Proof Systems, Large Functions and Combinatorics

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This short course will introduce and develop some basic ideas and results from mathematical logic which underpin fundamental concepts in computation, and which, quite possibly, will arise (explicitly or implicitly) in other courses at the summer school.

An existential statement with input-parameter x asserts the existence of an output witness y satisfying the required relationship to x. A proof of the statement contains information necessary to compute y from x, and furthermore the complexity of that computation should be reflected, somehow, in the complexity of the proof. Classical methods of proof theory provide uniform ways of extracting this information via well-known hierarchies of bounding functions (these are the *large* functions of the title). Somewhat surprisingly, they in turn have natural combinatorial significance, being intimately related to Ramsey's Theorem and other combinatorial results which, at first sight, have nothing to do with proof theory.

These connections will be explored and illustrated via a range of interrelated examples.

References

1. R.L. Graham, B.L. Rothschild and J.H. Spencer. *Ramsey Theory.* Wiley Interscience 1990.
2. A.S. Troelstra and H. Schwichtenberg. *Basic Proof Theory.* 2nd Ed., Cambridge Tracts in Theor. Comp. Sci., CUP 2000.
3. S. Wainer. *Basic Proof Theory with Applications to Computation.* H. Schwichtenberg (Ed) Logic of Computation, Springer ASI Series F 157, 1997, 349-394.