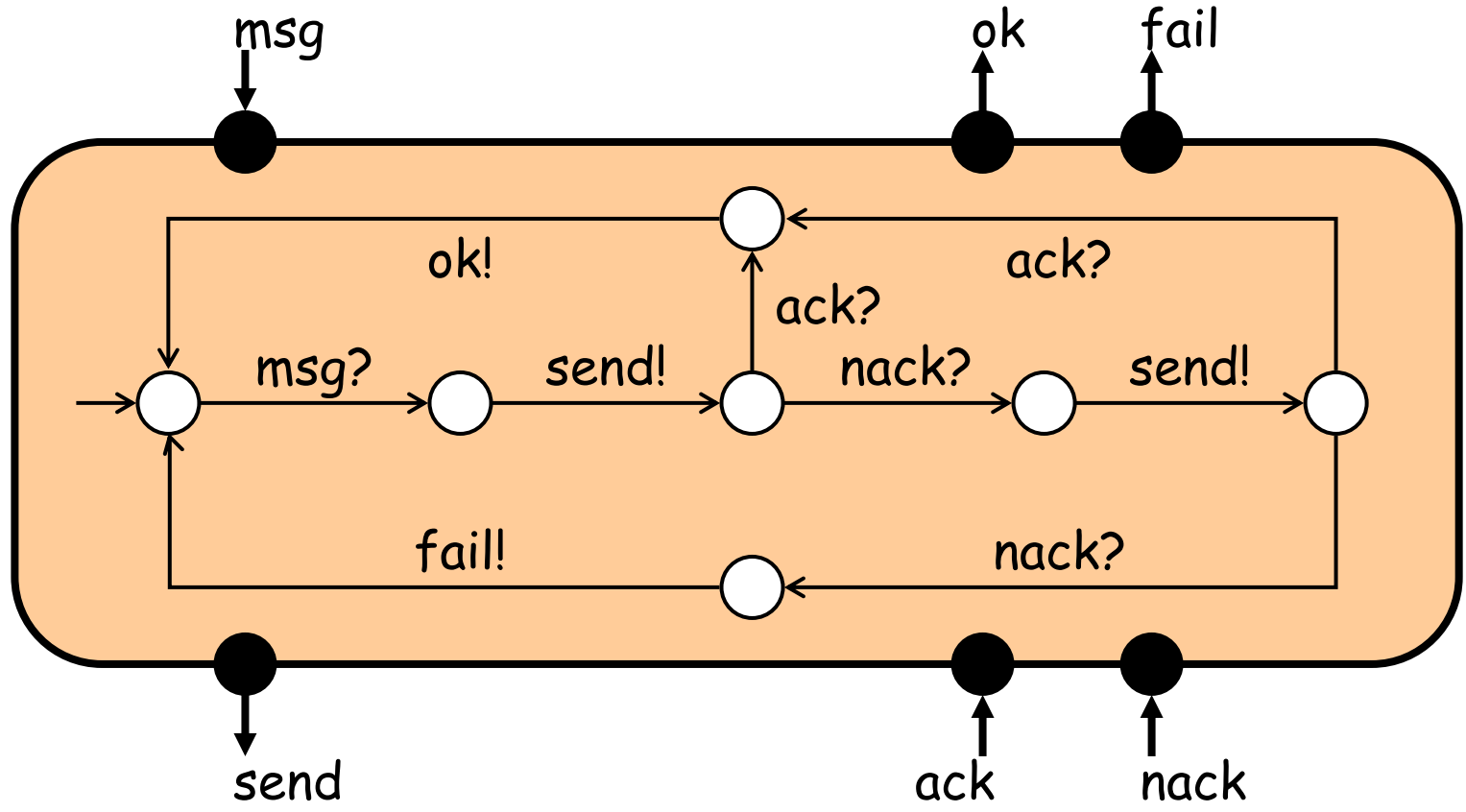
$A_F^O$  ... *output actions*

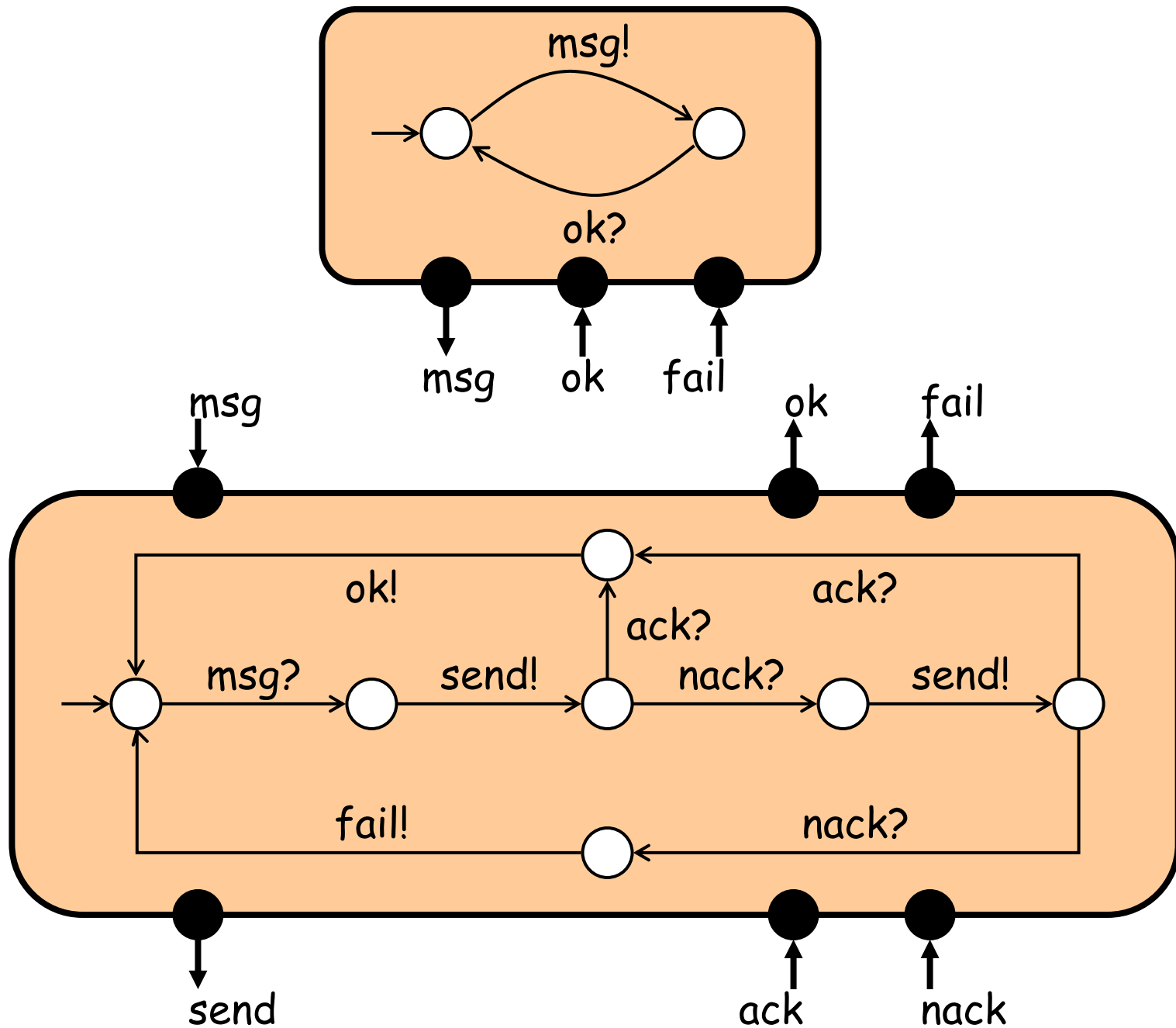
$A_F^H$  ... *internal (hidden) actions*

$T_F \subseteq Q_F \times A_F \times Q_F$  ... set of *transitions*

mutually disjoint

$$A_F = A_F^I \sqcup A_F^O \sqcup A_F^H$$







# The Product of Interface Automata

$F$  and  $G$  are composable if

$$A^H_F \dot{\wedge} A_G = ; \quad A^H_G \dot{\wedge} A_F = ; \quad A^I_F \dot{\wedge} A^I_G = ; \quad A^O_F \dot{\wedge} A^O_G = ;$$

If  $F$  and  $G$  are composable, then

$$Q_{F \dot{\wedge} G} = Q_F \dot{\wedge} Q_G$$

$$Q^O_{F \dot{\wedge} G} = Q^O_F \dot{\wedge} Q^O_G$$

$$A^I_{F \dot{\wedge} G} = (A^I_F [ A^I_G ] \setminus \text{shared}(F, G))$$

$$A^O_{F \dot{\wedge} G} = (A^O_F [ A^O_G ] \setminus \text{shared}(F, G))$$

$$A^H_{F \dot{\wedge} G} = A^H_F [ A^H_G [ \text{shared}(F, G) ]$$

$$\left. \begin{array}{l} A^I_{F \dot{\wedge} G} = (A^I_F [ A^I_G ] \setminus \text{shared}(F, G)) \\ A^O_{F \dot{\wedge} G} = (A^O_F [ A^O_G ] \setminus \text{shared}(F, G)) \\ A^H_{F \dot{\wedge} G} = A^H_F [ A^H_G [ \text{shared}(F, G) ] \end{array} \right\} \text{shared}(F, G) = A_F \dot{\wedge} A_G$$

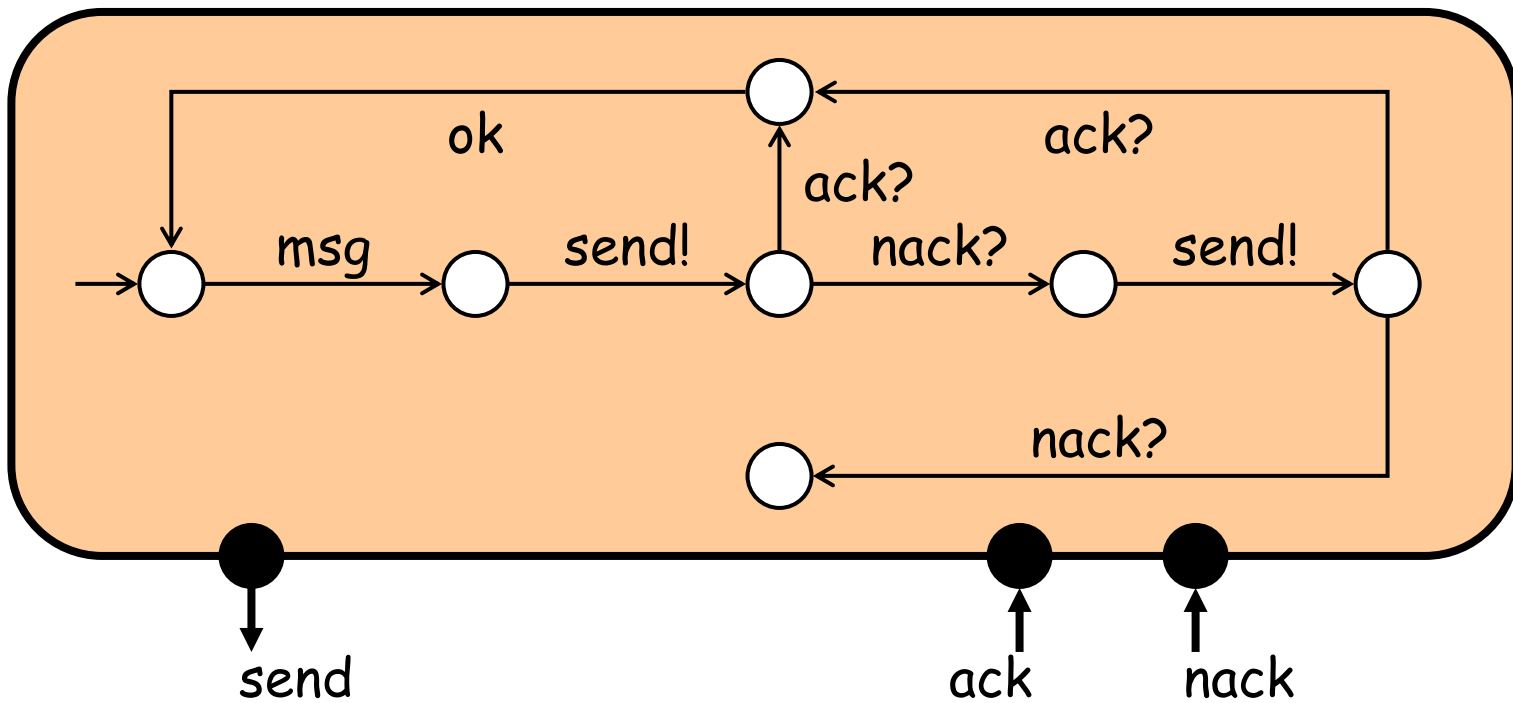
## The Product of Interface Automata, continued

$T_{F \times G} =$

$\{ ((f,g), a, (f',g)) : (f,a,f') \in T_F \wedge a \notin \text{shared}(F,G) \} \cup$

$\{ ((f,g), b, (f,g')) : (g,b,g') \in T_G \wedge b \notin \text{shared}(F,G) \} \cup$

$\{ ((f,g), c, (f',g')) : (f,c,f') \in T_F \wedge (g,c,g') \in T_G \wedge c \in \text{shared}(F,G) \}$



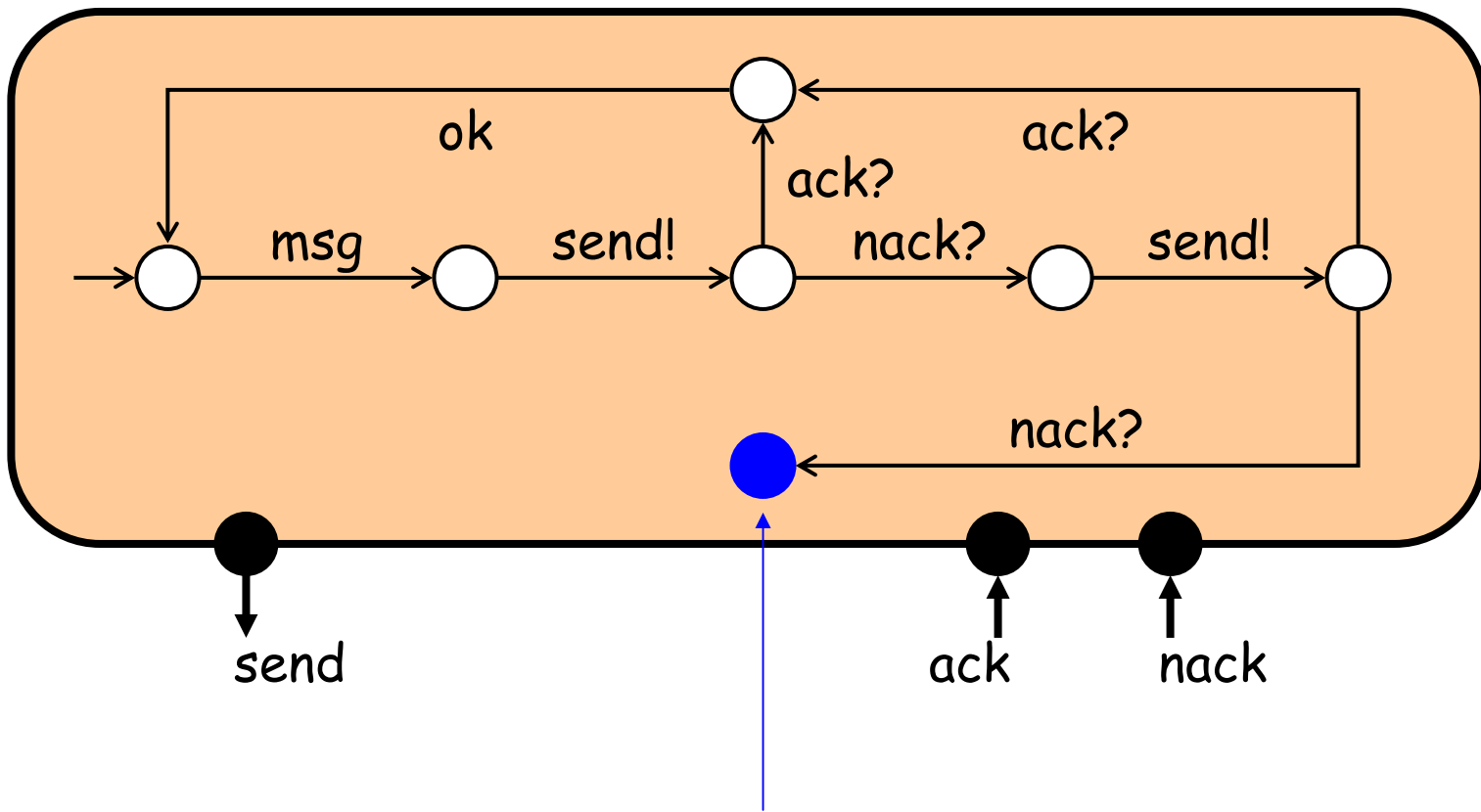
The product automaton.

# The Error States of the Product Automaton

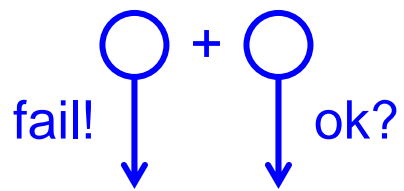
$$\begin{aligned} \text{Error}(F,G) = \{ (f,g) : & (\exists a \in \text{shared}(F,G)) \\ & (a \in A^O_F(f) \wedge a \notin A^I_G(g)) \cup \\ & (a \in A^O_G(g) \wedge a \notin A^I_F(f)) \} \end{aligned}$$

where  $A(q) = \{ a \in A : (\exists q') (q,a,q') \in T \}$

*Note: I/O automata are input enabling ( $A^I(q) = A^I$  for all  $q$ ) and therefore have no error states.*



ERROR state of the product.



# The Compatibility of Interface Automata

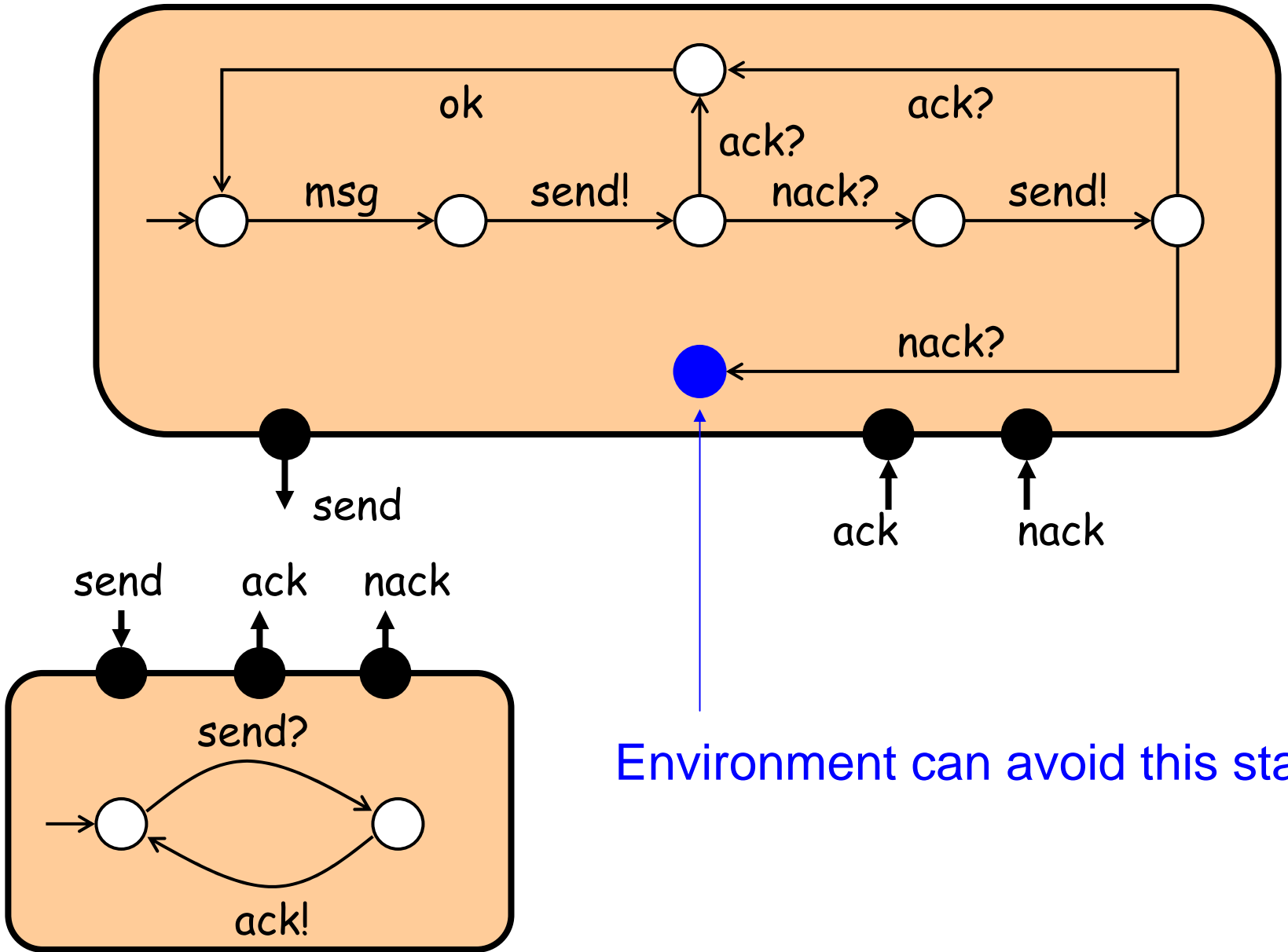
An *environment* for an interface automaton  $F$  is an interface automaton  $E$  such that

1.  $F$  is composable with  $E$
2.  $A'_F = A^O_E$
3.  $Error(F, E) = \emptyset$ ;

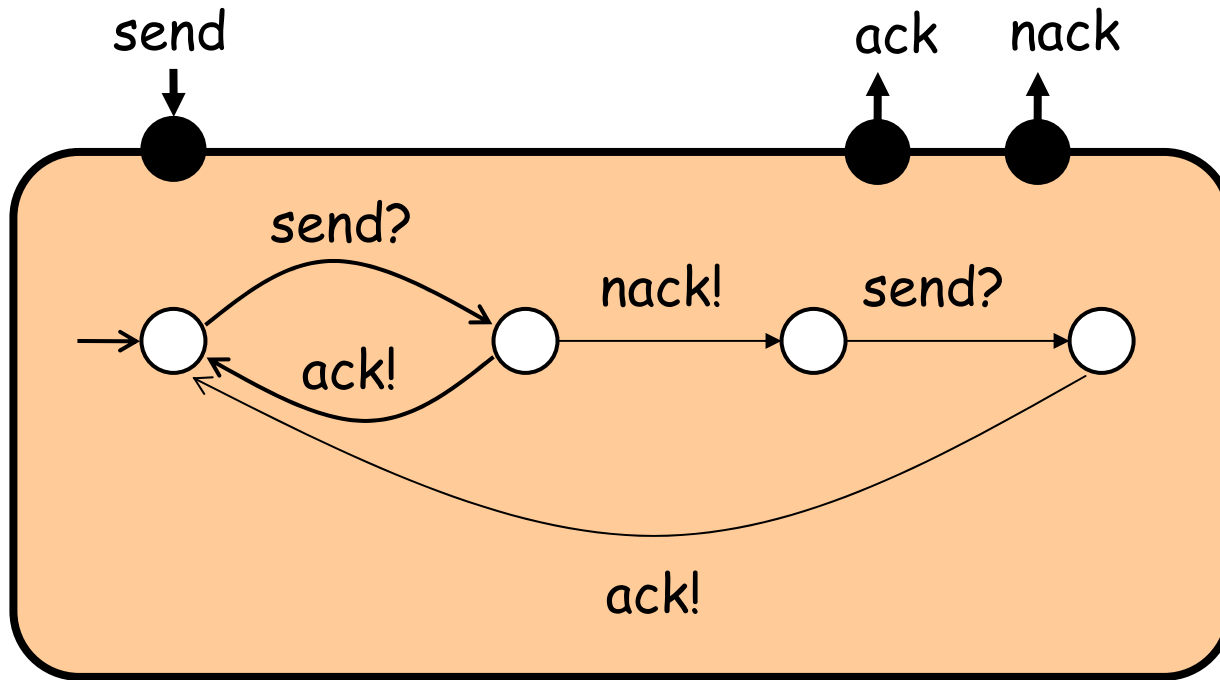
A *helpful* environment for two composable interface automata  $F$  and  $G$  is an environment  $E$  for the product  $F \times G$  such that

4. no state in  $Error(F, G) \times Q_E$  is reachable in  $(F \times G) \times E$

Two interface automata  $F$  and  $G$  are *compatible* if they are composable and there exists a helpful environment.



Environment can avoid this state.



The most general helpful environment.



# Computing the Composite Interface Automaton

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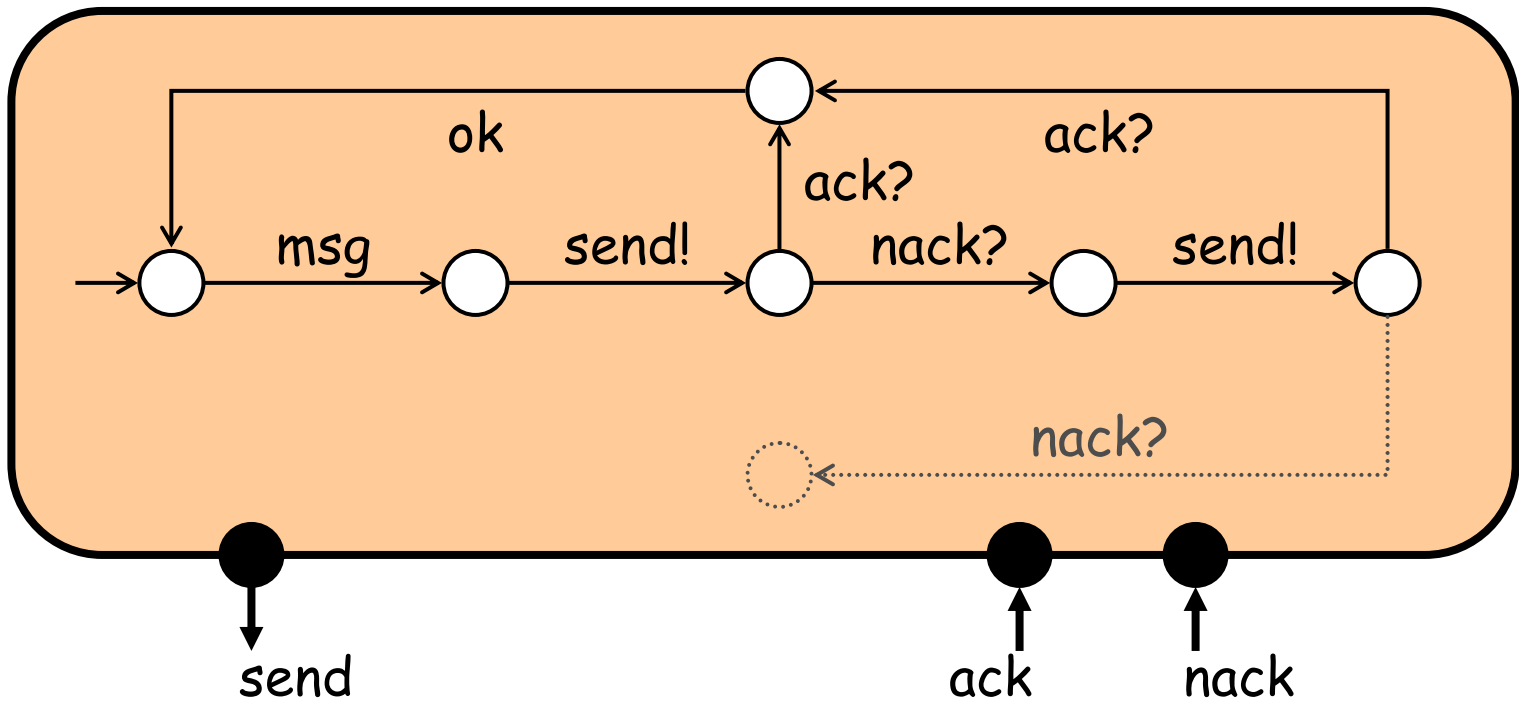
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This procedure computes the most general helpful environment as the most general strategy of the environment to avoid error states.



The composite interface automaton.