

# Computation Orchestration

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# Computation Orchestration

Given are **basic computing elements**. How to **compose** them?

- Computing elements are logic gates:  $\wedge$ ,  $\vee$ ,  $\neg$

Composition is a **circuit**.

- Computing elements are **functions**.

Composition is through **higher-order functions**.

- Computing elements are **processes**.

Composition is through **CCS or CSP operators**.

# Orc

Computing elements are **Sites**, such as

- function: **Compress MPEG file**
- method of an object: **LogOn procedure at a bank**
- monitor procedure: **read from a buffer**
- web service: **get a stock quote**
- transaction: **check account balance**
- distributed transaction: **move money from one bank to another**

## Structure of the Lectures

- Programming Notation: the composition operators, their usage
- Programming Methodology: Parallelism, Synchronization, Interrupt
- Semantics, Implementation
- Site Specification, Commitment, Revocation

### Other Possible Topics:

- Program Structuring
- Concurrency

## Lecture Material

Computation Orchestration: A Basis for Wide-Area Computing

<http://www.cs.utexas.edu/users/psp/Wide-area.pdf>

Exercises in your Handouts

I will give additional exercises during the lecture.

## Example: Airline

- Contact two airlines simultaneously for price quotes.
- Buy ticket from either airline if its quote is at most \$300.
- Buy the cheapest ticket if both quotes are above \$300.
- Buy any ticket if the other airline does not provide a timely quote.
- Notify client if neither airline provides a timely quote.

## Example: workflow

- An office assistant contacts a potential visitor.
- The visitor responds, sends the date of her visit.
- The assistant books an airline ticket and contacts two hotels for reservation.
- After hearing from the airline and any of the hotels: he tells the visitor about the airline and the hotel.
- The visitor sends a confirmation which the assistant notes.

## Example: workflow, contd.

After receiving the confirmation, the assistant

- confirms hotel and airline reservations.
- reserves a room for the lecture.
- announces the lecture by posting it at a web-site.
- requests a technician to check the equipment in the room.



## Wide-area Computing

Acquire data from remote services.

Calculate with these data.

Invoke yet other remote services with the results.

### Additionally

Invoke alternate services for failure tolerance.

Repeatedly poll a service.

Ask a service to notify the user when it acquires the appropriate data.

Download an application and invoke it locally.

Have a service call another service on behalf of the user.

## The Nature of Distributed Applications

Three major components in distributed applications:

### Persistent storage management

databases by the airline and the hotels.

### Specification of sequential computational logic

does ticket price exceed \$300?

### Methods for orchestrating the computations

contact the visitor for a second time only **after** hearing from the airline and one of the hotels.

**We look at only the third problem.**

# Orc

## A new kind of assignment

$$x \leftarrow f$$

where  $x$  is a variable and  $f$  is an Orc expression.

Evaluation of  $f$  yields zero or more values.

Assign the first value to  $x$ .

## An Orc expression is

- **Simple:** Site (Function call, method, web service, transaction)
- **Compound:**  $f \mid g$ ,  $f \gg g$ ,  $f * g$ ,  $\{ f \text{ where } x \leftarrow g \}$

## Simple Orc Expression

- $M$  is a news service,  $d$  a date. Download the news page for  $d$ .

$$x \in M(d)$$

- Side-effect: Book ticket at airline  $A$  for a flight described by  $c$ .

$$x \in A(c)$$

The returned value is the price and the confirmation number.

## Properties of Sites

- A site may not respond.

Its response at different times (for the same input) may be different.

- A site call may change states (of external servers) **tentatively** or **permanently**.

Tentative state changes are made permanent by **explicit** commitment.

## Structure of response

- The response from a site has:  
**value**, which the programmer can manipulate, and  
**pledge**, which the programmer cannot manipulate.
- Pledge is used to **commit** this site call.  
Pledge is **valid** for some time period.  
Value is meaningful during then.
- By committing a valid pledge (during the given period), the programmer establishes some fact.

## Nesting

- (Data Piping) Retrieve a news page for date  $d$  from  $M$  and email it to address  $a$ . Here,  $Email$  is a site.

$Email(a, M(d))$

- (Higher-order site) Call discovery service  $D$  with parameter  $x$  to locate a site; call that site with parameter  $y$ .

$Apply(D(x), y)$

## Simple Orc Expression: Sequencing

$M$ ,  $N$ ,  $R$  are sites for 3 professors.

$s$  is a set of possible meeting times.

$M(s)$  is a subset of  $s$ , the times when  $M$  can meet.

$M(N(R(s)))$  is the possible meeting times of all three professors.



## Parallel, Strict evaluation

Arguments of a site call are evaluated in parallel.

A site is called only after **all** its arguments have been evaluated.

## Fork-join parallelism

$A(c)$  and  $B(c)$  return ticket prices from airlines  $A$  and  $B$ .

$Min$  returns the minimum of its arguments.

$Min(A(c), B(c))$ :

Compute  $A(c)$  and  $B(c)$  in parallel.

Call  $Min$  when both quotes are available.

## Predefined sites

- *Fail* never responds.
- $let(x, y, \dots)$  returns a tuple of argument values as soon they are available.  $let(\theta)$  is *skip*.
- *random* returns a random number (in a specified range), instantaneously.
- *fst* returns the value of the first argument as soon all argument values are available.
- $timer(t)$ , where  $t$  is a non-negative integer, returns a signal exactly after  $t$  time units.
- $timer(t, x)$  is  $fst(x, timer(t))$ ; returns  $x$  after  $t$  time units.

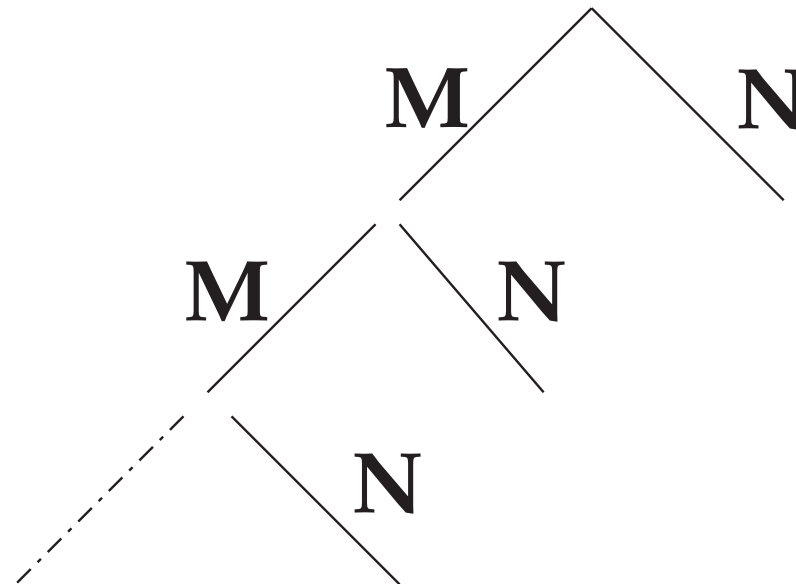
## Composing Expressions

- (Alternation)  $f \mid g$ : evaluate  $f$  and  $g$  in parallel; values of  $f \mid g$  are those from  $f$  and from  $g$ .
- (Piping)  $f \gg g$ : Evaluate  $g$  for **all** values of  $f$ ; values of  $f \gg g$  are those from  $g$ .
- (Iteration)  $f * g$ : values from  $g$  after zero or more piping steps of  $f$ .

$$\begin{aligned}
 & f * g \\
 = & g \mid (f \gg (f * g)) \\
 = & g \mid (f \gg (g \mid f \gg (g \mid f \gg \dots)))
 \end{aligned}$$

- (Definition)  $\{ f \text{ where } x \in g \}$

## Pictorial Depiction of $M * N$



Values of  $M * N$  are the ones returned by  $N$ .

## Binding power

| has the lowest binding power.

>> and \* have equal binding powers.

$$f * g \mid h \gg g \equiv (f * g) \mid (h \gg g)$$

Example of Orc expression:

$$G(q) \gg ( \langle M(q) \mid R(\theta, q) \gg G(\theta) \rangle * S(\theta) )$$

## Default Parameter

- $M \gg N(x, \theta)$
- $(M \mid S) \gg (N(x, \theta) \mid R(\theta))$
- Start computation of  $f$  with value  $v$  for  $\theta$ :  
 $let(v) \gg f$ .
- Start an iteration where  $x_0 = v$  and  $x_{i+1} = M(x_i)$ .  
Values returned are  $N(x_i)$ , for  $i \geq 0$ .

$$let(v) \gg (M(\theta) * N(\theta))$$

## Properties of the timer

$$\begin{aligned} x \in \text{timer}(t) \mid \text{timer}(u) &\equiv x \in \text{timer}(t), \text{ given } t \leq u, \\ \text{timer}(t) \gg \text{timer}(u) &\equiv \text{timer}(t + u) \end{aligned}$$



## Alternation, Piping

- Assign the first value from  $M(c)$  or  $N(d)$  to  $z$ .

$$z : \in M(c) \mid N(d)$$

- assign to  $z$  the value from  $M$  if it arrives before  $t$ , 0 otherwise.

$$z : \in M \mid \text{timer}(t, 0)$$

- Interruption

$$f \mid \text{Interrupt.get}$$

- Make four requests to site  $M$ , in intervals of one time unit each.

$$M \mid \text{timer}(1) \gg M \mid \text{timer}(2) \gg M \mid \text{timer}(3) \gg M$$

## Priority

Request  $M$  and  $N$  for values. Give priority to  $M$ .

- Allocate one extra time unit for  $M$  to respond.

$$z:\in M \mid \text{timer}(1) \gg N \quad \text{or} \quad z:\in M \mid N \gg \text{timer}(1, \theta)$$

- Accept the response from  $M$  if it arrives within one time unit, else accept the first response.

$$z:\in M \mid \text{fst}(N, \text{timer}(1))$$

## Iteration

- Call  $M$  forever at unit time intervals until it returns a value.

$$z:\in timer(1) * M$$

which is

$$z:\in M \mid timer(1) \gg (M \mid timer(1) \gg (M \mid \dots))$$

- Same as above, but stop calling after 10 time units.

$$z:\in timer(1) * M \mid timer(10)$$

## Iteration; Contd.

- Site  $M$  returns stock price of company  $abc$   
Site  $C(x)$ : returns  $x$  if  $x < 20$ ; silent otherwise.

$$M * C(\theta)$$

either never returns a value (if  $abc$  never falls below 20)  
or returns a value lower than 20. Initially,  $\theta \geq 20$ .

- **Variation:** Poll  $M$  once every hour for 6-hours:

$$timer(1) * \langle M \gg C(\theta) \rangle \mid timer(6)$$

## Definition within Orc expression

- A machine is assembled from two parts,  $u$  and  $v$ .
- Two vendors for each part:  $u1$  and  $u2$  for  $u$ , and  $v1$  and  $v2$  for  $v$ .
- Solicit quotes from all vendors.
- Accept the first quote for each part.
- Compute the machine cost to be 20% above the sum of the part costs.

$$\begin{array}{l}
 \text{cost}:\in \{ (u + v) \times 1.2 \\
 \quad \text{where} \\
 \quad \quad u:\in u1 \mid u2 \\
 \quad \quad v:\in v1 \mid v2 \\
 \quad \}
 \end{array}$$

## General Orc Statements

$$z:\in \{ f(\dots x \dots y \dots) \\ \text{where} \\ \quad x:\in g \\ \quad y:\in h \\ \}$$

**Example:**  $M$ ,  $N$ ,  $R$ ,  $S$  are sites.

$$z:\in \{ (M(x) \mid N(y)) \gg M(y) \gg \{ M(y) \\ \text{where} \\ \quad x:\in R(y) \mid N(y) \\ \quad y:\in \{ R \mid N(t) \\ \quad \quad \text{where } t:\in S \\ \} \\ \}$$

# Syntax

statement ::= defn

defn ::= variable :∈ expr

expr ::= term  
 | expr | expr  
 | expr >> expr  
 | expr \* expr  
 | { expr where defn }

term ::= site([parameter])

parameter ::= variable |  $\theta$

[parameter] is a list of parameters, possibly empty.

## Free and Bound Variables

- Variable assigned in a **statement** is the **goal** variable.
- Variables named in an expression are **global** or **local**.
- Free variables:

$$\mathit{free}(M(L)) = \{x \mid x \in L, x \neq \theta\}$$

$$\mathit{free}(f \mathit{op} g) = \mathit{free}(f) \cup \mathit{free}(g), \text{ where } \mathit{op} \in \{ |, \gg, * \}$$

$$\mathit{free}(\{f \text{ where } x:\in g\}) = (\mathit{free}(f) - \{x\}) \cup \mathit{free}(g)$$

- In  $\{f \text{ where } x:\in g\}$ , any free occurrence of  $x$  in  $f$  is bound to the variable shown.
- $z:\in f$  is **well-formed** if all the free variables in  $f$  are global variables.



## Flat Expression

**Flat expression:** without a **where** clause.

**Non-flat expression:**

Flat expression is a regular expression of language theory.

Terms are symbols.

## Syntactic Conventions: Omit Braces; group **where**

$$\{ \{ f \text{ where } x \in g \} \text{ where } y \in h \}$$

is

$$f \text{ where } x \in g, y \in h$$

or,  $\{ f \text{ where } x \in g, y \in h \}$

## Syntactic Conventions: Nested Site Calls

- $Email(a, M(d))$  is not an expression.

It means:

$$\{ Email(a, u) \text{ where } u:\in M(d) \}$$

- We allow  $R(f, g)$  where  $f$  and  $g$  are expressions.

It means :  $\{ R(x, y) \text{ where } x:\in f, y:\in g \}$

- $timer(f, g)$  is (after  $f_0$  time units return  $g_0$ )

$$\{ fst(x, y) \text{ where } x:\in g, y:\in \{ timer(u) \text{ where } u:\in f \} \}$$

## Argument Evaluation in Nested Site Calls

Consider  $Q(N(x), N(x), N(x))$ .

For the first two arguments:

evaluate  $N(x)$  once and use the value for both.

For the last argument:

reevaluate  $N(x)$ .

$$\left\{ \begin{array}{l} Q(u, u, v) \\ \text{where} \\ u \in N(x) \\ v \in N(x) \end{array} \right\}$$

## Operational Semantics

$$\begin{array}{ccc}
 z:\in \{A(x) \mid B(y)\} & \gg & \{C(p, \theta)\} \\
 \text{where} & & \text{where} \\
 \begin{array}{l} x:\in M \mid R \\ y:\in N \end{array} & & p:\in N \\
 \} & & \}
 \end{array}$$

- Execute the defns of  $x$ ,  $y$  and evaluate  $A(x) \mid B(y)$ , all in parallel.
- Suspend evaluation of  $A(x) \mid B(y)$  until  $x$  or  $y$  gets a value.
- When  $x$  gets a value, resume evaluation of  $A(x)$ .
- When  $y$  gets a value, resume evaluation of  $B(y)$ .
- Suppose  $A(x)$  returns  $v$ . Evaluate  $C(p, v)$ . Start with  $p:\in N$ .

## Execution Rules

- **State**: Variable, value pair. Value for  $\theta$  in every state.  
 $p.x$  is the value of  $x$  in state  $p$ .
- In the initial state only globals and  $\theta$  have values.
- Rules describe the **bag** of values computed for expression  $f$ , by structural induction on  $f$ .

```

expr      ::=   term
            |   expr | expr
            |   expr >> expr
            |   expr * expr
            |   { expr where defn }

```

## Execution Rules; Starting state $p$

- term  $M(x, y)$ : call  $M$  with parameters  $p.x$  and  $p.y$ .
  - $M$  never responds: computation never terminates.
  - $M$  responds with value  $v$ : Only one result state  $q$ ,  
 $q.\theta = v$  and  $q.x = p.x$  for all other  $x$ .

- $f \mid g$ : evaluate  $f$  and  $g$  in state  $p$ , in parallel.

The (bag of) result states are the ones returned by both  $f$  and  $g$ .

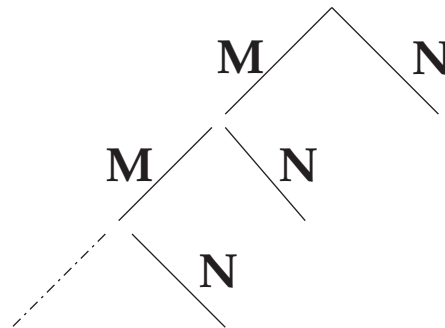
- $f \gg g$ : evaluate  $f$  in  $p$ . For each result state  $q$ , evaluate  $g$  in  $q$ .

Value computed for  $f$  is in  $q.\theta$ . Value of  $f \gg g$  are the states returned by  $g$ .

## Execution Rule for $f * g$ in state $p$

Evaluate both  $f$  and  $g$  in  $p$  and in any  $q$  returned by  $f$ .

The result states are the ones returned by  $g$ .



Values are returned by  $N$ .



**Execution Rule for  $\{ f \text{ where } x:\in g \}$  in state  $p$** 

- Evaluate  $f$  and  $g$  in state  $p$ , in parallel.
- When  $g$  returns state  $q$ , augment  $p$  with  $(x, q.\theta)$ .
- In evaluating  $f$ , if we need the value of  $x$ :  
wait until the value is available (in an augmented state).
- Result states of  $\{ f \text{ where } x:\in g \}$  are from  $f$ .  
Remove the tuple for  $x$  from the state because  $x$  is not defined outside this scope.

**Execution Rule for  $z:\in f$  in state  $p$** 

- Evaluate  $f$  in  $p$ .
- Any result state,  $q$ , has the same set of variables as  $p$ , and  $p.x = q.x$ , for all  $x$  except  $\theta$ ; the result of evaluation is  $q.\theta$ .
- Let  $r$  be the first result state. Augment  $p$  by  $(z, r.\theta)$ , and return this as the result state of  $z:\in f$ .
- If  $f$  never responds, state  $p$  is never augmented.

## Fork-Join parallelism

$$\begin{array}{l|l} z:\in \text{fst}(\text{true}, x) & \text{fst}(\text{false}, x) \\ \text{where} & \text{where} \\ x:\in \text{timer}(1) & x:\in \text{timer}(2) \end{array}$$

$z$  is assigned *true* after 1 time unit.

## Angelic Nondeterminism

In  $(M \mid N) \gg R$ ,  $R$  may be called twice. We have

$$(M \mid N) \gg R = M \gg R \mid N \gg R,$$

More generally, Right Distributivity of  $\gg$  over  $\mid$ :

$$(f \mid g) \gg h = (f \gg h \mid g \gg h)$$

## Demonic Nondeterminism; where clause

$$(N \mid R) \gg M$$

is *not* equivalent to

$$\text{let}(x) \gg M$$

**where**

$$x:\in N \mid R$$

## Idempotence and Left Distributivity do not hold

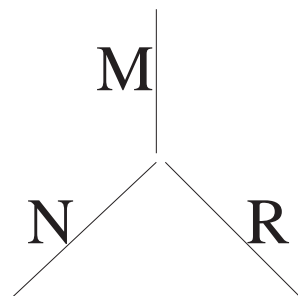
Following laws do not hold.

(Idempotence of  $|$ )

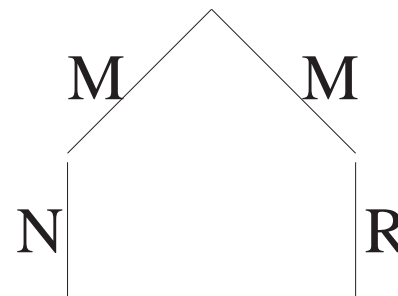
$$f | f = f$$

(Left Distributivity of  $\gg$  over  $|$ )

$$f \gg (g | h) = (f \gg g) | (f \gg h)$$



(a)



(b)

Figure 1: Schematic for  $M \gg (N | R)$  and  $M \gg N | M \gg R$

## Parallel or

Let sites  $M$  and  $N$  return booleans. Compute their **parallel or**.

$$z:\in \text{ift}(x) \mid \text{ift}(y) \mid \text{or}(x, y)$$

where

$$x:\in M$$

$$y:\in N$$

Similarly, evaluate any function  $f$  of the form

$$f(x, y) = \begin{cases} p(x) & \text{if } c(x) \\ q(y) & \text{if } d(y) \\ r(x, y) & \text{otherwise} \end{cases}$$

## Eight queens

- **configuration**: placement of queens in the last  $i$  rows.
- Represent a configuration by a list of integers  $j$ ,  $0 \leq j \leq 7$ .
- **Valid configuration**: no queen captures another.
- $check(x:xs)$ : Given  $xs$  valid, return  
 $x : xs$ , if it is valid  
remain silent, otherwise.



## Eight queens; Contd.

*let*( $\square$ )

$\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$   
 $\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$   
 $\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$   
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 $\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$   
 $\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$   
 $\gg \langle \text{check}(0 : \theta) \mid \text{check}(1 : \theta) \mid \text{check}(2 : \theta) \dots \mid \text{check}(7 : \theta) \rangle$

---

$\text{let}(\square) \gg \langle \gg i : 0 \leq i \leq 7 : \langle \mid j : 0 \leq j \leq 7 : \text{check}(j : \theta) \rangle \rangle$   
 $\rangle$

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## Local object

- Call sites  $M$ ,  $N$  and  $R$ .
- Terminate after receiving two response.

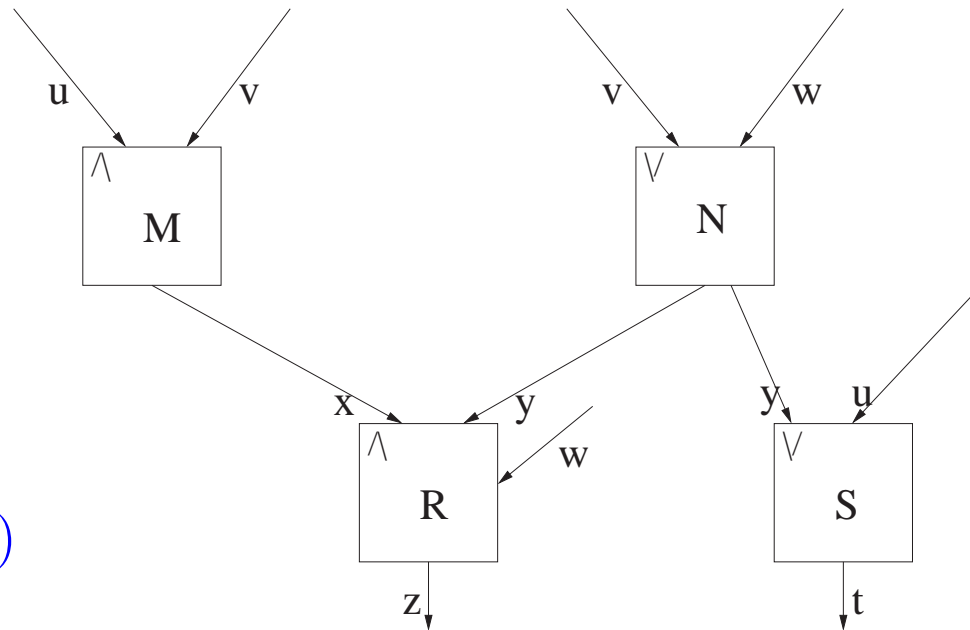
Object  $count$  with integer state. Initially,  $0$ .

- $count.incr$  increments state;
- returns a signal if state  $\geq 2$ , otherwise, remains silent.

$c:\in$

$M \gg count.incr$   
 $| N \gg count.incr$   
 $| R \gg count.incr$

# And-Or graph



$r: \in \text{let}(z, t)$

**where**

$z: \in R(x, y, w)$

$t: \in S(y) \mid S(u)$

**where**

$x: \in M(u, v)$

$y: \in N(v) \mid N(w)$

## Airline

- Return any quote, from  $A$  or  $B$ , provided it is below  $300$ .
- If neither quote is below  $300$ , then return the cheapest quote or any quote available by time  $t$ .
- If no quote is available by  $t$ , return  $\infty$ .

$Min$  returns the minimum of its argument values.

$threshold(x)$  returns  $x$  if  $x$  is below  $300$ ; silent otherwise.

$z \in threshold(x) \mid threshold(y) \mid Min(x, y)$

**where**

$x \in A \mid timer(t, \infty)$

$y \in B \mid timer(t, \infty)$

## Workflow: Visit Coordination

- $Email(p, s)$ : contact  $p$  with dates  $s$ ; response is date  $d$  from  $s$ .
- $Hotel(d)$ : booking from hotel.
- $Airline(d)$ : booking from airline.
- $Ack(p, t)$ : similar to  $Email$ ; response is an acknowledgment.
- $Confirm(t)$ : confirm reservation  $t$  (for hotel or airline).
- $Room(d)$ : reserve room for  $d$ . Response  $q$ : room number, time.
- $Announce(p, q)$ : announce the lecture.
- $AV(q)$ : contact technician with room and time information in  $q$ .

## Workflow; Contd.

$z: \in \text{let}(b)$

**where**

$b: \in \text{Ack}(p, h, f)$

**where**

$h: \in \text{Hotel}(d)$

$f: \in \text{Airline}(d)$

**where**

$d: \in \text{Email}(p, s)$

$\gg \text{let}(c, e)$

**where**

$c: \in \text{Confirm}(h)$

$e: \in \text{Confirm}(f)$

$\gg \text{let}(u, v)$

**where**

$u: \in \text{Announce}(p, q)$

$v: \in \text{AV}(q)$

**where**

$q: \in \text{Room}(d)$

## Interrupt handling

- Orc statement can not be directly interrupted.
- *Interrupt* site: a monitor.
- *Interrupt.set*: to interrupt the Orc statement
- *Interrupt.get*: responds after *Interrupt.set* has been called.

---


$$z \in f$$


---

is changed to

$$z \in f \mid \text{Interrupt.get}$$

## Processing Interrupt

$$z:\in \{ f(x, y) \\ \text{where } x:\in g, y:\in h \}$$

If  $f$  is interrupted, call  $M$  and  $N$  with parameters  $x$  and  $y$ , respectively, to cancel the effects of  $g$  and  $h$ .

$$z:\in \text{Normal}(t) \mid \text{Interr}(t) \gg \text{let}(X, Y) \\ \text{where} \\ X:\in M(x) \\ Y:\in N(y)$$

where

$$t :\in f(x, y) \mid \text{Interrupt.get}$$

where

$$x :\in g \\ y :\in h$$



## Phase Synchronization

Process starts its  $(k + 1)^{th}$  phase only after all processes have completed their  $k^{th}$  phases.

Consider  $M \gg f$  and  $N \gg g$ .

$\{let(x, y)$

**where**

$x \in M$

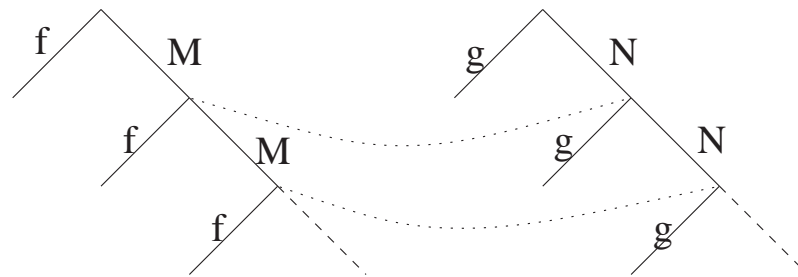
$y \in N\}$

$\gg$

$fst(\theta) \gg f \mid snd(\theta) \gg g$

## Phase Synchronization; Contd.

Synchronize  $M * f$  and  $N * g$ .



$\{let(x, y)$

**where**

$x \in M$

$y \in N\}$

\*

$fst(\theta) \gg f \mid snd(\theta) \gg g$

## Heat transfer computation over a grid

- Value  $x_{ij}$  at point  $(i, j)$  in a phase is the average of its neighbors' values in the previous phase.
- Site *average* returns the average of its arguments.
- Site *converge* returns its argument value if the values have converged sufficiently, otherwise, it remains silent.

$z \in \{let(x)$

where

$\langle \forall i, j :: x_{ij} \in average(\theta_{i-1,j}, \theta_{i+1,j}, \theta_{i,j-1}, \theta_{i,j+1}) \rangle$

}

\* *converge*( $\theta$ )

## Environment

An environment is a set of tuples. Each tuple has:

- a **name** (of a variable or  $\theta$ ),
- a *val* component (its value) and
- a *clock* component, the time at which this value was computed.

**Example:**  $p: \langle (x, \text{false}, 27), (y, \text{true}, 12), (\theta, 13, 20) \rangle$

$p.x.val = \text{false}$  and  $p.x.clock = 27$ .

An environment is a statement about a computation:  
the value of  $x$ , computed at time **27**, is *false*, and the value of  $y$ ,  
computed at time **12**,  $\dots$ .

## Relation over Bags of Environments

Each **expression** and **defn** is a binary relation over bags of environments.

Notation:  $P$ ,  $Q$  are bags of environments.  $\cup$  is bag union.

Write  $P f Q$  and  $P (z:\in f) Q$ .

**Coercion rule:** 
$$\frac{\langle \forall p : p \in P : \{p\} f Q_p \rangle}{P f \langle \cup p : p \in P : Q_p \rangle}$$

Consequently, we need only consider  $\{p\} f Q$ .

Note:  $\{\} f \{\}$

## Relation over Bags of Environments; Contd.

$\{p\} f Q$ : evaluation of  $f$  started in  $p$  at time  $p.\theta.\text{clock}$  yields **all** environments in  $Q$  in some computation.

$Q$  may be empty: non-terminating computation.

$Q$  may have duplicates: as in evaluating  $M \mid M$ .

## Example

$p$ :  $\langle (x, \text{false}, 27), (y, \text{true}, 12), (\theta, 13, 20) \rangle$ .

$\{p\}$   $\text{let}(x, y)$   $\{ \langle (x, \text{false}, 27), (y, \text{true}, 12), (\theta, (\text{false}, \text{true}), 27) \rangle \}$

$\{p\}$   $\text{timer}(2)$   $\{ \langle (x, \text{false}, 27), (y, \text{true}, 12), (\theta, \text{SIGNAL}, 22) \rangle \}$

$\{p\}$   $u \in \text{let}(x, y)$   
 $\{ \langle (x, \text{false}, 27), (y, \text{true}, 12), (u, (\text{false}, \text{true}), 27), (\theta, 13, 20) \rangle \}$

$\{p\}$   $\text{let}(z)$   $\{ \}$ , because  $z$  is not defined in  $p$ .

$\{p\}$   $u \in \text{let}(z)$   $\{p\}$

## Semantics of Term

Evaluate  $M(L)$  in environment  $p$ . Result is at most one environment.

- $x \in L$  and  $x \notin p$ : no result environment.
- Otherwise: call  $M$  with values  $p.x.val$  for all  $x$  in  $L$ , at maximum of  $p.\theta.clock$  and clock values of all parameters in  $L$ .
- If  $M$  responds with value  $v$  at time  $t$ , the result environment is  $q$ , where

$$q.x = p.x, \text{ for all } x \text{ in } p, x \neq \theta$$

$$q.\theta.val = v$$

$$q.\theta.clock = t$$



## Axioms about terms

Notation:  $p \setminus x$ : remove the tuple for  $x$  from  $p$ .  $x \notin p \Rightarrow p \setminus x = p$ .

- $\{p\} M(L) \{\}$ , if  $x \in L$  and  $x \notin p$ .
- Given  $\{p\} M(L) \{q\}$ :
  - $q.x = p.x$ , for all  $x$  in  $p$ ,  $x \neq \theta$ , and  $q.\theta.\text{clock} \geq p.\theta.\text{clock}$
  - if  $x \notin L$ , then  $\{p \setminus x\} M(L) \{q \setminus x\}$
  - if  $x \in L$ , let  $p' = p \setminus x$  except  
 $p'.\theta.\text{clock} = \max(p.\theta.\text{clock}, p.x.\text{clock})$ .  
 Then,  $\{p'\} M(L[x := p.x.\text{val}]) \{q \setminus x\}$
- (parameters may be renamed) For  $y \notin p$  and  $y \notin L$ ,

$$\{p\} M(L) \{q\} \equiv \{p[x := y]\} M(L[x := y]) \{q[x := y]\}$$

## Semantics of some sites

- $\{p\}$  *Fail*  $\{\}$

- $\{p\}$  *let*( $x, y$ )  $\{q\}$ :

$$q.\theta.val = (p.x.val, p.y.val)$$

$$q.\theta.clock = \max(p.\theta.clock, p.x.clock, p.y.clock)$$

In particular,  $\{p\}$  *let*( $\theta$ )  $\{p\}$ .

- $\{p\}$  *random*  $\{q\}$ :

$$q.\theta.val = \text{a number from the specified range}$$

$$q.\theta.clock = p.\theta.clock$$

## Semantics of some sites; Contd.

- $\{p\} \text{fst}(x, y) \{q\}$ :

$$q.\theta.val = p.x.val$$

$$q.\theta.clock = \max(p.\theta.clock, p.x.clock, p.y.clock)$$

- $\{p\} \text{timer}(t) \{q\}$ :

$$q.\theta.val = \text{SIGNAL}$$

$$q.\theta.clock = p.\theta.clock + t$$

## Exercise: Properties of the timer

$$\begin{aligned} x \in \text{timer}(t) \mid \text{timer}(u) &\equiv x \in \text{timer}(t), \text{ given } t \leq u, \\ \text{timer}(t) \gg \text{timer}(u) &\equiv \text{timer}(t + u) \end{aligned}$$

## Semantics of Defn

- $$\frac{\{p\} f \{ \}}{\{p\} (z:\in f) \{p\}}$$
- $$\frac{\{p\} f Q, q \text{ ismin } Q}{\{p\} (z:\in f) \{p + (z, q.\theta)\}}$$

$q \text{ ismin } Q$ :  $q \in Q$  and  $q.\theta.\text{clock} \leq r.\theta.\text{clock}$  for every  $r$  in  $Q$ .

$+$  denotes expansion of an environment by a tuple.

## Semantics of Expression

$$\bullet \frac{\{p\} (x:\in g) \{q\}, \{q\} f Q}{\{p\} \{f \text{ where } x:\in g\} (Q \setminus x)}$$

$$\bullet \frac{\{p\} f Q, \{p\} g R}{\{p\} (f \mid g) (Q \cup R)}$$

$$\bullet \frac{\{p\} f Q, Q g R}{\{p\} (f \gg g) R}$$

$$\bullet \frac{\{p\} f Q, Q f^* R}{\{p\} f^* (\{p\} \cup R)}$$

$$f * g \equiv f^* \gg g$$

$$f^* \equiv f * \mathbf{1}, \text{ where } \mathbf{1} = \text{let}(\theta).$$

## Notes

- $\gg$  is relational composition.
- $|$  is **not** relational union,

$P f Q$  does not imply  $P (f | g) Q$ .

Under relational union  $M$  and  $M | M$  would be identical. We treat them differently.

# Kleene Algebra

(Zero and  $|$ )

$$f | \mathbf{0} = f$$

(Commutativity of  $|$ )

$$f | g = g | f$$

(Associativity of  $|$ )

$$(f | g) | h = f | (g | h)$$

(Idempotence of  $|$ )

$$f | f = f$$

(Associativity of  $\gg$ )

$$(f \gg g) \gg h = f \gg (g \gg h)$$

(Left zero of  $\gg$ )

$$\mathbf{0} \gg f = \mathbf{0}$$

(Right zero of  $\gg$ )

$$f \gg \mathbf{0} = \mathbf{0}$$

(Left unit of  $\gg$ )

$$\mathbf{1} \gg f = f$$

(Right unit of  $\gg$ )

$$f \gg \mathbf{1} = f$$

(Left Distributivity of  $\gg$  over  $|$ )

$$f \gg (g | h) = (f \gg g) | (f \gg h)$$

(Right Distributivity of  $\gg$  over  $|$ )

$$(f | g) \gg h = (f \gg h) | (g \gg h)$$

(Recursive Expansion of Kleene star)

$$f^* = \mathbf{1} | f \gg f^*$$



## Corollaries

(Left Distributivity of  $*$  over  $|$ )  $f * (g | h) = (f * g | f * h)$   
 (Regrouping  $*$  over  $\gg$ )  $(f * g) \gg h = f * (g \gg h)$

## Additional Properties of Non-flat Expressions

- (Narrowing the scope) Given that  $x$  is not free in  $g$ :

$$\begin{aligned} \{g \text{ where } x:\in h\} &= g \\ \{f \mid g \text{ where } x:\in h\} &= \{f \text{ where } x:\in h\} \mid g \\ \{f \gg g \text{ where } x:\in h\} &= \{f \text{ where } x:\in h\} \gg g \end{aligned}$$

- (Bound variable renaming) In the following,  $y$  is not free in  $f$  or  $g$ .

$$\{f \text{ where } x:\in g\} = \{f[x := y] \text{ where } y:\in g\}$$

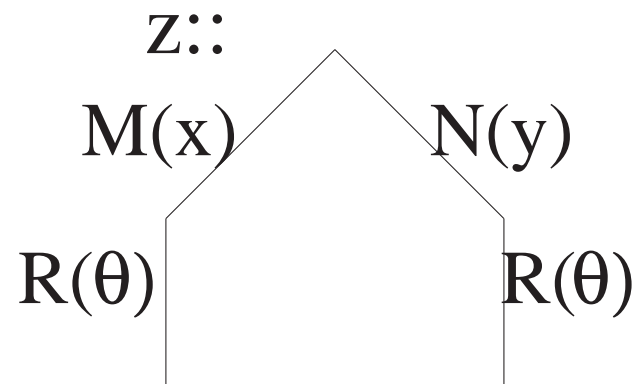
- (Independent defn)  $y$  is not free in  $g$  and  $x$  is not free in  $h$ .

$$\begin{aligned} & \{\{f \text{ where } x:\in g\} \text{ where } y:\in h\} \\ = & \{\{f \text{ where } y:\in h\} \text{ where } x:\in g\} \end{aligned}$$

## Implementation

- **Compile** the statement into a (set of) finite state automata
- **explore** the automata to compute the goal variable.

For  $z \in (M(x) \mid N(y)) \gg R(\theta)$



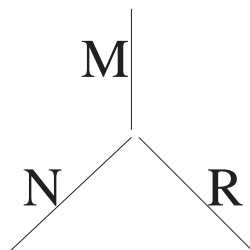
## Difficulties In Automata construction

- How to compile a non-flat expression:

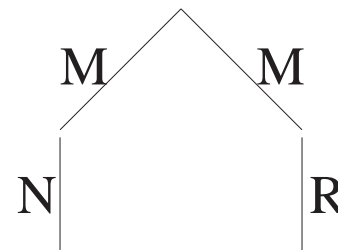
$$z:\in \{M(x) \mid N \text{ where } x:\in R\}$$

- We cannot use standard procedures. In automata theory,

$$M \gg (N \mid R) \equiv M \gg N \mid M \gg R$$



(a)



(b)

- **Non-determinism:**  $M$  is called twice in  $M \gg N \mid M \gg R$ .



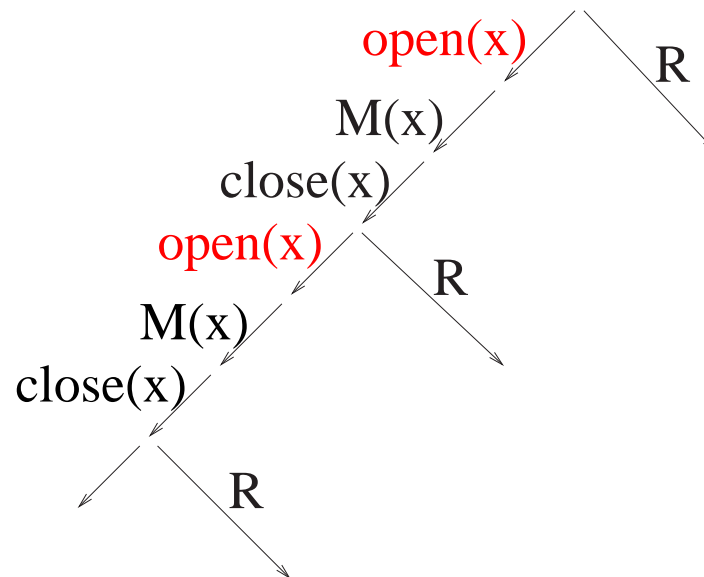
## *open* and *close*

- *open* and *close* are treated differently from usual sites.
- Calling *open*( $x$ ) starts a new computation of  $x$  on a **clone**.
- A state includes the values for the global variables and  $\theta$ , but **reference** to **clone** for local variable.
- *close*( $x$ ) removes **reference** to  $x$  from the state.

# clone

Several clones of an fsa may be simultaneously in existence.

$z \in \{ M(x) \text{ where } x \in N \} * R$  has the flat program  
 $z \in \langle \text{open}(x) \gg M(x) \gg \text{close}(x) \rangle * R$   
 $x \in N$



## Finite State Automata (fsa) from Flat Program

- An Orc fsa is a finite directed graph.
- Its edges are labeled with terms (including *open* and *close*).
- A pair of nodes in the fsa may have multiple edges between them, possibly with the same label.
- Two distinguished nodes: **begin** and **end**.
- No incoming edge to **begin** node; no outgoing edge of **end**.
- For every edge, there is a path from **begin** to **end** that includes the edge.



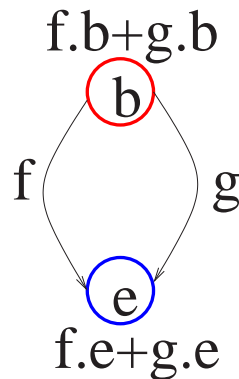
## Recursive fsa Construction

**merge** of  $x$ ,  $y$  is  $x + y$ : incoming, outgoing edges of  $x$ ,  $y$ .

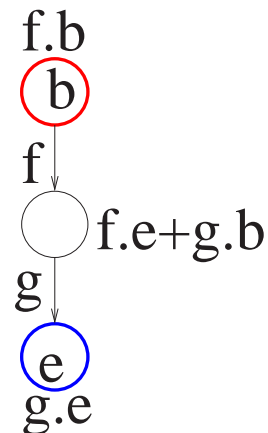
$f.b$  and  $f.e$  for **begin** and **end** nodes of fsa for  $f$ .

term  $M(L)$ : The fsa has one edge from **begin** to **end**, labeled  $M(L)$ .

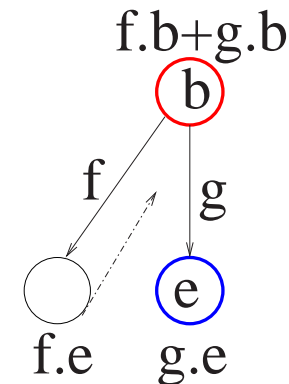
For edge from  $(f * g).b$  to  $x$  with label  $r$ : make edge  $(f.e, x)$  with  $r$ .



(a)  $f \mid g$

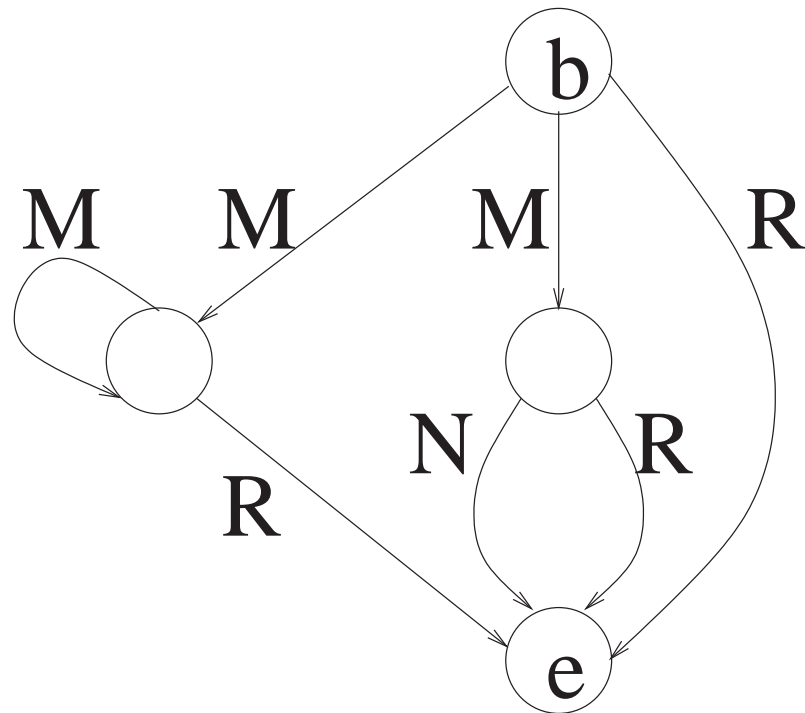


(b)  $f \gg g$



(c)  $f * g$

fsa for  $M \gg (N \mid R) \mid M * R$



## Notes on fsa

- Traditional deterministic fsa construction is P-space complete.  
Orc fsa construction is linear (see exercise).
- Each fsa can be unrolled to a (possibly infinite) tree.

## fsa exploration; token

- **token** associated with an edge of a clone of an fsa.
- Corresponds to a single step of computation.
- **token** has a **state** and a **parent**. parent is a token in the same clone or *NIL*.
- token processes label  $M(x)$  on edge  $e$ :  
wait until  $x$  has a value, then call site  $M$ . On receiving  $v$  from  $M$ :
  - creates children tokens on all successor edges of  $e$ ,
  - bequeaths to them the state with  $\theta$  value  $v$ ,
  - if  $e$  has no successor edge, reports  $v$  as the value of the computation.

## fsa exploration; token; contd.

Assume: single outgoing edge from each **begin** node (**begin edge**).

Ensure by adding an edge with label **1**.

- The token processes label  $open(x)$  on edge  $e$ :
  - initiate computation on a clone of the fsa for  $x$
  - return reference to the clone as part of the state
- $close(x)$ : remove the reference to the clone for  $x$  from the state.
- To initiate computation on a clone in state  $s$ :  
place a token on its **begin edge** with state  $s$  and  $NIL$  parent.

## Site

- Any level of granularity in site, from simple message transmission to business-business transaction involving many servers
- May spawn new processes, start servers and change database contents
- May interact with peripheral devices, including displays and keyboards

## Site Specification

The specification of site  $M$  is a predicate  $p$  over a triple  $(x, y, t)$ :

$x$  is the value of actual parameters,

$y$  is the result returned by  $M$ ,

$t$  is an (absolute) time instant.

## Two stage Site Operation

A site operates in two stages.

- **response**: Client calls. Site returns  $y$  or remains silent.  
also returns a **pledge** which is invisible to the Orc statement.  
pledge carries a **deadline** by which it should be committed.
- **commit**: If the caller commits the pledge at time  $t$  before the deadline,  
the site executes its commit stage,  
which establishes predicate  $p(x, y, t)$ .



## Examples of Site Specification

- Function  $f$ :  
returns  $y$  where  $y = f(x)$ . The deadline is irrelevant.  
predicate:  $y = f(x)$ . No commitment needed.
- Site *postOffice*:  
called with description of a parcel and returns  $y$ , the cost of delivery.  
The deadline is the instant  $t$ , the time of response.  
predicate: the cost of delivery of  $x$  at time  $t$  is  $y$ .  
needs no commitment to establish this predicate.

## Examples of Site Specification; Contd.

- Object *count*: has an integer value *count.v* and two methods, *incr* and *read*.
- Initially,  $count.v = 0$ .
- *incr*:  $count.v := count.v + 1$ ;  
*read*: returns  $v = count.v$ .  
The moment of response, *t*, is the deadline.
- predicate:  $count.v \geq v$  beyond *t*. No commitment required.
- Another spec:  $count.v = v$  at the moment of commitment *s*,  $s \leq t$ .  
Implement specification by: lock *count.v* until the moment of commitment or *t*, whichever comes first.

## Examples of Site Specification; Contd.

- Transaction sells 80 shares of stock *pqr* if price is above \$25 **and** buys 100 shares of stock *abc* if it is below \$20 a share.
- Both price conditions are met before the transaction returns a signal.
- The deadline is very short.
- If the client commits within the deadline, establishes the predicate:  
**client has bought and sold the requisite number of shares for the given prices at the moment of commitment.**

## Examples of Site Specification; Contd.

- A site call may cause state change during the response stage.
- An airline issues a price quote and changes its state during the response stage, even if no call commits.
- Its only obligation is to issue a ticket at the given price if the client commits within the deadline.

## Type of pledge

- **Instant pledge:** site makes immediate commitment for the caller  
The pledge can only be revoked next.  
Calling site *email* sends an email, without waiting for commitment.  
Common in concurrent computing.
- **Deferred pledge:** Commitment by deadline establishes the associated predicate.

## Structure of pledge

A pledge has:

- An **id**,
- A **deadline**, an absolute time instant by which it has to be committed or revoked (if already committed),
- A set of **pertinent arguments**, the arguments of the site call which must be committed in order to commit this pledge.

$$z: \in \text{Min}(x, y)$$

where

$$x: \in A$$

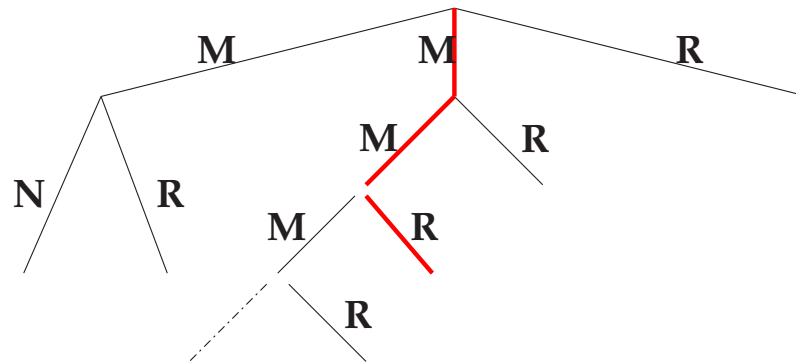
$$y: \in B$$

Pertinent argument from  $\text{Min}$  is the cheaper of  $x$  and  $y$ .

## Critical Path

Response from a terminal edge,  $e$ , is assigned as the variable value.

Edge  $e$  defines the **Critical Path**.



Commit all pledges and their pertinent pledges along the critical path.  
Transitive closure needed.

Revoke all other instant pledges.

## Definitions

For any clone  $c$ :

$child(c)$  =  $\{d \mid d \text{ is a clone spawned by an } open \text{ in } c\}$

$pert(c)$  =  $\{d \mid d \text{ is not a global variable or } \theta, \text{ and } d \text{ is a pertinent argument in a deferred pledge along } c\text{'s critical path}\}$

$instant(c)$  = instant pledge ids received in  $c$

$deferred(c)$  = deferred pledge ids received in  $c$

$instant^+(c)$  = instant pledge ids received along the critical path in  $c$

$deferred^+(c)$  = deferred pledge ids received along the critical path in  $c$



## Transitive closure definitions

For any clone  $c$ :

$all\_instant(c)$  instant pledge ids received in  $c$  and its descendants.

$all\_deferred(c)$  deferred pledge ids received in  $c$  and its descendants.

$all\_instant^+(c)$  and  $all\_deferred^+(c)$ , limited to the critical paths.

$$\begin{aligned}
 all\_instant(c) &= instant(c) \\
 &\quad \cup \langle \cup d : d \in child(c) : all\_instant(d) \rangle \\
 all\_deferred(c) &= deferred(c) \\
 &\quad \cup \langle \cup d : d \in child(c) : all\_deferred(d) \rangle \\
 all\_instant^+(c) &= instant^+(c) \\
 &\quad \cup \langle \cup d : d \in pert(c) : all\_instant^+(d) \rangle \\
 all\_deferred^+(c) &= deferred^+(c) \\
 &\quad \cup \langle \cup d : d \in pert(c) : all\_deferred^+(d) \rangle
 \end{aligned}$$

## Pledges to be Committed, Revoked

For any clone  $c$ :

$all^+(c)$ : pledges necessary and sufficient to commit  $c$  (received by  $c$  and its descendants).

$$all^+(c) = all\_instant^+(c) \cup all\_deferred^+(c)$$

Instant pledges in  $all^+(c)$  are already committed.

$pos(c)$ : pledges which remain to be committed in  $all^+(c)$ ; i.e.,

$$pos(c) = all\_deferred^+(c)$$

$neg(c)$ : pledges which need to be revoked, i.e., already committed and not part of  $all^+(c)$ ,

$$\begin{aligned} neg(c) &= all\_instant(c) - all^+(c), \text{ i.e.,} \\ neg(c) &= all\_instant(c) - all\_instant^+(c) \end{aligned}$$

## Commitment and Revocation Algorithm

- Client: sends to the appropriate sites,  
 $commit_{init}$  for pledges in  $pos(c)$ ,  
 $revoke_{init}$  for pledges in  $neg(c)$
- Site: responds with  
 $ack_{init}$  if it is ready to commit/revoke the pledge or  
 $nack_{init}$  if it can not.  
Treat failure to respond (timely) as  $nack_{init}$ .

## Commitment and Revocation Algorithm; Contd.

- Client:
  - If all responses are  $ack_{init}$ , sends  $commit_{final}$  to sites corresponding to  $pos(c)$  and  $revoke_{final}$  to sites corresponding to  $neg(c)$ .
  - Otherwise, sends  $abort_{final}$  to all sites.
- Site:
  - commit after receiving  $commit_{final}$
  - revoke after receiving  $revoke_{final}$ ,
  - recovery computation after receiving  $abort_{final}$ .

## Explicit Commit and Revoke

To handle transactions of differing deadlines.

Commit and Revoke sites.

A successful call to *Commit*( $x, y$ ):

returns *true* and guarantees that  $x$  and  $y$  are committed with their pertinent variables. Similarly, *Revoke*.

Make explicit the commit in  $z:\in f$

$$z:\in \left\{ \begin{array}{l} \textit{Commit}(y) \gg \textit{let}(y) \\ \textbf{where} \\ y:\in f \end{array} \right\}$$

## Example

Reserve a hotel room and an airline ticket.

The hotel responds after a long delay but gives a long deadline.

The airline usually responds quickly but gives a short deadline.

### Strategy

- Contact the hotel. After it responds, contact the airline.
- Airline responds before the hotel's deadline:
  - if its quote is excessively high: cancel vacation plan and assign  $\infty$  to the goal variable.
  - Otherwise: commit to both and return the sum of the quotes as the goal variable value.

## Example; Contd.

- Airline does not respond before the hotel's deadline: commit to the hotel; and wait 1 unit for the airline response.
  - Airline responds before the new deadline:
    - if its quote is excessive**: revoke the hotel commitment and cancel vacation plans.
    - Otherwise**: commit to the airline and return the sum of the quotes as the goal variable value.
  - Airline does not respond before the new deadline: revoke the hotel commitment and cancel vacation plans.

## Example; Contd.

$$z:\in$$

$$\left\{ \begin{array}{l} exc(a) \gg let(\infty) \\ | \neg exc(a) \gg Commit(a, h) \gg Plus(a, h) \\ | let(t_0) \gg Commit(h) \gg \\ \quad ( ( exc(a) | let(t_1)) \gg Revoke(h) \gg let(\infty) \\ \quad | \neg exc(a) \gg Commit(a) \gg Plus(a, h) \\ \quad ) \end{array} \right.$$

**where**

$$\begin{array}{l} (h, d) :\in Hotel \\ a \quad :\in let(h) \gg Airline \\ t_0 \quad :\in timer(d) \\ t_1 \quad :\in let(t_0) \gg timer(1) \end{array}$$

$$\left. \vphantom{\begin{array}{l} (h, d) :\in Hotel \\ a \quad :\in let(h) \gg Airline \\ t_0 \quad :\in timer(d) \\ t_1 \quad :\in let(t_0) \gg timer(1) \end{array}} \right\}$$