

Verification, Performance Analysis and Controller Synthesis of Real Time Systems

— *using UPPAAL* —

Kim Guldstrand Larsen



BRICS
Basic Research
in Computer Science



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

UPPAAL Branches

- Real Time Verification

CLASSIC

- Real Time Scheduling & Performance Evaluation

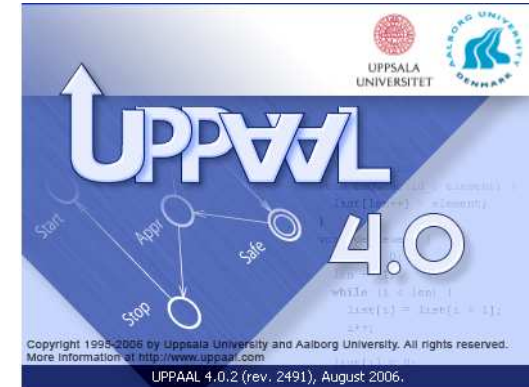
CORA

- Real Time Controller Synthesis

TIGA

- Real Time Testing

TRON



Modeling Formalism
 Theory
 Alg.& Datastr.
 Applications
 Open Problems
 DEMO's

Slides, Reading Material, Challenges,..

www.cs.aau.dk/~kgl/Marktoberdorf08

The screenshot shows a Mozilla Firefox browser window with the address bar containing <http://www.cs.aau.dk/~kgl/Marktoberdorf08/>. The slide content includes:

Real Time Systems

Verification, Performance Analysis, and Controller Synthesis

using UPPAAL

Below the text are six images: three yellow cartoon characters, a black and white portrait of a man with glasses, and two more yellow cartoon characters.

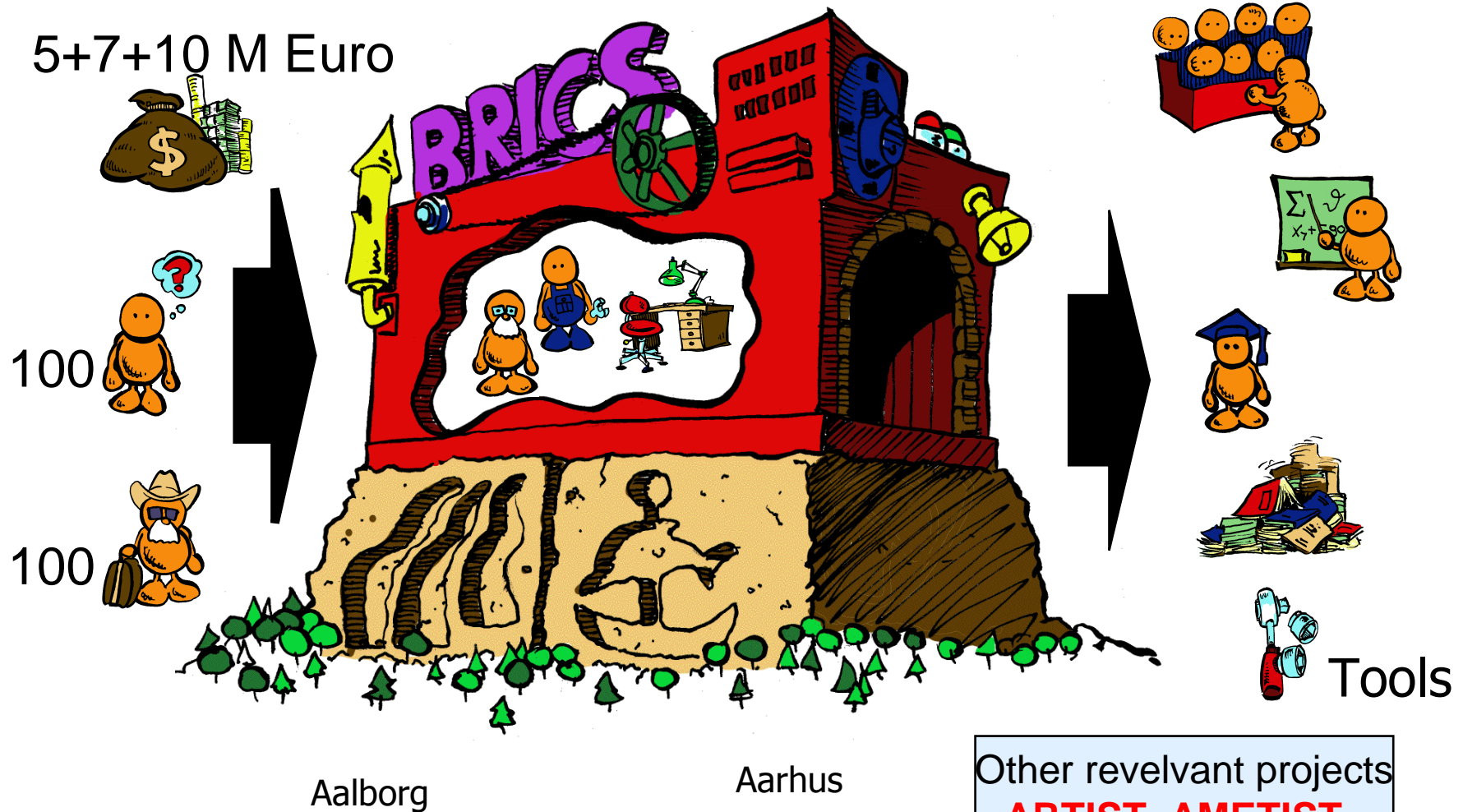
Kim Guldstrand Larsen
 CISS, Aalborg University, DENMARK

International Summer School
 Marktoberdorf
 August 5-17, 2008

.../Material.html

BRICS Machine

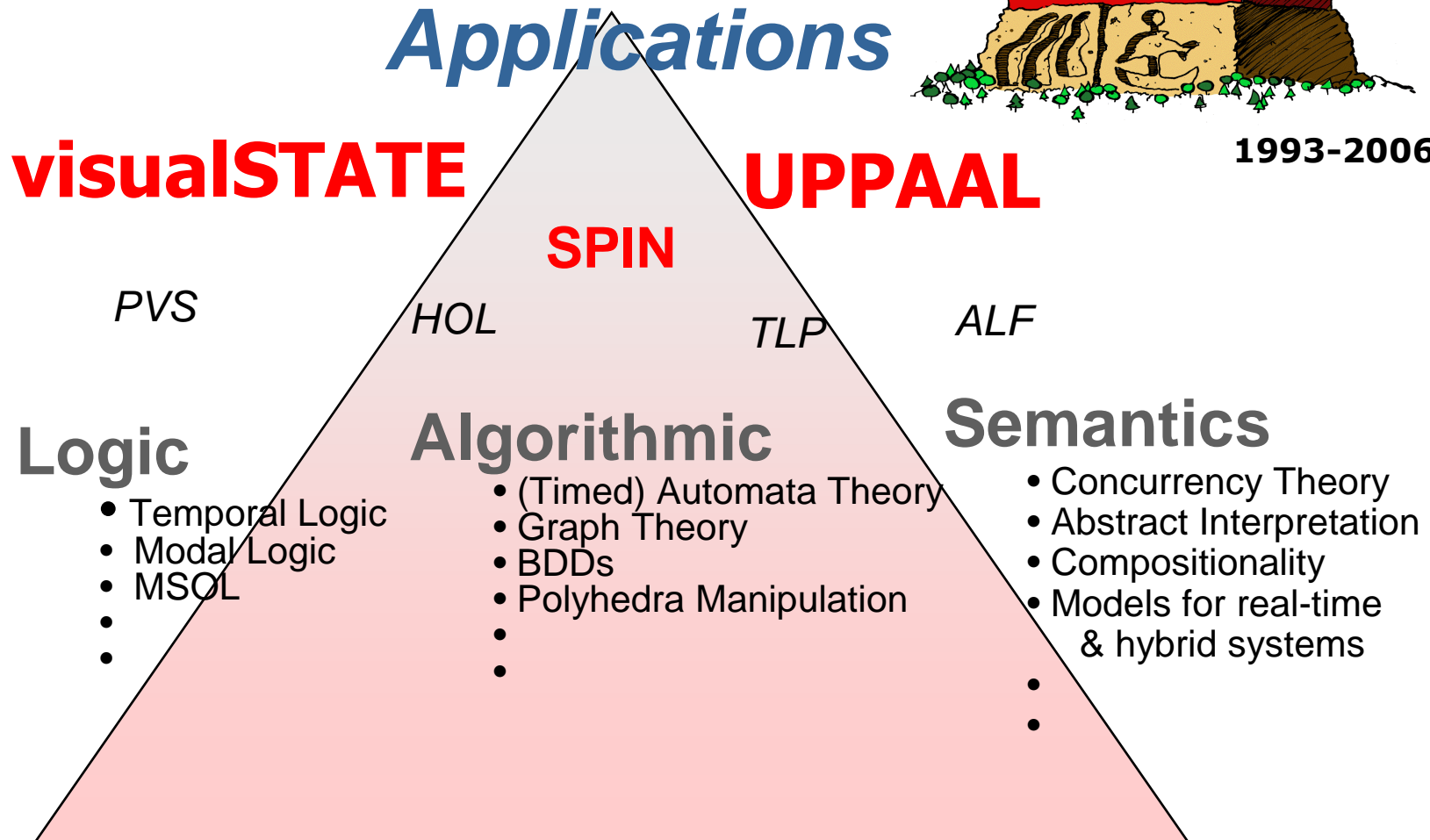
Basic Research in Computer Science, 1993-2006



Tools and BRICS



1993-2006



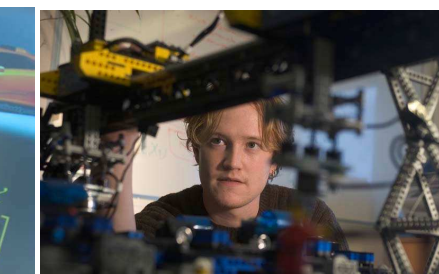
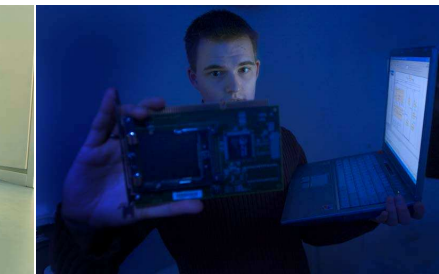
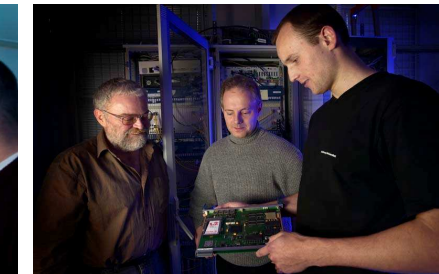
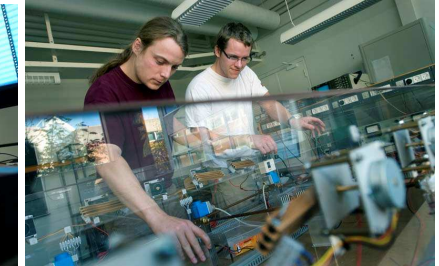
CISS

Center for Embedded Software Systems

- National Competence Center 2002 sponsored by:

- Ministry of Tech. & Res.
- North Jutland
- Aalborg City
- Aalborg University

- 40 projects
- 20 CISS employees
- 25 CISS ass. Res.
- 20 industrial PhDs





European Network of Excellence



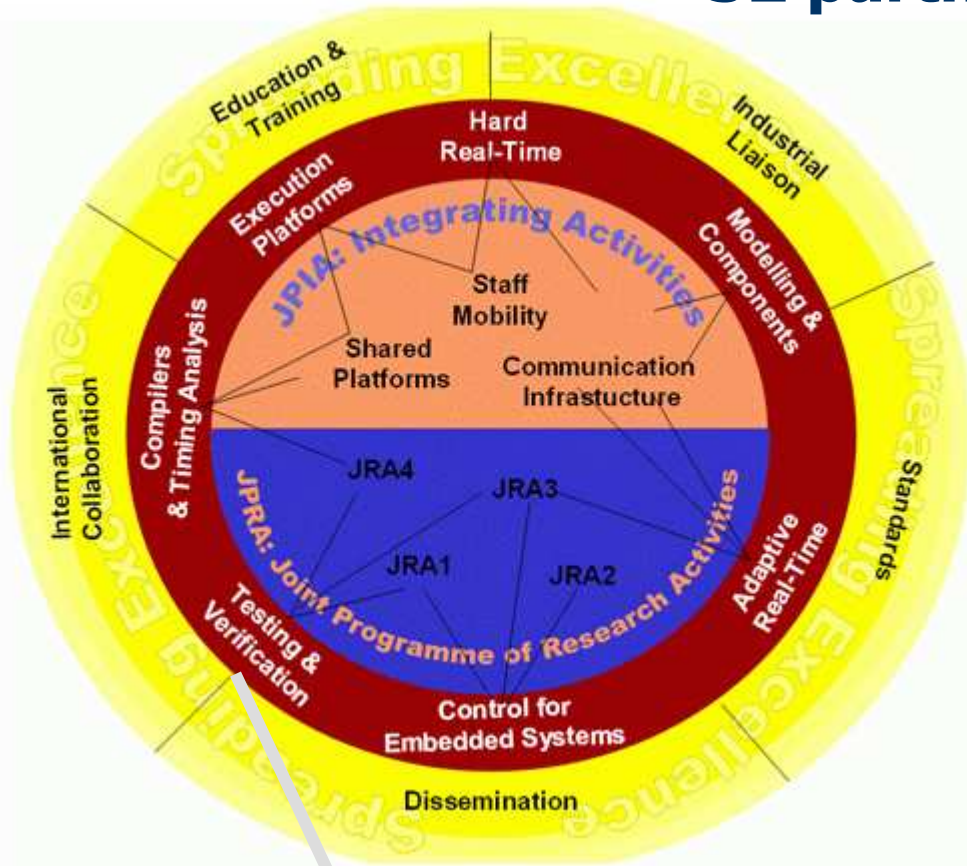
32 partners



ARTEMIS



Joseph Sifakis
Co-winner of Turing Award 2007
ARTIST Director



Testing & Verification
CISS coordinator

Why Verification and Testing

- 30-40% of production time is currently spend on elaborate, ad-hoc testing:
 - Errors expensive and difficult to fix!
 - The potential of existing/improved testing methods and tools is enormous!
 - Time-to-market may be shortened considerable by verification and performance analyses of early designs!

Why Verification and Testing

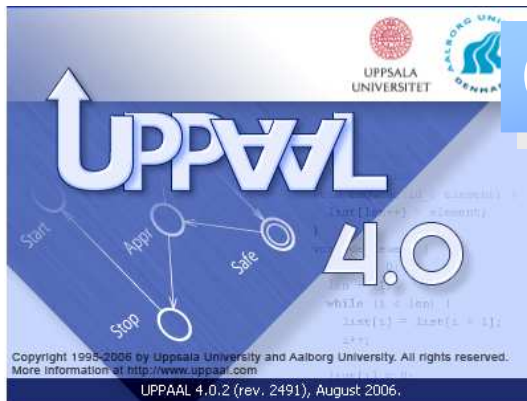
■ IMPORTANCE for EMBEDDED SYSTEMS

- Often safety critical
- Often economical critical
- Hard to patch



■ CHALLENGES for EMBEDDED SYSTEMS

- Correctness of embedded systems depend crucially on use of *resources*
 e.g. real-time, memory, bandwidth, energy.
- Need for *quantitative models*



CLASSIC

Modelling, Specification, and Verification

— *using UPPAAL* —

Kim Guldstrand Larsen



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Collaborators

@UPPsala

- Wang Yi
- Paul Pettersson
- John Håkansson
- Anders Hessel
- Pavel Krcal
- Leonid Mokrushin
- Shi Xiaochun



@AALborg

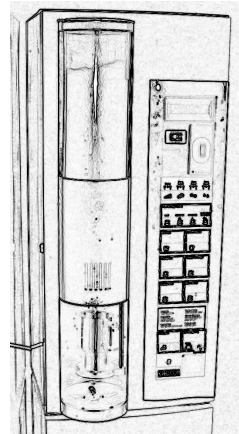
- Kim G Larsen
- Gerd Behrman
- Alexandre David
- Jacob I. Rasmussen
- Brian Nielsen
- Arne Skou
- Marius Mikucionis
- Thomas Chatain



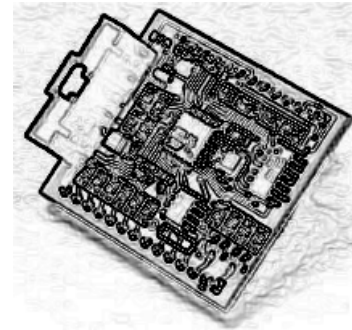
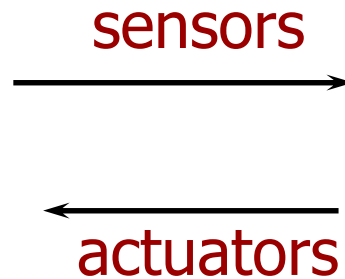
@Elsewhere

- Emmanuel Fleury, Didier Lime, Johan Bengtsson, Fredrik Larsson, Kåre J Kristoffersen, Tobias Amnell, Thomas Hune, Oliver Möller, Elena Fersman, Carsten Weise, David Griffioen, Ansgar Fehnker, Frits Vandraager, Theo Ruys, Pedro D'Argenio, J-P Katoen, Jan Tretmans, Judi Romijn, Ed Brinksma, Martijn Hendriks, Klaus Havelund, Franck Cassez, Magnus Lindahl, Francois Laroussinie, Patricia Bouyer, Augusto Burgueno, H. Bowmann, D. Latella, M. Massink, G. Faconti, Kristina Lundqvist, Lars Asplund, Justin Pearson...

Real Time Systems



Plant
Continuous



Controller Program
Discrete

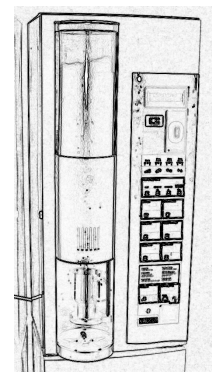
Eg.: Realtime Protocols
Pump Control
Air Bags
Robots
Cruise Control
ABS
CD Players
Production Lines

Real Time System

A system where correctness not only depends on the logical order of events but also on their **timing!!**

Real Time Model Checking

Plant
Continuous



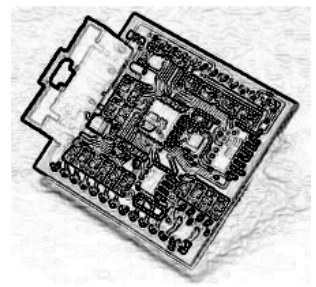
sensors



actuators

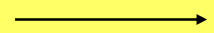
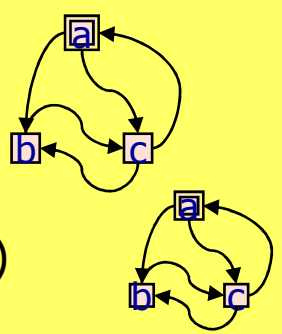


Controller Program
Discrete

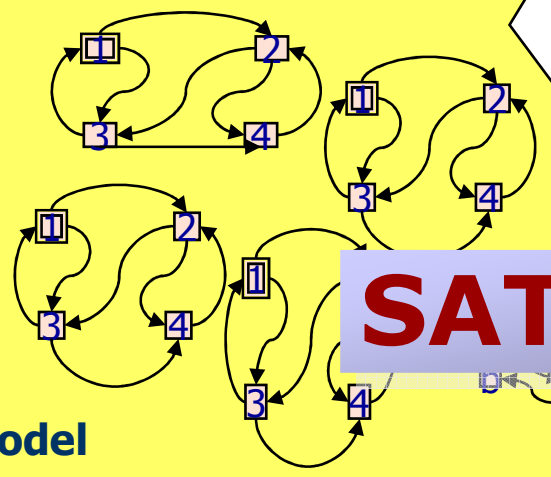


Model
of
tasks
(automatic?)

Model
of
environment
(user-supplied /
non-determinism)



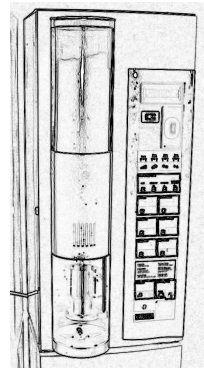
UPPAAL Model



SAT \emptyset ??

Real Time Control Synthesis

Plant
Continuous



sensors

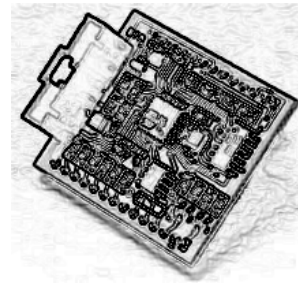


actuators



Controller Program

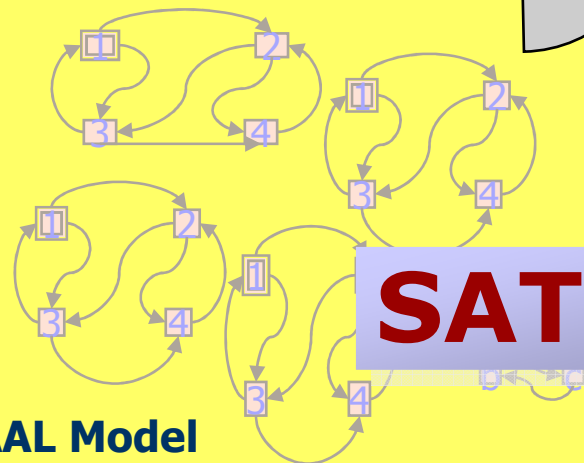
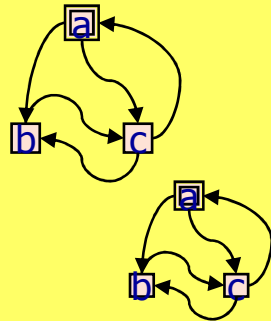
Discrete



Synthesis
of
tasks
(automatic)



Model
of
environment
(user-supplied)



Partial UPPAAL Model

SAT \emptyset !!

Timed Automata

Alur & Dill 1989



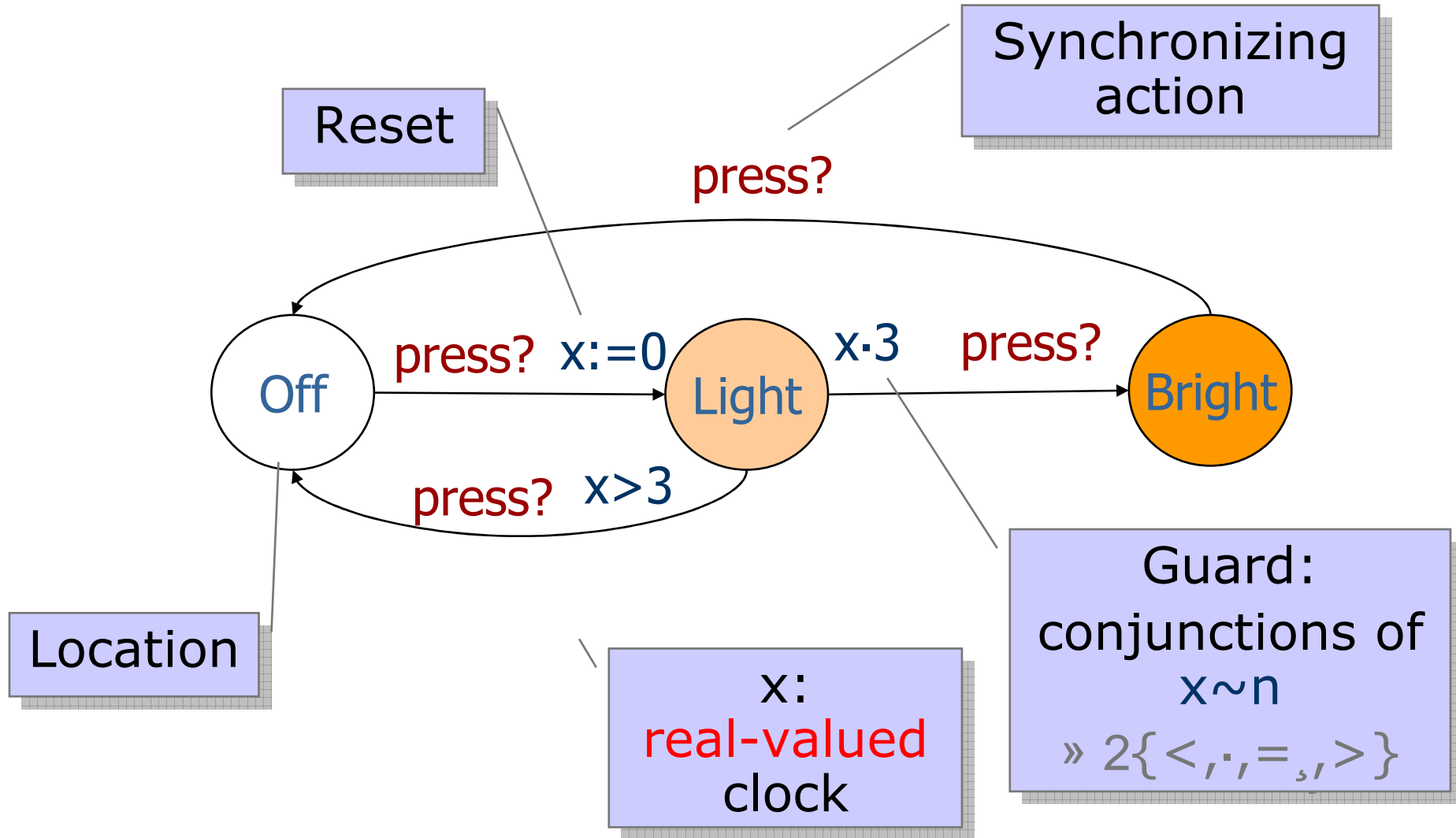
BRICS

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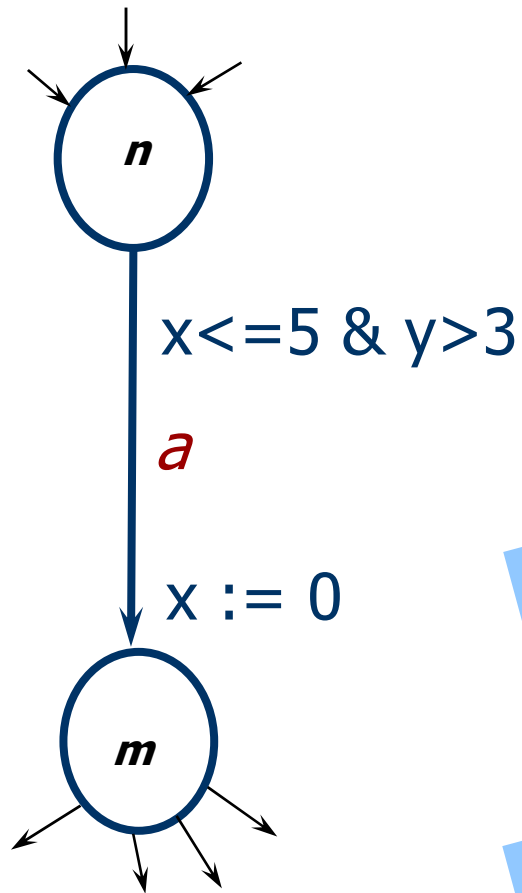


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Timed Automata



Timed Automata *semantics*



State

(*location* , $x=v$, $y=u$) where v,u are in \mathbf{R}

Transitions

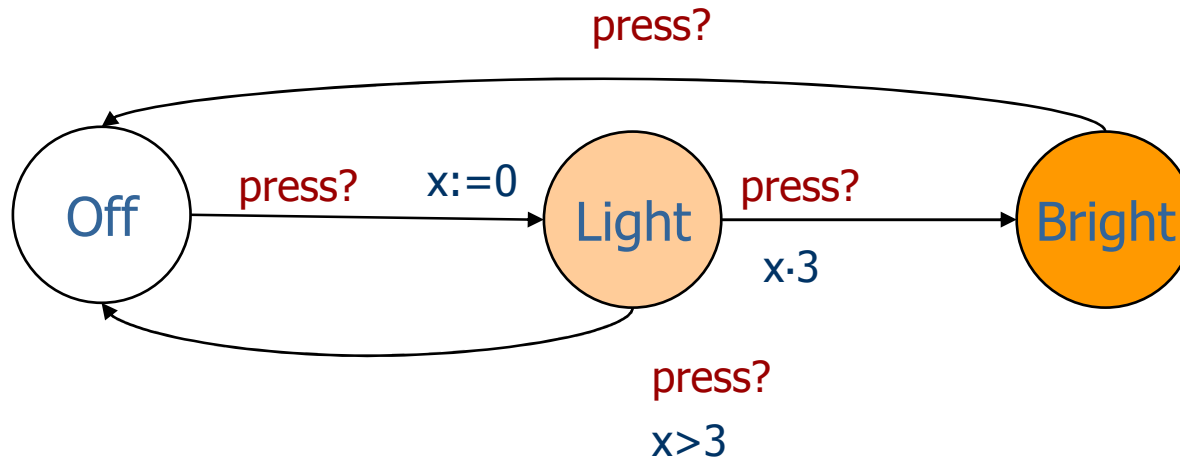
(n , $x=2.4$, $y=3.1415$) \xrightarrow{a} (m , $x=0$, $y=3.1415$)

Discrete Trans

(n , $x=2.4$, $y=3.1415$) $\xrightarrow{e(1.1)}$ (n , $x=3.5$, $y=4.2415$)

Delay Trans

Timed Automata

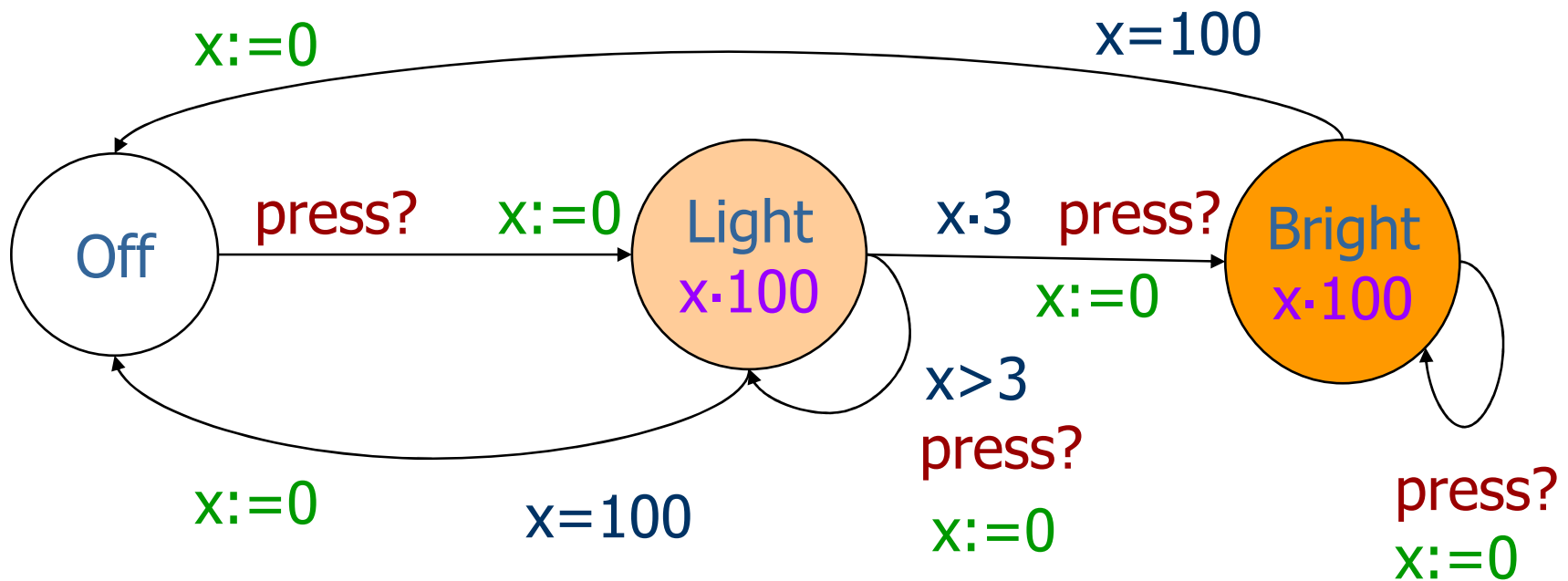


Transitions:

	→	(Off , x=0)
delay 4.32	→	(Off , x=4.32)
press?	→	(Light , x=0)
delay 2.51	→	(Light , x=2.51)
press?	→	(Bright , x=2.51)

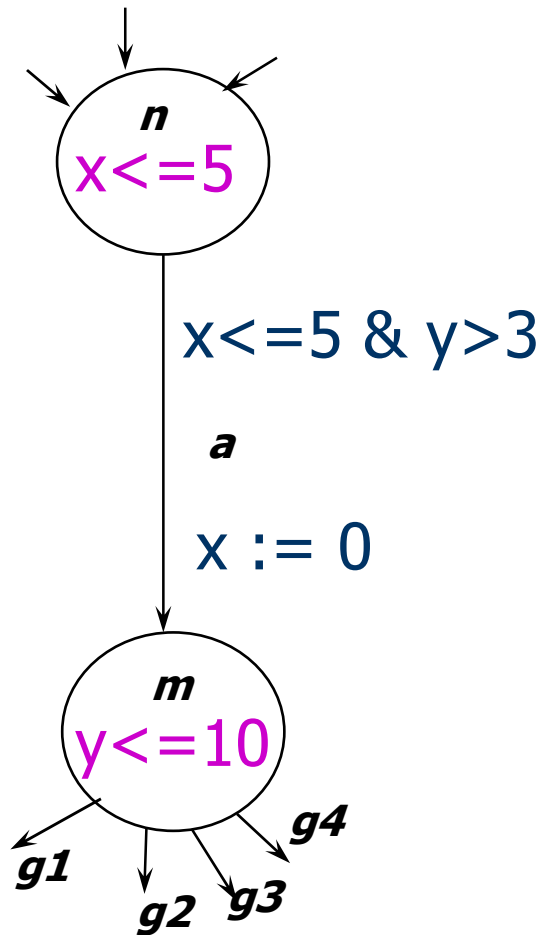
Intelligent Light Control

Using Invariants



Timed Automata *semantics*

Invariants



Transitions

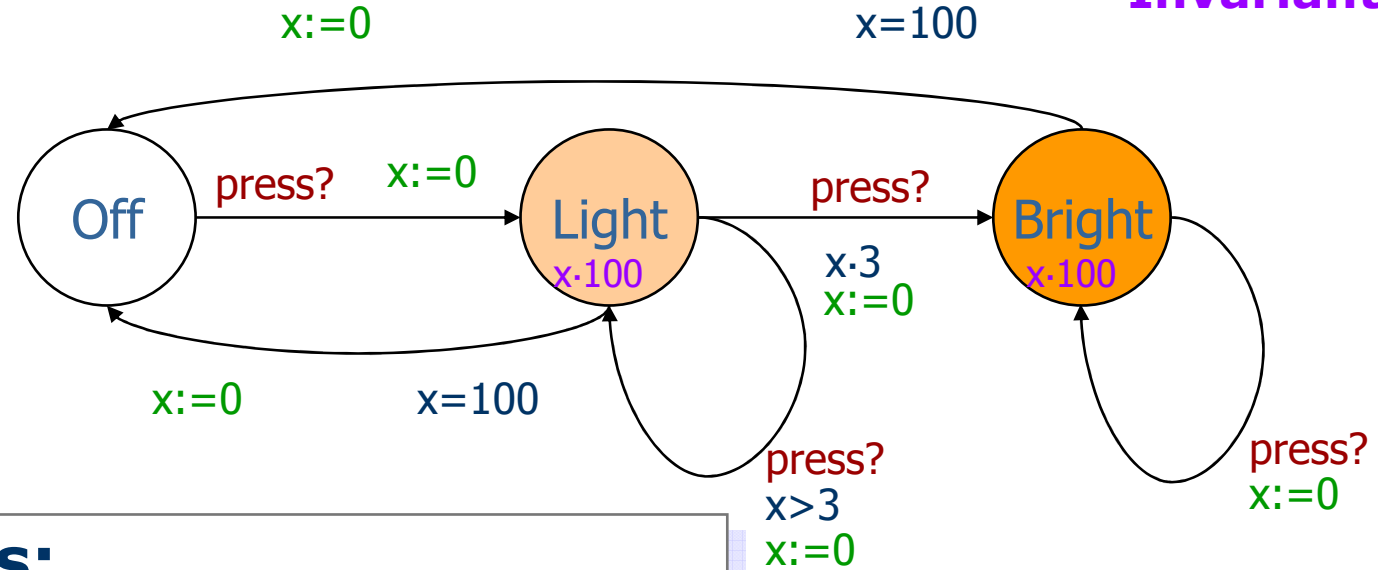
$$(n, x=2.4, y=3.1415) \xrightarrow{e(3.2)}$$

$$(n, x=2.4, y=3.1415) \xrightarrow{e(1.1)} (n, x=3.5, y=4.2415)$$

Invariants ensure progress !!

Intelligent Light Control

Invariants



Transitions:

- (Off , x=0)
- delay 4.32 → (Off , x=4.32)
- press? → (Light , x=0)
- delay 4.51 → (Light , x=4.51)
- press? → (Light , x=0)
- delay 100 → (Light , x=100)
- τ → (Off , x=0)

Invariants ensures progress

Note:
 (Light , x=0) delay ~~X~~103 →

Constraints

Definition

Let X be a set of clock variables. The set $\mathcal{B}(X)$ of *clock constraints* ϕ is given by the grammar:

$$\phi ::= x \leq c \mid c \leq x \mid x < c \mid c < x \mid \phi_1 \wedge \phi_2$$

where $c \in \mathbb{N}$ (or \mathbb{Q}).

Clock Valuations and Notation

Definition

The set of *clock valuations*, \mathbb{R}^C is the set of functions $C \rightarrow \mathbb{R}_{\geq 0}$ ranged over by u, v, w, \dots

Notation

Let $u \in \mathbb{R}^C$, $r \subseteq C$, $d \in \mathbb{R}_{\geq 0}$, and $g \in \mathcal{B}(X)$ then:

- $u + d \in \mathbb{R}^C$ is defined by $(u + d)(x) = u(x) + d$ for any clock x
- $u[r] \in \mathbb{R}^C$ is defined by $u[r](x) = 0$ when $x \in r$ and $u[r](x) = u(x)$ for $x \notin r$.
- $u \models g$ denotes that g is satisfied by u .

Timed Automata

Definition

A timed automaton A over clocks C and actions Act is a tuple (L, l_0, E, I) , where:

- L is a finite set of locations
- $l_0 \in L$ is the initial location
- $E \subseteq L \times \mathcal{B}(X) \times Act \times \mathcal{P}(C) \times L$ is the set of edges
- $I : L \rightarrow \mathcal{B}(X)$ assigns to each location an invariant

Semantics

Definition

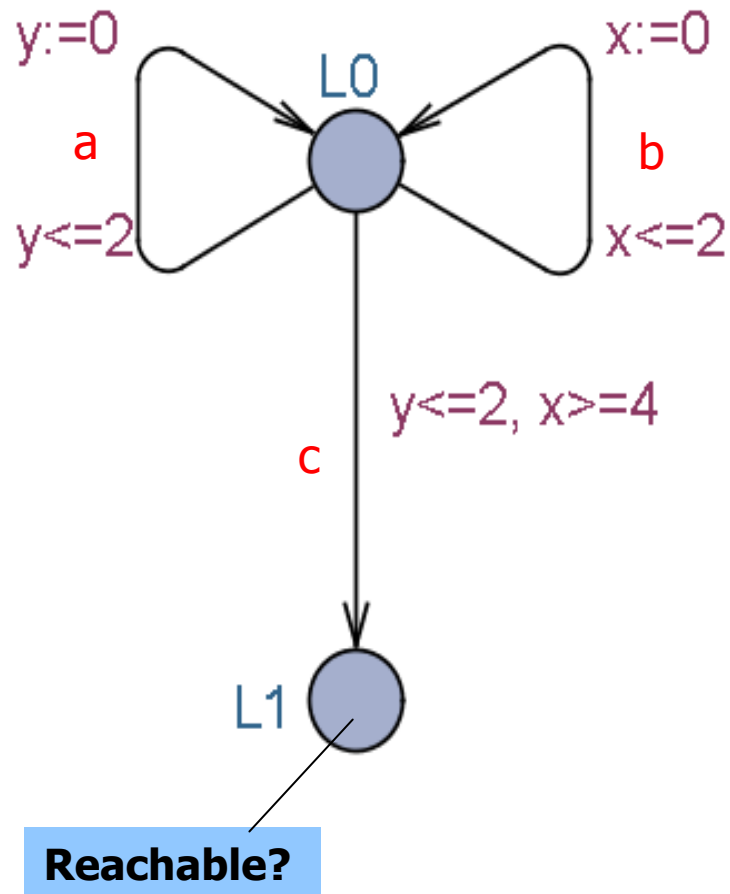
The semantics of a timed automaton A is a labelled transition system with state space $L \times \mathbb{R}^C$ with initial state $(l_0, u_0)^*$ and with the following transitions:

- $(l, u) \xrightarrow{\epsilon(d)} (l, u + d)$ iff $u \in I(l)$ and $u + d \in I(l)$,
- $(l, u) \xrightarrow{a} (l', u')$ iff there exists $(l, g, a, r, l') \in E$ such that
 - $u \models g$,
 - $u' = u[r]$, and
 - $u' \in I(l')$

* $u_0(x) = 0$ for all $x \in C$

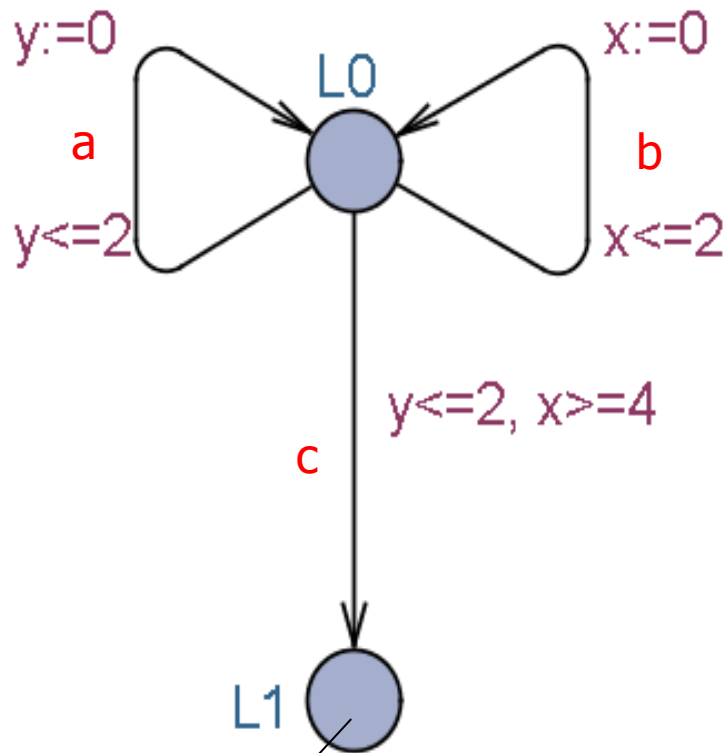
Example

With two clocks

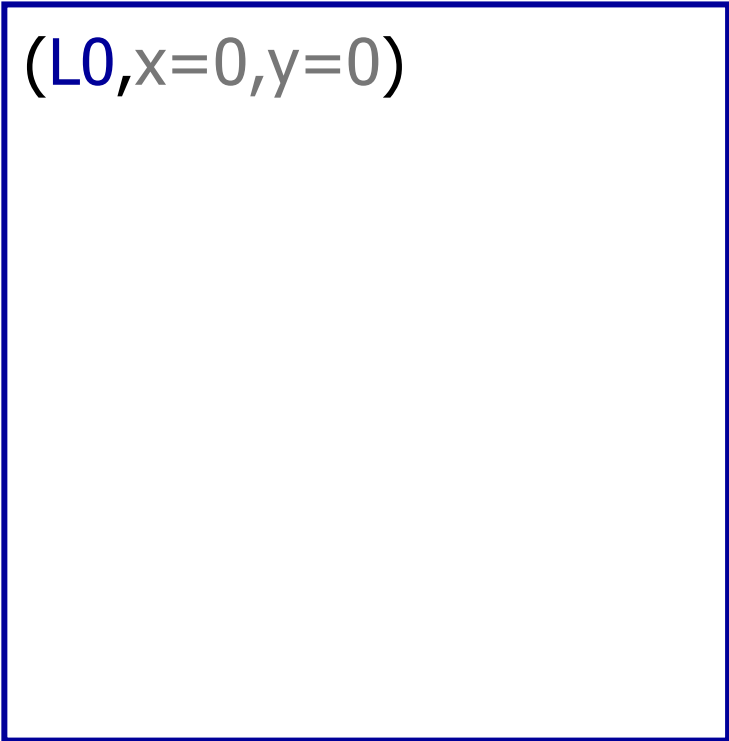
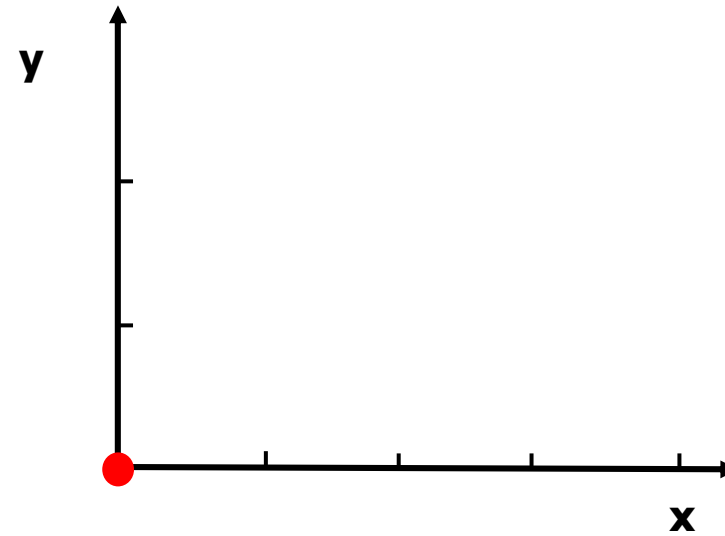


Example

With two clocks

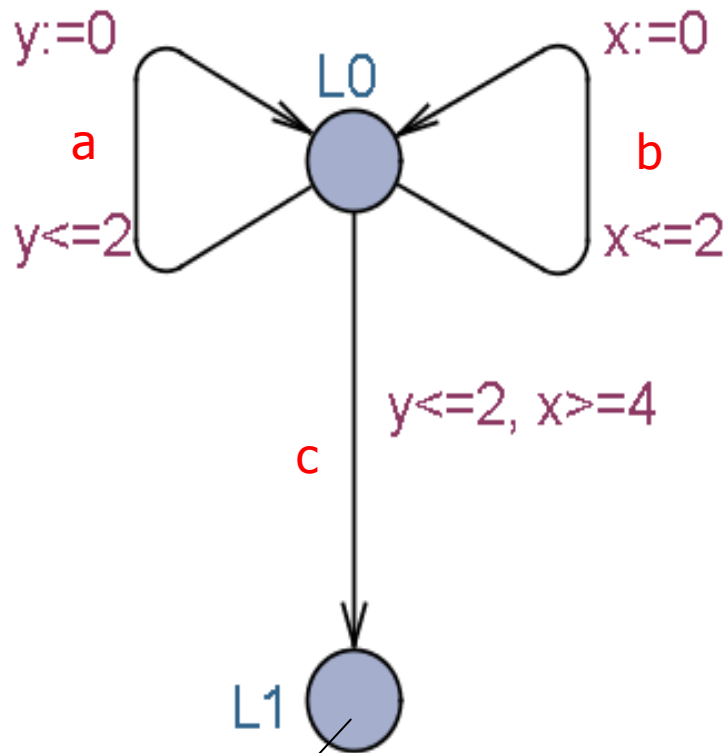


Reachable?

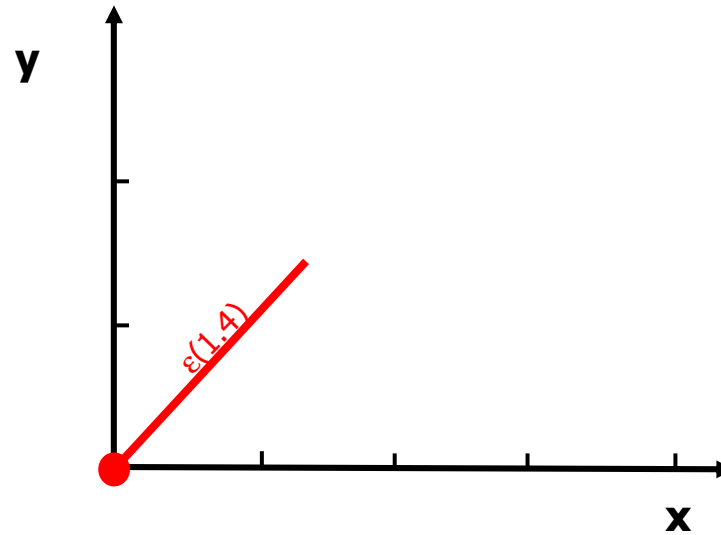


Example

With two clocks



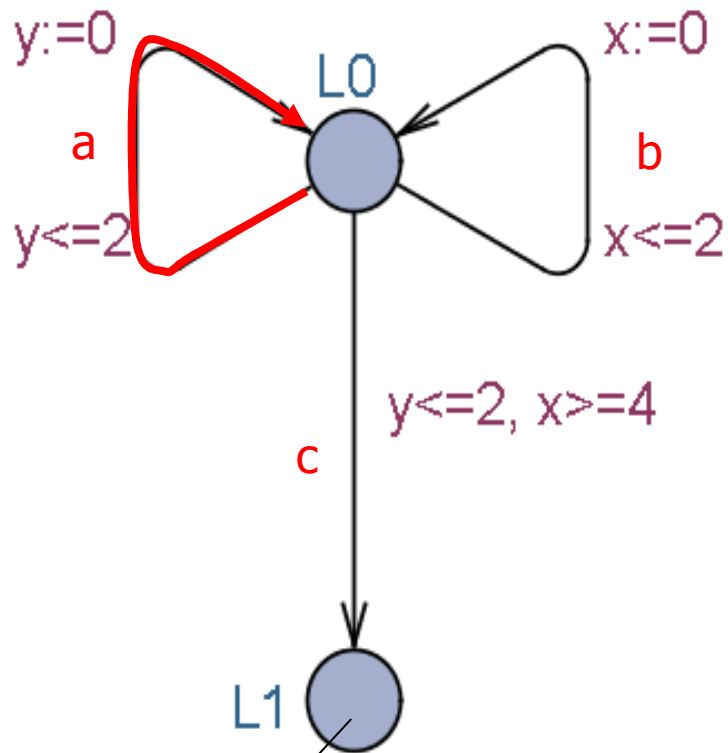
Reachable?



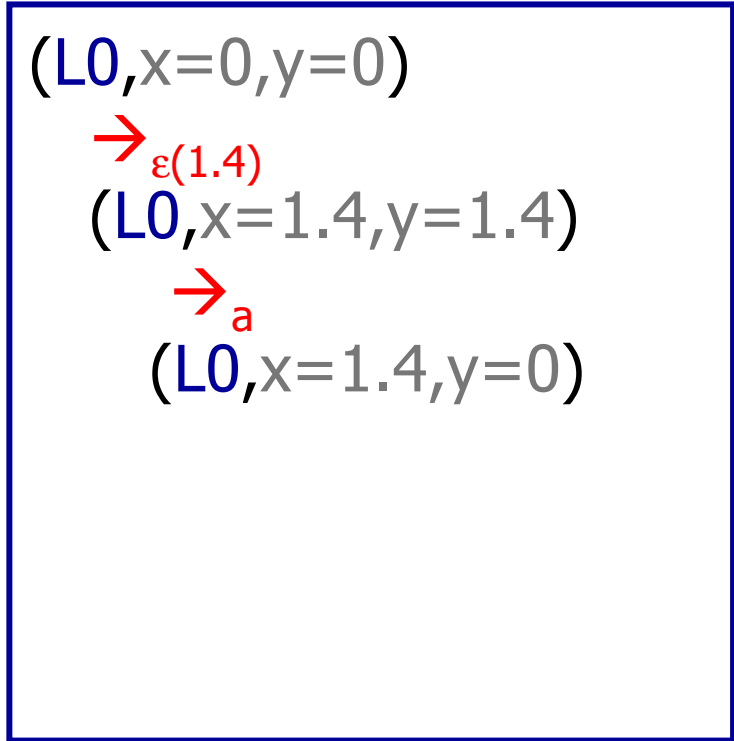
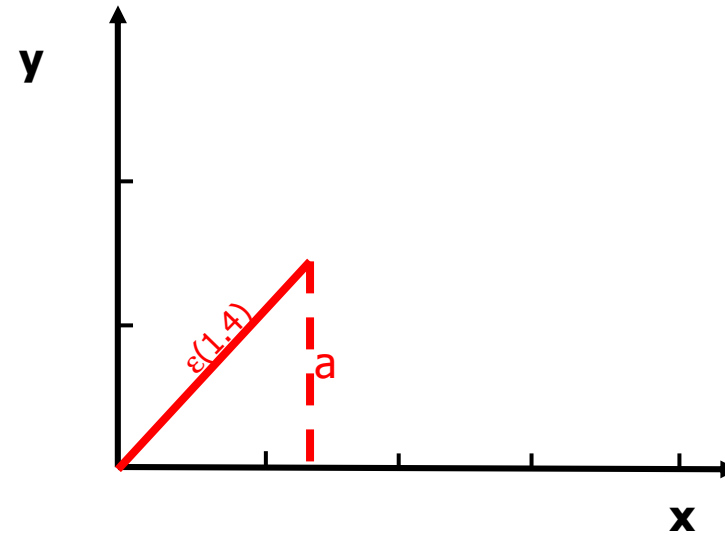
$(L0, x=0, y=0)$
 $\xrightarrow{\epsilon(1.4)}$
 $(L0, x=1.4, y=1.4)$

Example

With two clocks

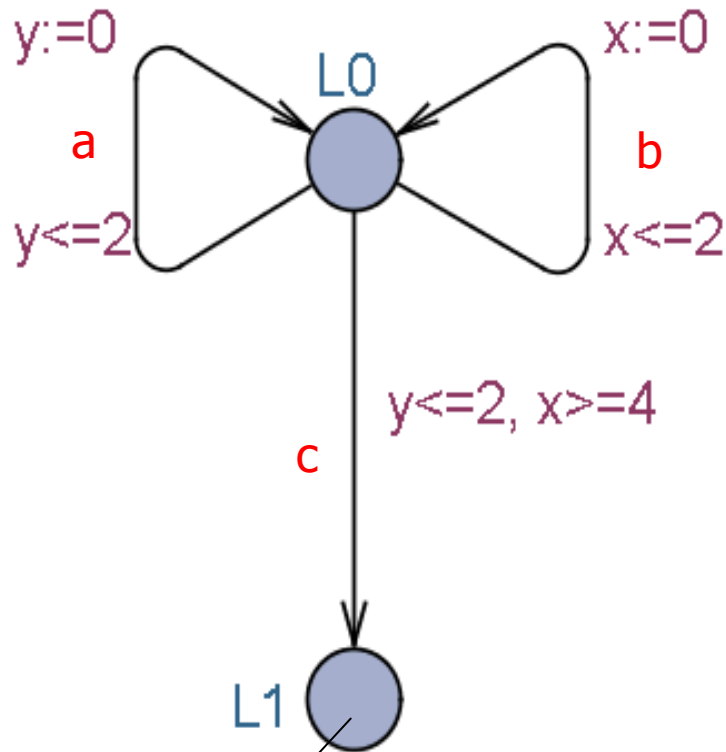


Reachable?

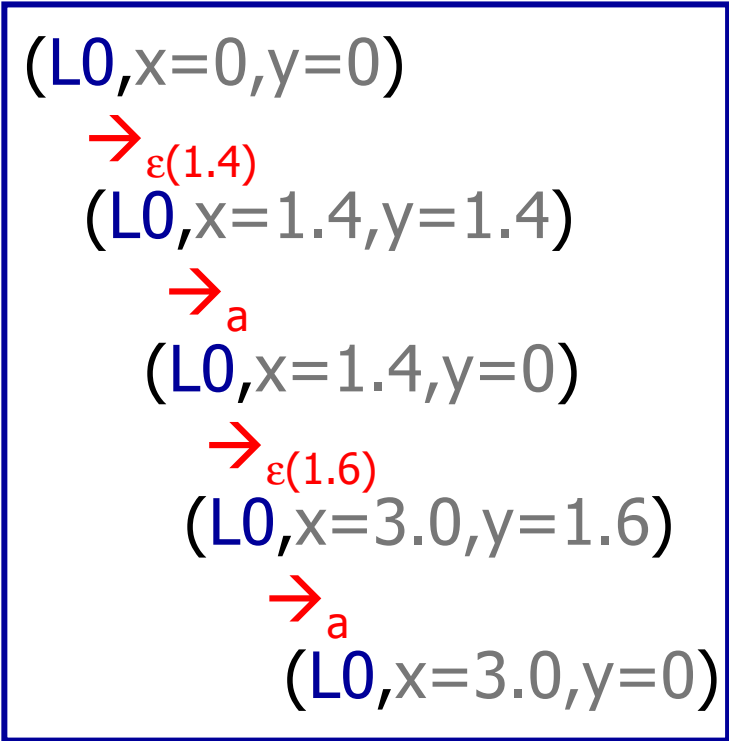
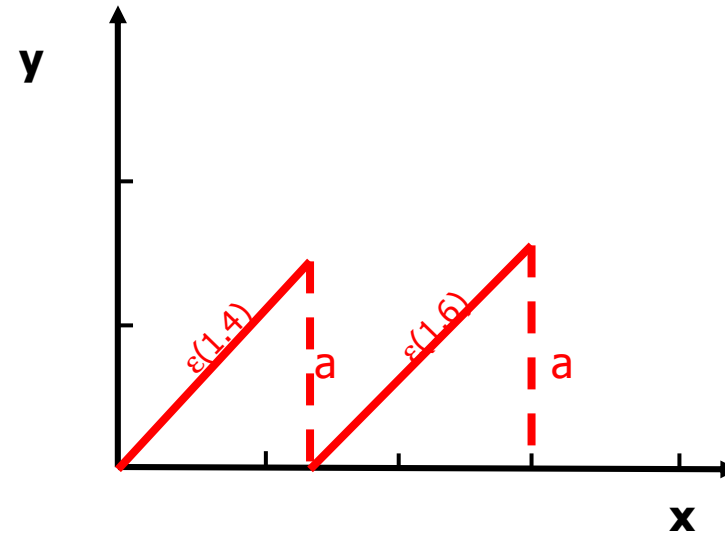


Example

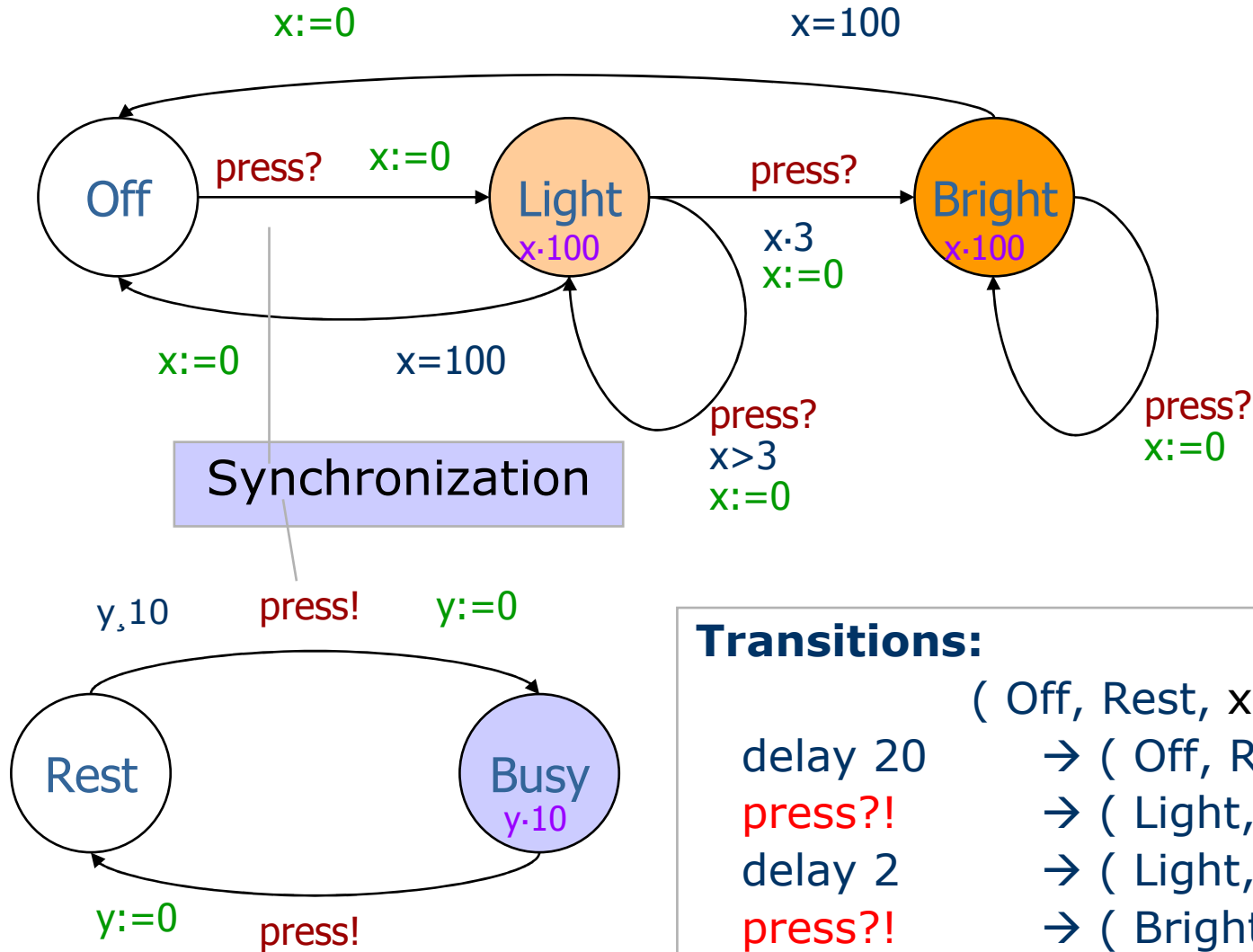
With two clocks



Reachable?



Networks Light Controller & User



Transitions:

(Off, Rest, $x=0$, $y=0$)

delay 20 → (Off, Rest, $x=20$, $y=20$)

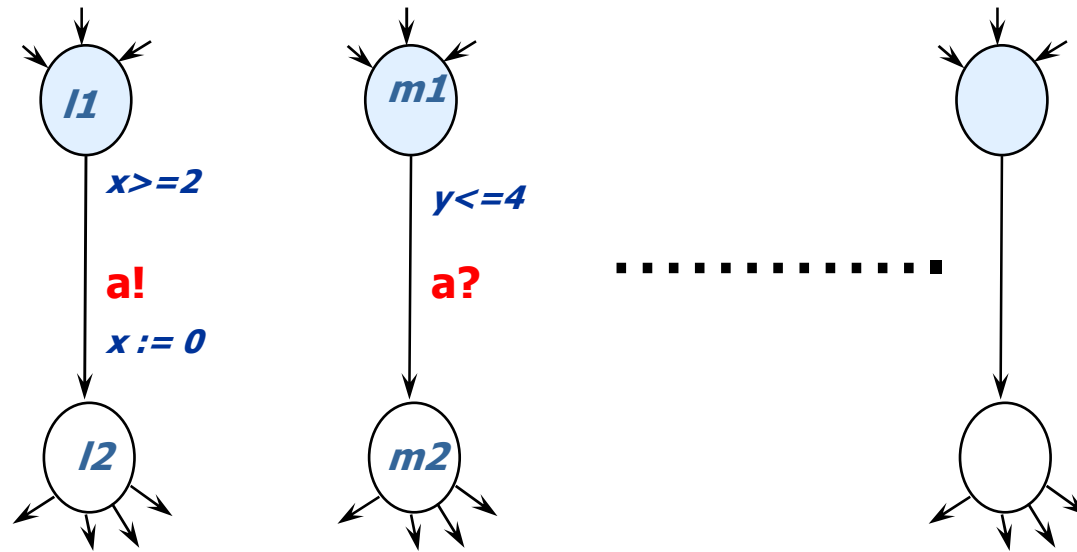
$press?! \rightarrow$ (Light, Busy, $x=0$, $y=0$)

delay 2 → (Light, Busy, $x=2$, $y=2$)

$press?! \rightarrow$ (Bright, Rest, $x=0$, $y=0$)

Networks of Timed Automata

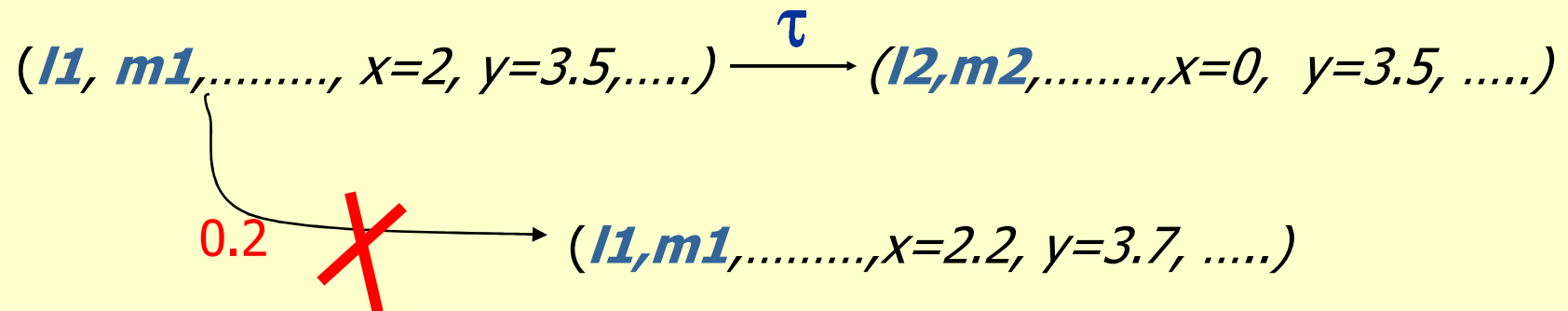
(a'la CCS)



Two-way synchronization on *complementary* actions.

Closed Systems!

Example transitions



If **a** URGENT CHANNEL

Network Semantics

$$T_1 \parallel_X T_2 = (S_1 \times S_2, \rightarrow, s_0^1 \parallel_X s_0^2) \quad \text{where}$$

$$\frac{S_1 \xrightarrow{\mu} S_1'}{S_1 \parallel_X S_2 \xrightarrow{\mu} S_1' \parallel_X S_2} \qquad \frac{S_2 \xrightarrow{\mu} S_2'}{S_1 \parallel_X S_2 \xrightarrow{\mu} S_1 \parallel_X S_2'}$$

$$\frac{S_1 \xrightarrow{a!} S_1' \quad S_2 \xrightarrow{a?} S_2'}{S_1 \parallel_X S_2 \xrightarrow{\tau} S_1' \parallel_X S_2'}$$

$$\frac{S_1 \xrightarrow{e(d)} S_1' \quad S_2 \xrightarrow{e(d)} S_2'}{S_1 \parallel_X S_2 \xrightarrow{e(d)} S_1' \parallel_X S_2'}$$

Network Semantics

(URGENT synchronization)

+ Urgent synchronization

$$T_1 \parallel_X T_2 = (S_1 \times S_2, \rightarrow, s_0^1 \parallel_X s_0^2) \quad \text{where}$$

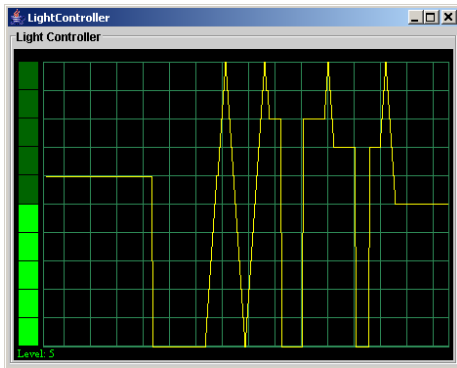
$$\frac{s_1 \xrightarrow{\mu} \rightarrow_1 s_1'}{s_1 \parallel_X s_2 \xrightarrow{\mu} \rightarrow s_1 \parallel_X s_2}$$

$$\frac{s_2 \xrightarrow{\mu} \rightarrow_2 s_2'}{s_1 \parallel_X s_2 \xrightarrow{\mu} \rightarrow s_1 \parallel_X s_2'}$$

$$\frac{s_1 \xrightarrow{a!} \rightarrow_1 s_1' \quad s_2 \xrightarrow{a?} \rightarrow_2 s_2'}{s_1 \parallel_X s_2 \xrightarrow{\tau} \rightarrow s_1' \parallel_X s_2'}$$

$\forall d' < d, \forall u \in \text{UAct}:$
 $\neg (s_1 \xrightarrow{e(d')} \rightarrow u? \wedge s_2 \xrightarrow{e(d')} \rightarrow u!)$

$$\frac{s_1 \xrightarrow{e(d)} \rightarrow_1 s_1' \quad s_2 \xrightarrow{e(d)} \rightarrow_2 s_2'}{s_1 \parallel_X s_2 \xrightarrow{e(d)} \rightarrow s_1 \parallel_X s_2'}$$



Light Control Interface



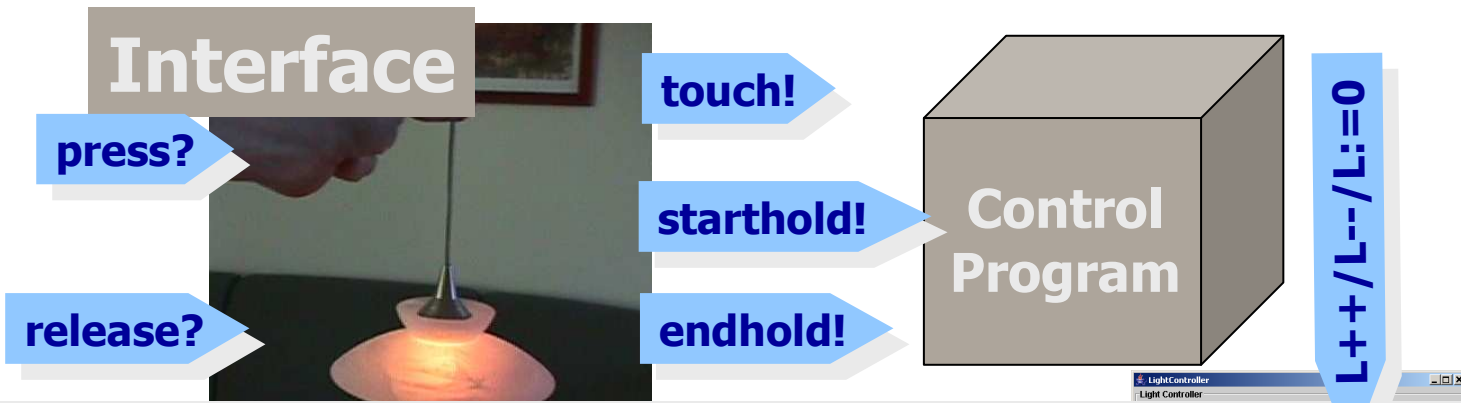
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Light Control Interface

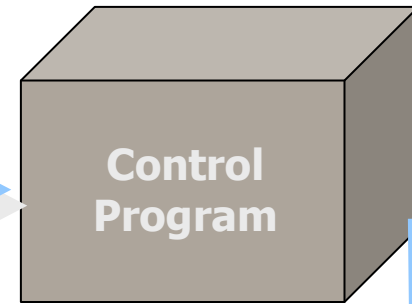
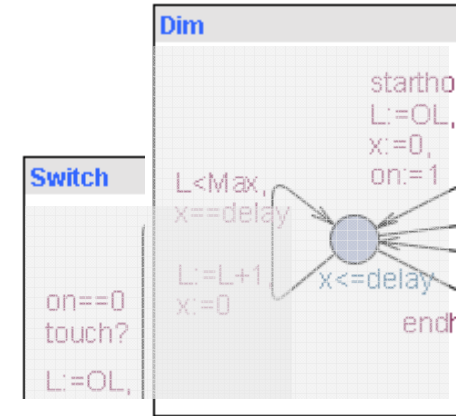
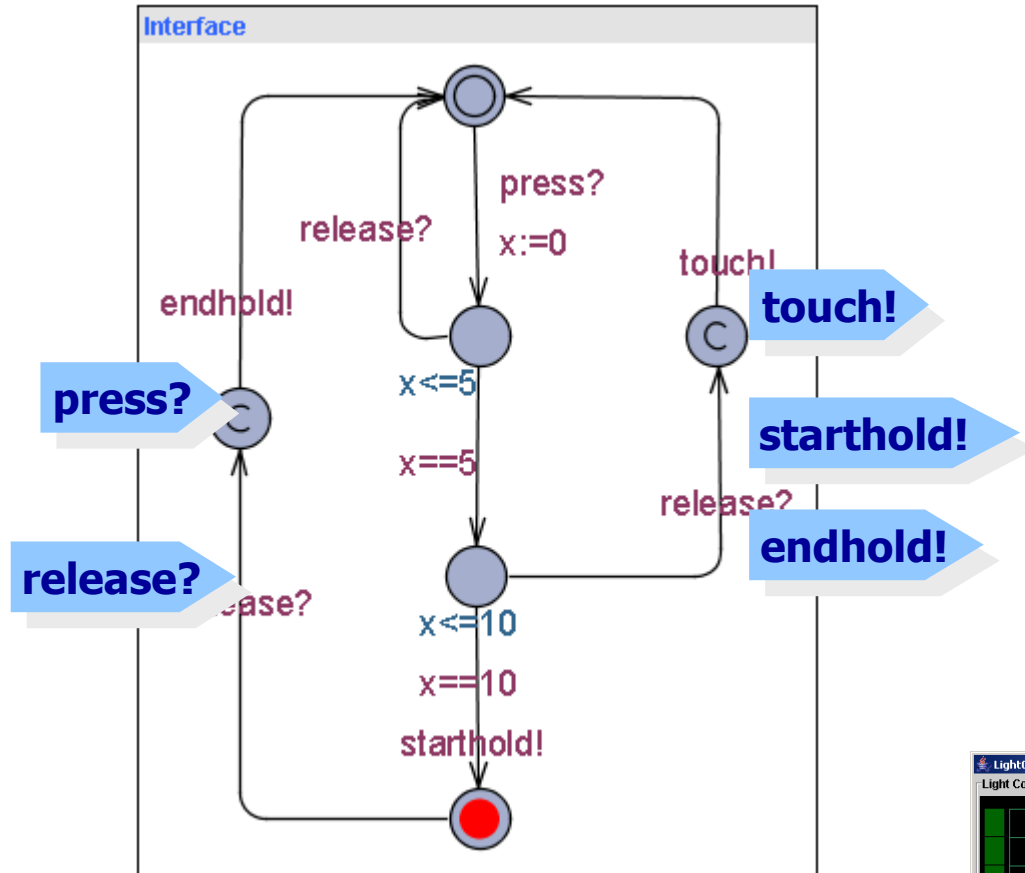
press? d release? \rightarrow touch! $0.5 \cdot d - 1$
 press? 1 \rightarrow starthold!
 press? d release? \rightarrow endhold! $d > 1$



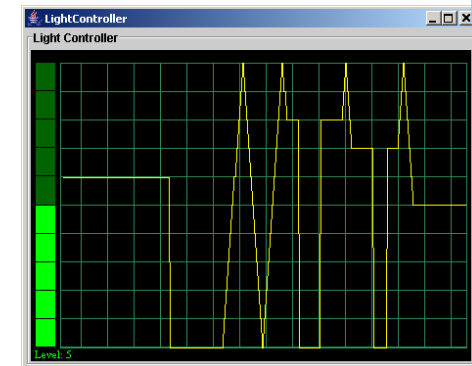
Light Control Interface



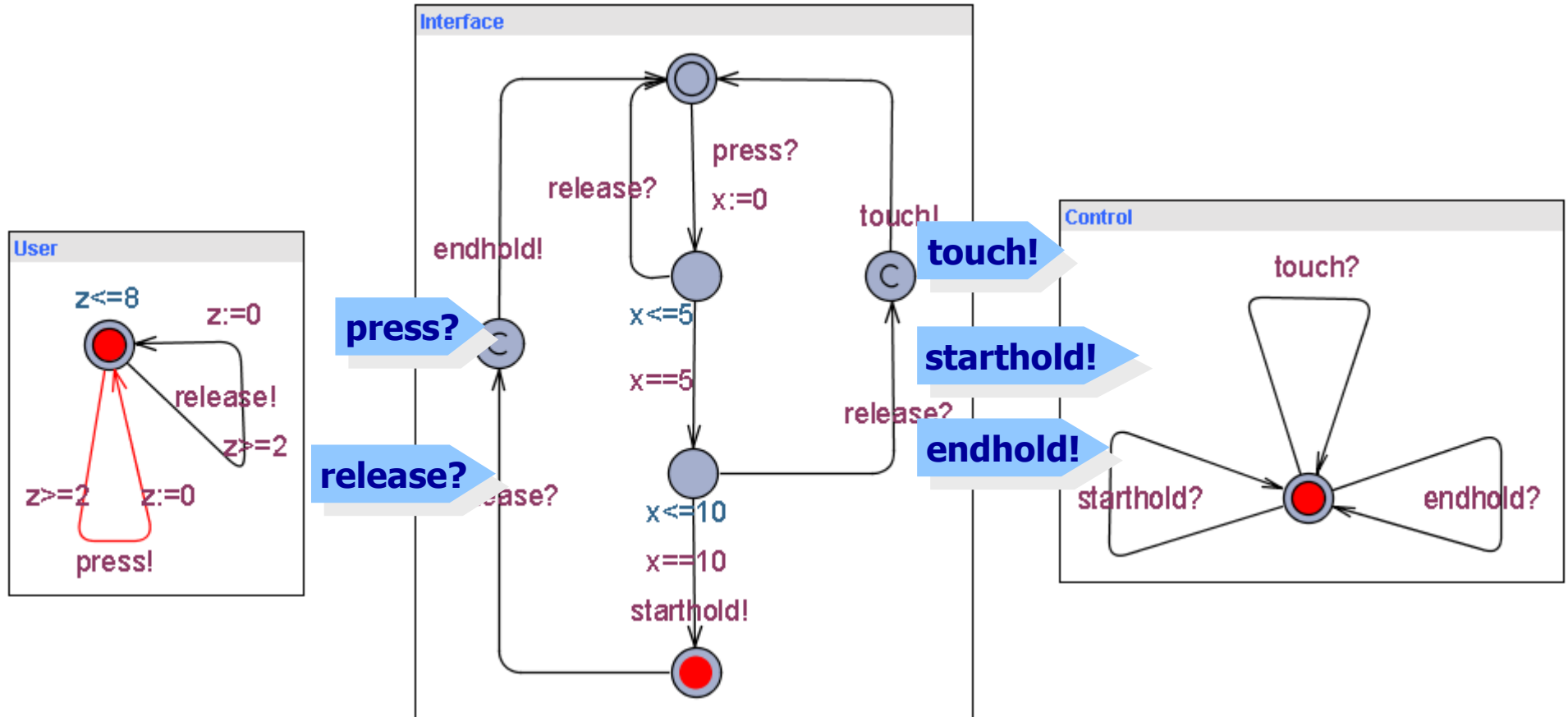
User



L++/L--/L:=0

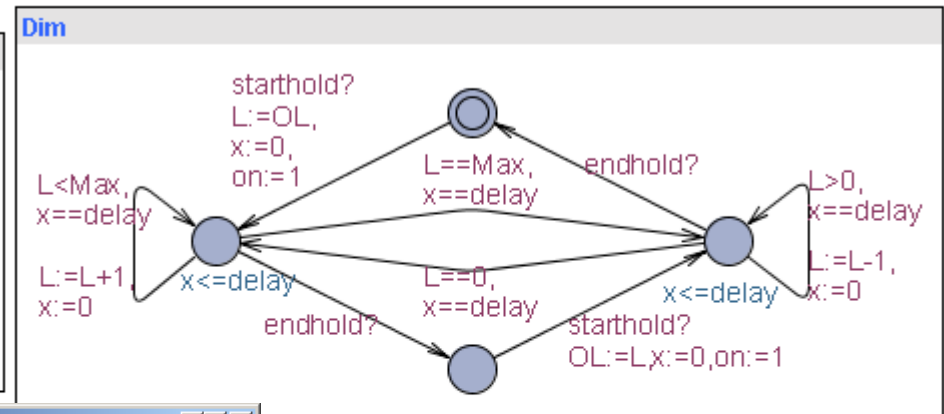
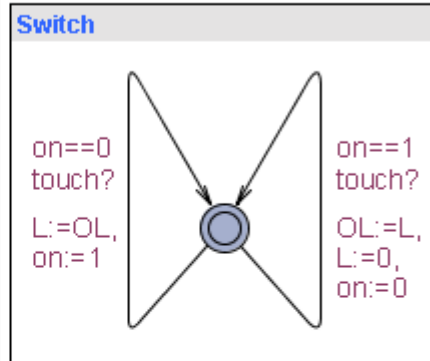
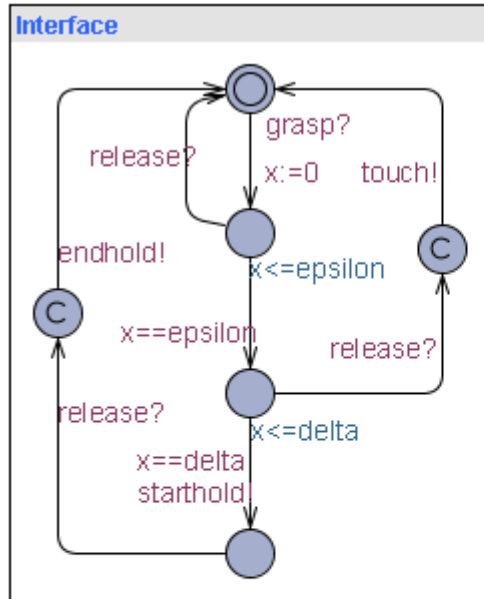


Light Control Network

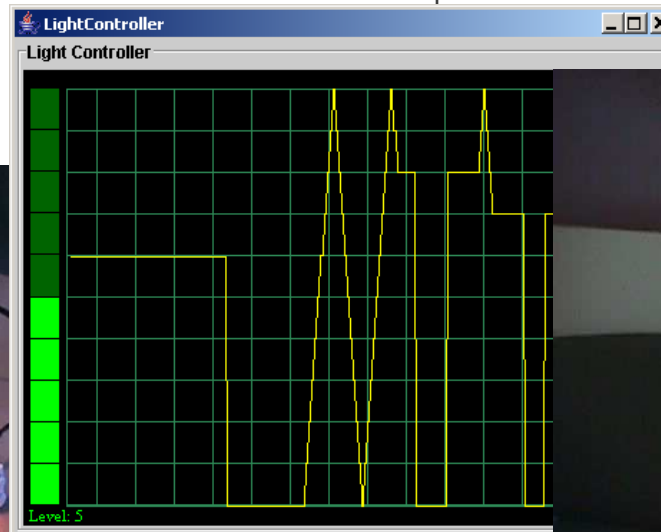
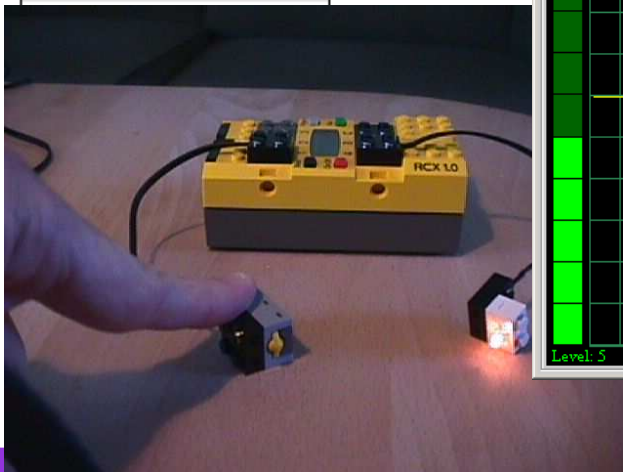


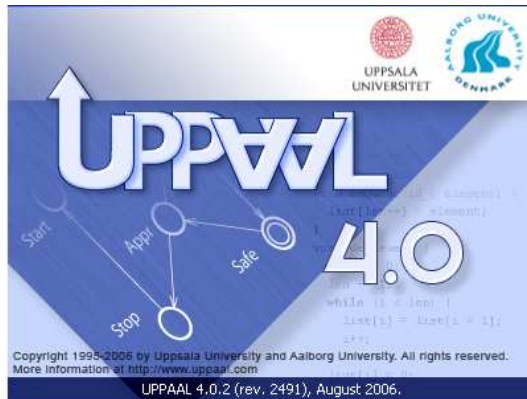
Validation

Light Controller



User





Timed Automata Modeling and Decidability

Kim Guldstrand Larsen



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Overview

- BRICK Sorting

- Reachability Checking
 - Region Construction

- Bisimulation Checking
- Model Checking
- Trace Inclusion Checking

Brick Sorting



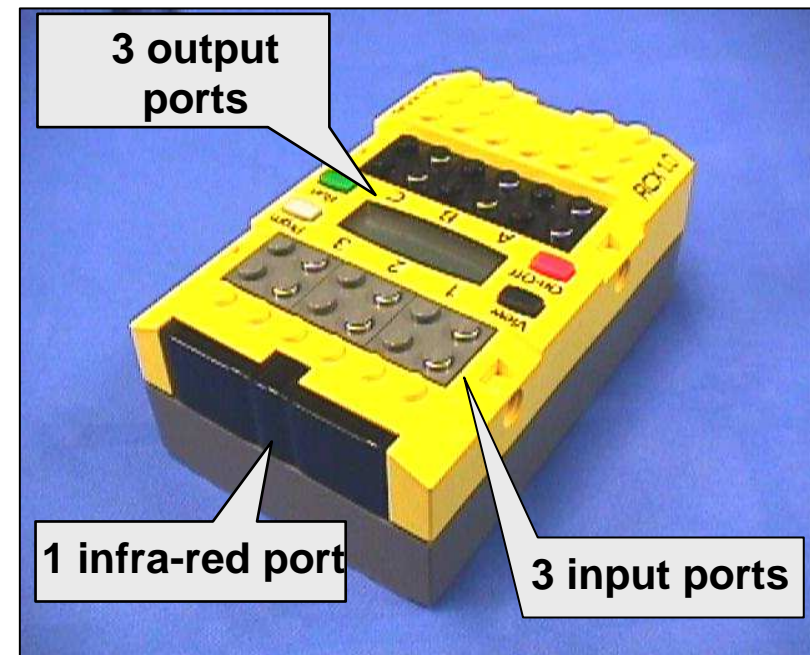
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in Computer Science



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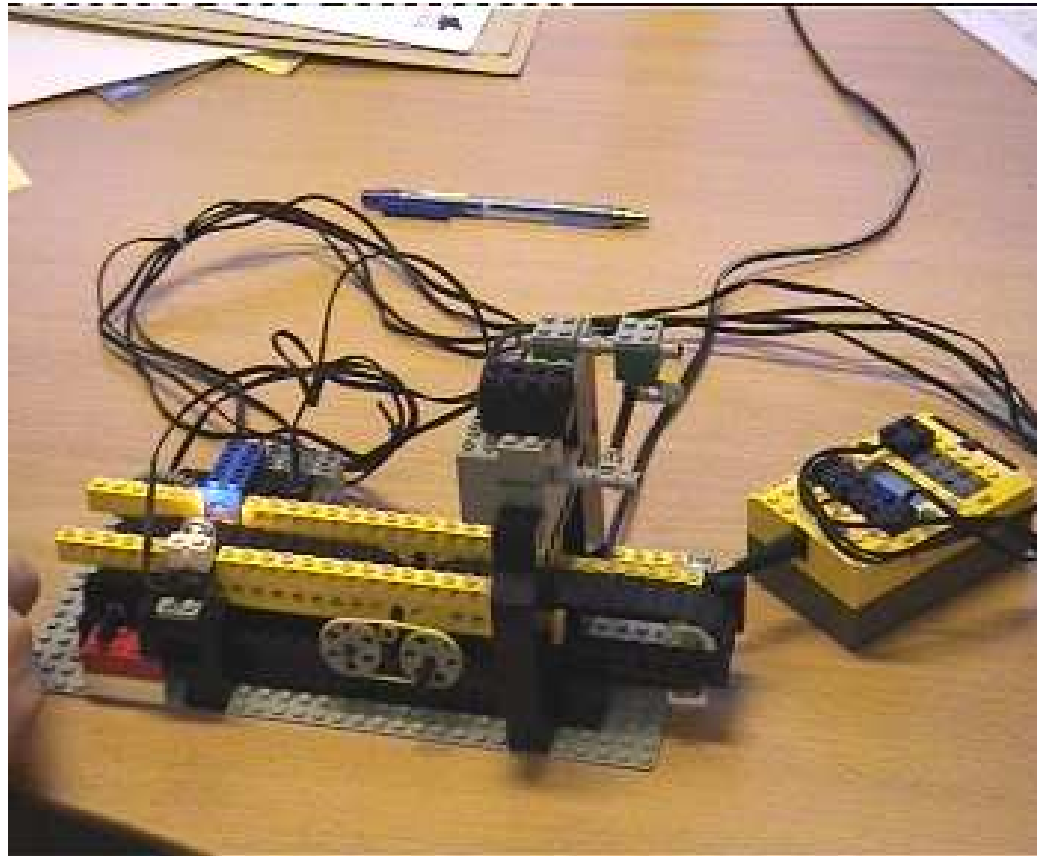
LEGO Mindstorms/RCX

- **Sensors:** temperature, light, rotation, pressure.
- **Actuators:** motors, lamps,
- **Virtual machine:**
 - 10 tasks, 4 timers, 16 integers.
- **Several Programming Languages:**
 - NotQuiteC, Mindstorm, Robotics, legOS, etc.



A Real Real Timed System

The Plant
Conveyor Belt
&
Bricks



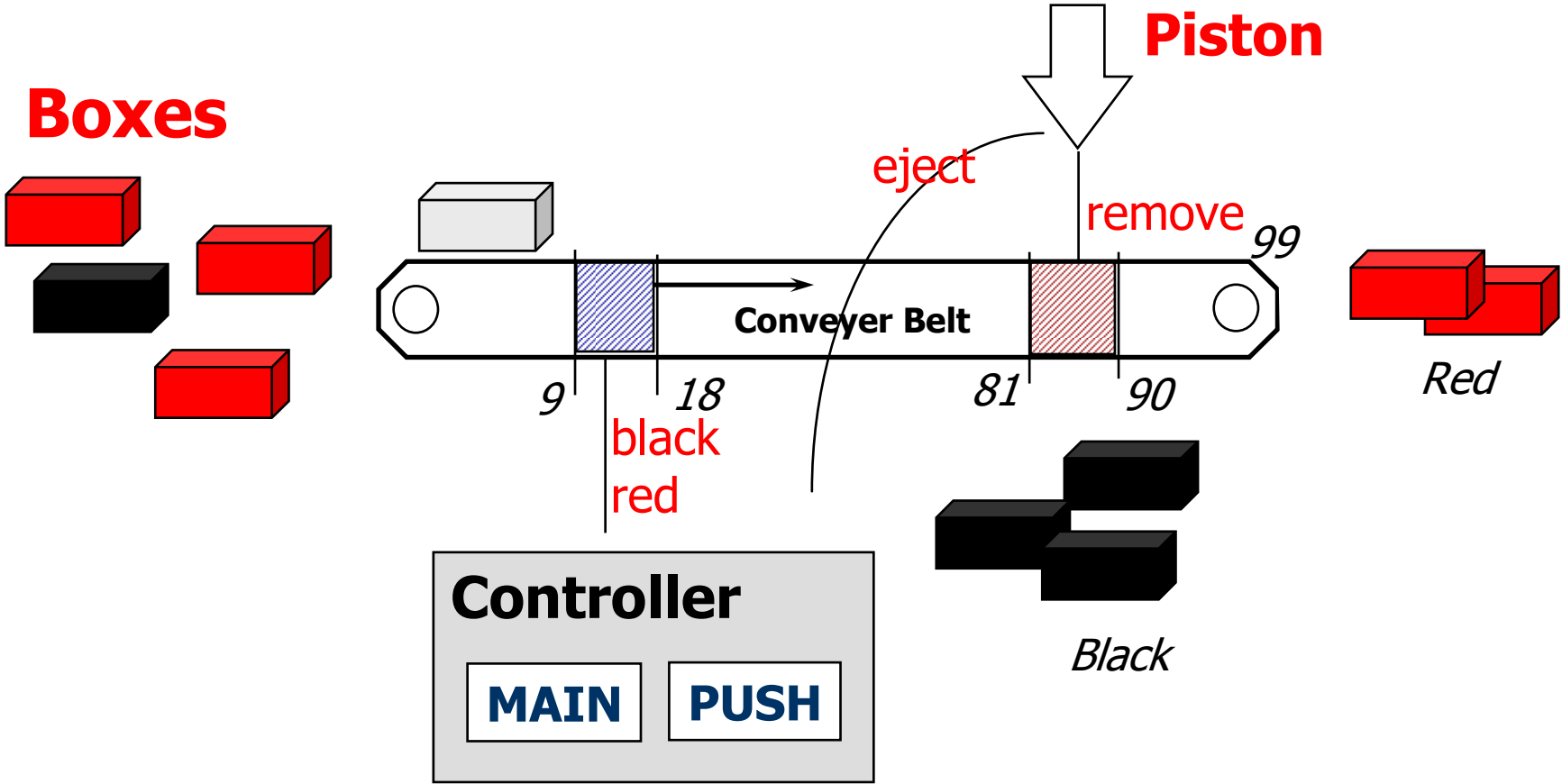
**Controller
Program**
LEGO MINDSTORM

What is the CONTROL program doing?

First UPPAAL model

Sorting of Lego Boxes

Ken Tindell



NQC programs

```
int active;
int DELAY;
int LIGHT_LEVEL;
```

```
task MAIN{
  DELAY=75;
  LIGHT_LEVEL=35;
  active=0;
  Sensor(IN_1, IN_LIGHT);
  Fwd(OUT_A,1);
  Display(1);

  start PUSH;

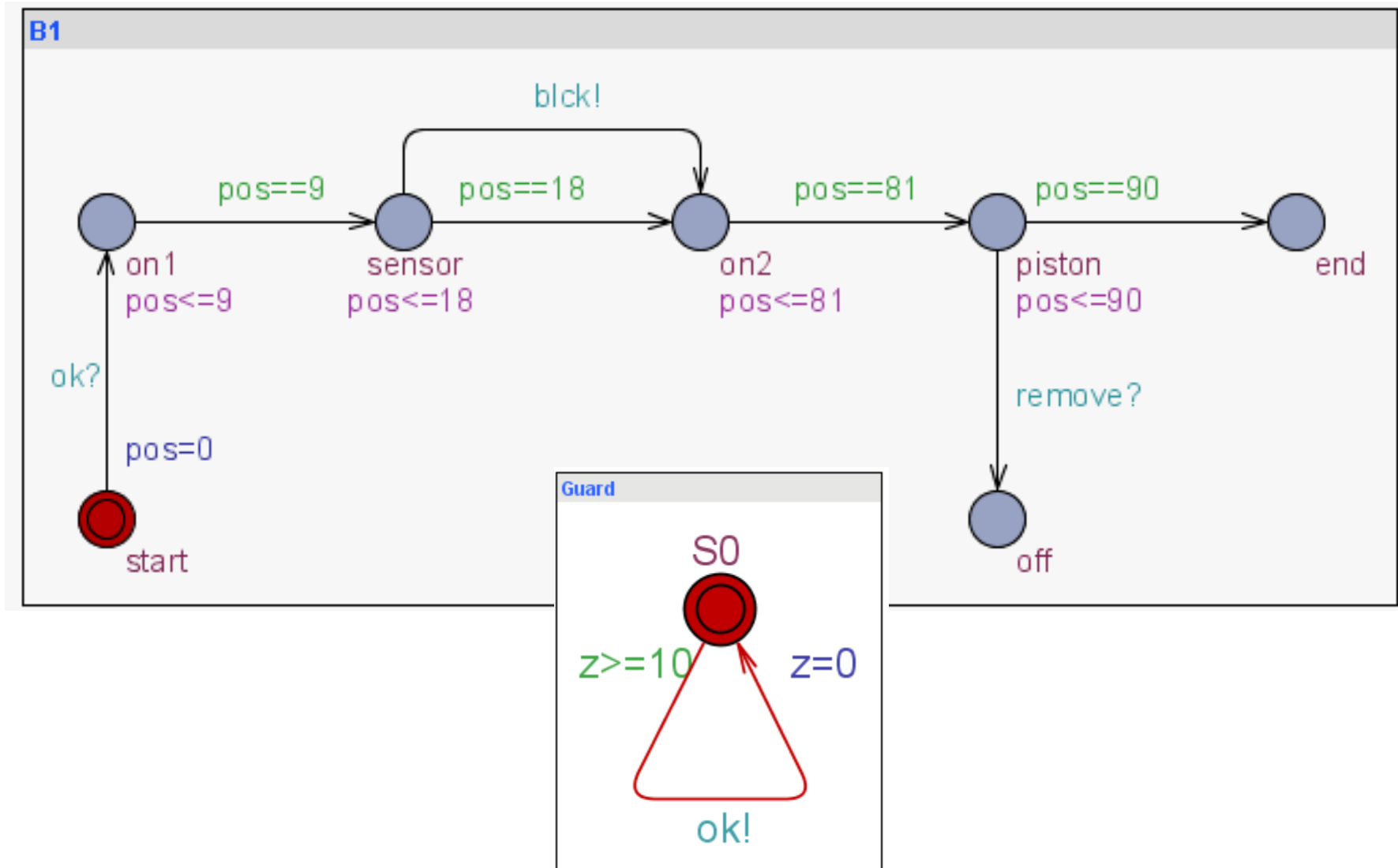
  while(true){

wait(IN_1<=LIGHT_LEVEL);
  ClearTimer(1);
  active=1;
  PlaySound(1);

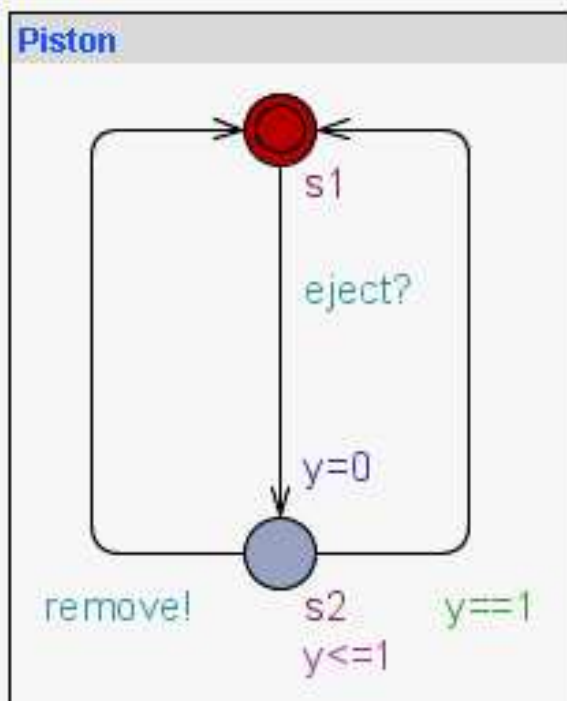
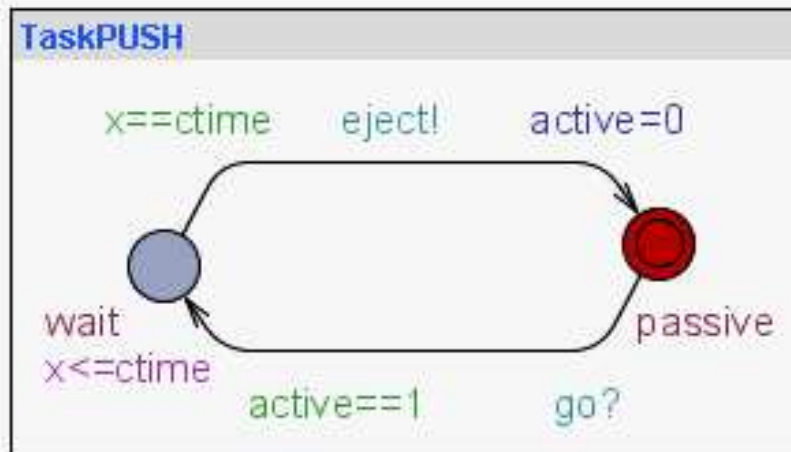
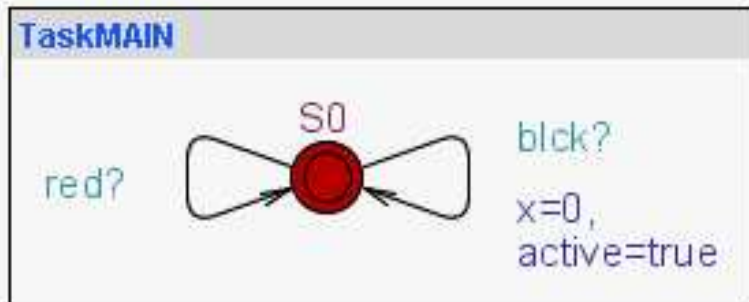
wait(IN_1>LIGHT_LEVEL);
  }
}
```

```
task PUSH{
  while(true){
    wait(Timer(1)>DELAY && active==1);
    active=0;
    Rev(OUT_C,1);
    Sleep(8);
    Fwd(OUT_C,1);
    Sleep(12);
    Off(OUT_C);
  }
}
```

A Black Brick & The Guard



Control Tasks & Piston



GLOBAL DECLARATIONS:

const int ctime = 75;

int[0,1] active;

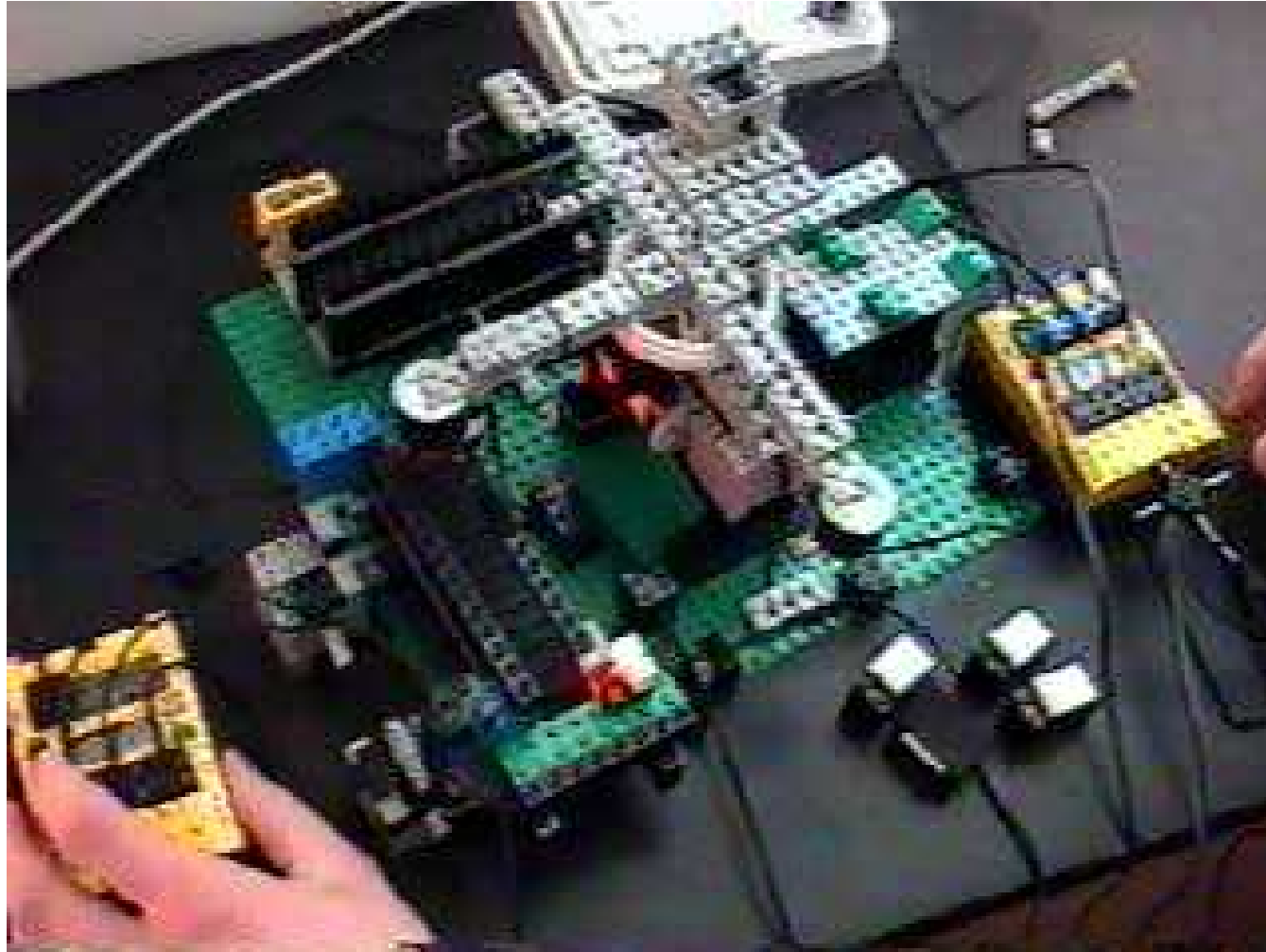
clock x, time;

chan eject, ok;

chan blk, red, remove, go;

The Production Cell in LEGO

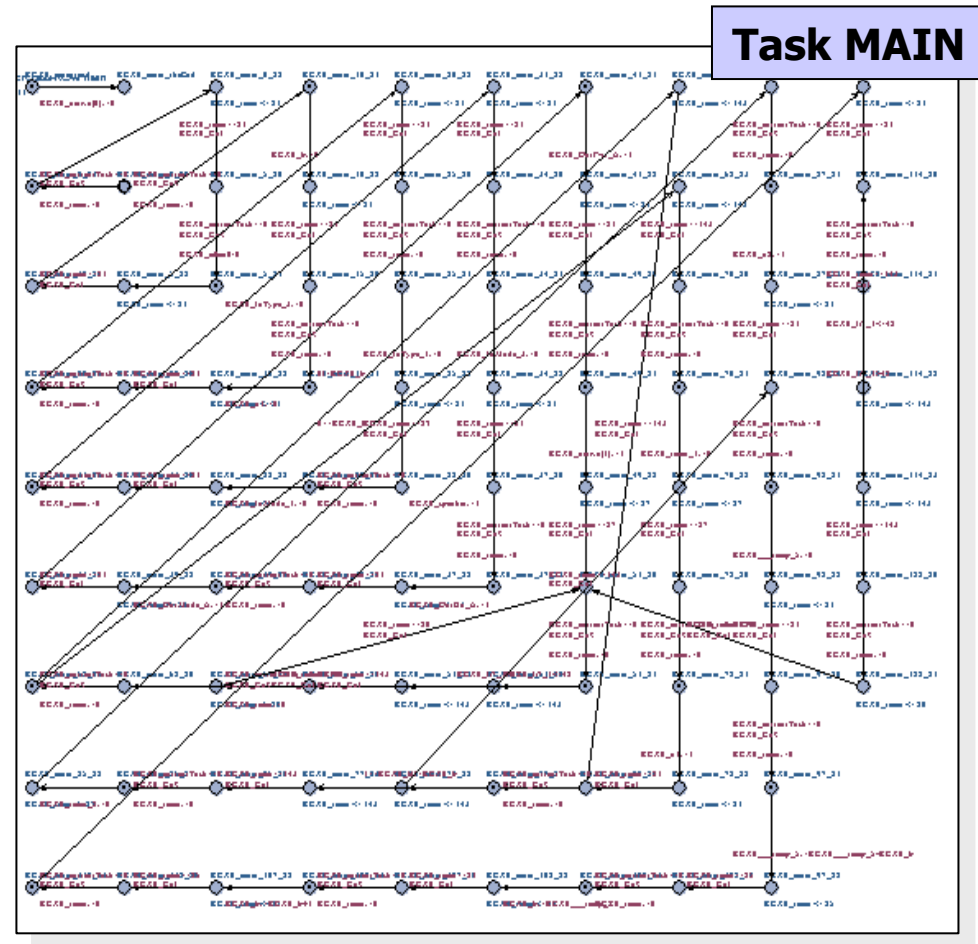
Course at DTU, Copenhagen



Rasmus Crüger Lund
Simon Tune Riemanni

From RCX to UPPAAL – and back

- Model includes Round-Robin Scheduler.
- Compilation of RCX tasks into TA models.
- Presented at ECRTS 2000 in Stockholm.
- From UPPAAL to RCX: Martijn Hendriks.



UPPAAL

Home

[Home](#) | [About](#) | [Documentation](#) | [Download](#) | [Examples](#) | [Bugs](#)

UPPAAL is an integrated tool environment for modeling, validation and verification of real-time systems modeled as networks of timed automata, extended with data types (bounded integers, arrays, etc.).

The tool is developed in collaboration between the [Department of Information Technology](#) at Uppsala University, Sweden and the [Department of Computer Science](#) at Aalborg University in Denmark.

Download

The current official release is UPPAAL 3.4.11 (Jun 23, 2005). A release of UPPAAL **3.6 alpha 3** (dec 20, 2005) is also available. For more information about UPPAAL version 3.4, we refer to this [press release](#).

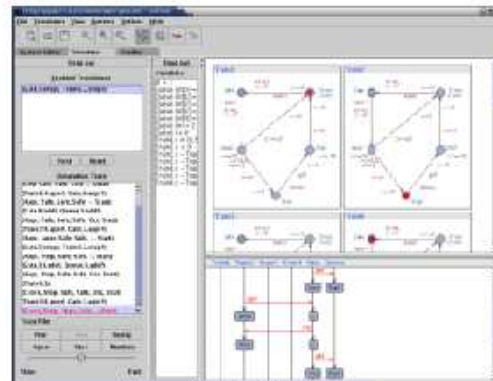


Figure 1: UPPAAL on screen.

License

The UPPAAL tool is **free** for non-profit applications. For information about commercial licenses, please email [sales\(at\)uppaal\(dot\)com](mailto:sales(at)uppaal(dot)com).

To find out more about UPPAAL, read this short [introduction](#). Further information may be found at this web site in the pages [About](#), [Documentation](#), [Download](#), and [Examples](#).

Mailing Lists

UPPAAL has an open [discussion forum](#) group at Yahoo!Groups intended for users of the tool. To join or post to the forum, please refer to the information at the [discussion forum](#) page. Bugs should be reported using the [bug tracking system](#). To email the development team directly, please use [uppaal\(at\)list\(dot\)it\(dot\)uu\(dot\)se](mailto:uppaal(at)list(dot)it(dot)uu(dot)se).



UPPSALA
UNIVERSITET



AALBORG UNIVERSITY

Decidability

The Region Construction



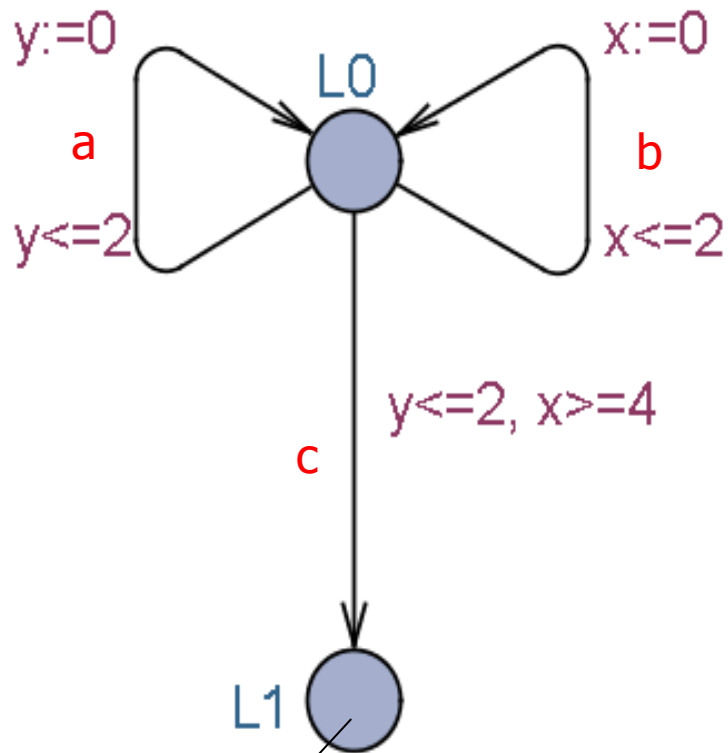
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Basic Research
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Reachability ?



OBSTACLE:
 Uncountably infinite state space

$L \times \mathbb{R}^C$
 locations clock-valuations

Reachable from initial state ($L_0, x=0, y=0$) ?

Constraints

Definition

Let X be a set of clock variables. The set $\mathcal{B}(X)$ of *clock constraints* ϕ is given by the grammar:

$$\phi ::= x \leq c \mid c \leq x \mid x < c \mid c < x \mid \phi_1 \wedge \phi_2$$

where $c \in \mathbb{N}$ (or \mathbb{Q}).

Clock Valuations and Notation

Definition

The set of *clock valuations*, \mathbb{R}^C is the set of functions $C \rightarrow \mathbb{R}_{\geq 0}$ ranged over by u, v, w, \dots

Notation

Let $u \in \mathbb{R}^C$, $r \subseteq C$, $d \in \mathbb{R}_{\geq 0}$, and $g \in \mathcal{B}(X)$ then:

- $u + d \in \mathbb{R}^C$ is defined by $(u + d)(x) = u(x) + d$ for any clock x
- $u[r] \in \mathbb{R}^C$ is defined by $u[r](x) = 0$ when $x \in r$ and $u[r](x) = u(x)$ for $x \notin r$.
- $u \models g$ denotes that g is satisfied by u .

Timed Automata

Definition

A timed automaton A over clocks C and actions Act is a tuple (L, l_0, E, I) , where:

- L is a finite set of locations
- $l_0 \in L$ is the initial location
- $E \subseteq L \times \mathcal{B}(X) \times Act \times \mathcal{P}(C) \times L$ is the set of edges
- $I : L \rightarrow \mathcal{B}(X)$ assigns to each location an invariant

Semantics

Definition

The semantics of a timed automaton A is a labelled transition system with state space $L \times \mathbb{R}^C$ with initial state $(l_0, u_0)^*$ and with the following transitions:

- $(l, u) \xrightarrow{\epsilon(d)} (l, u + d)$ iff $u \in I(l)$ and $u + d \in I(l)$,
- $(l, u) \xrightarrow{a} (l', u')$ iff there exists $(l, g, a, r, l') \in E$ such that
 - $u \models g$,
 - $u' = u[r]$, and
 - $u' \in I(l')$

* $u_0(x) = 0$ for all $x \in C$

Derived Relations and Reachability

$$(l, u) \xrightarrow{\delta} (l', u') \quad \text{iff} \quad \exists d > 0. (l, u) \xrightarrow{\epsilon(d)} (l', u').$$

$$(l, u) \xrightarrow{\alpha} (l', u') \quad \text{iff} \quad \exists a \in Act. (l, u) \xrightarrow{a} (l', u')$$

$$(l, u) \rightsquigarrow (l', u') \quad \text{iff} \quad (l, u) (\xrightarrow{\delta} \cup \xrightarrow{\alpha})^* (l', u')$$

Definition

The set of reachable locations, $Reach(A)$, of a timed automaton A is defined as:

$$l \in Reach(A) \equiv^{\Delta} \exists u. (l_0, u_0) \rightsquigarrow (l, u)$$

Time Abstracted Bisimulation

Definition

Let $G \subseteq L$ be a set of goal locations. An equivalence relation R on $L \times \mathbb{R}^C$ is a TAB wrt G if whenever $(l, u)R(n, v)$ the following holds:

1. $l \in G$ iff $n \in G$,
2. whenever $(l, u) \xrightarrow{\delta} (l', u')$ then $(n, v) \xrightarrow{\delta} (n', v')$ with $(l', u')R(n', v')$
3. whenever $(l, u) \xrightarrow{a} (l', u')$ then $(n, v) \xrightarrow{a} (n', v')$ with $(l', u')R(n', v')$

Stable Quotient

Definition

Let R be a TAB wrt G . The induced *quotient* has classes of R , $\pi \in (L \times \mathbb{R}^C / R)$, as states. For classes π, π' the transitions are

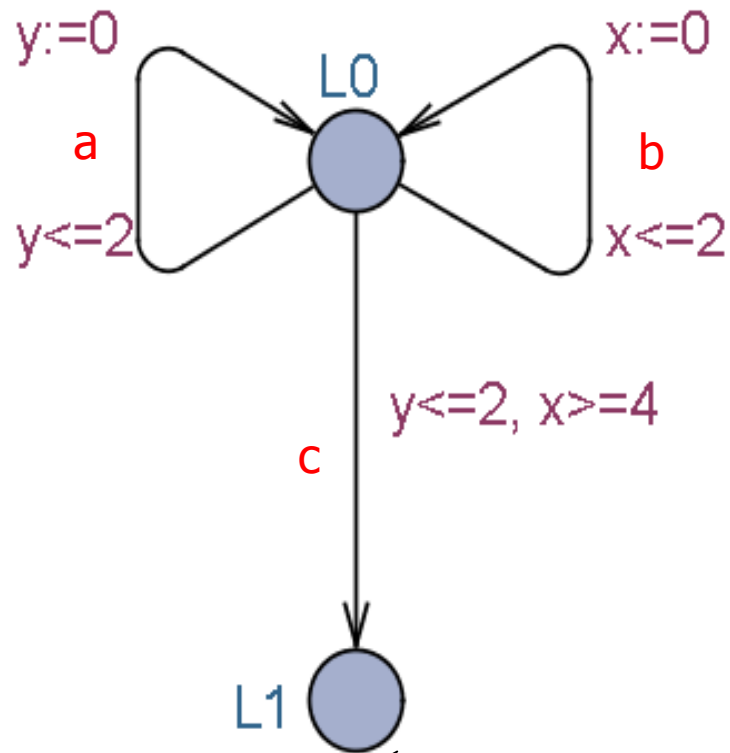
- $\pi \xrightarrow{\delta} \pi'$ iff $(l, u) \xrightarrow{\delta} (l', u')$ for some $(l, u) \in \pi, (l', u') \in \pi'$.
- $\pi \xrightarrow{a} \pi'$ iff $(l, u) \xrightarrow{a} (l', u')$ for some $(l, u) \in \pi, (l', u') \in \pi'$.

Theorem

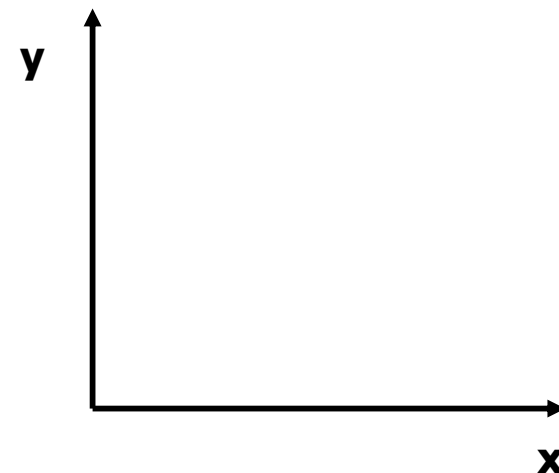
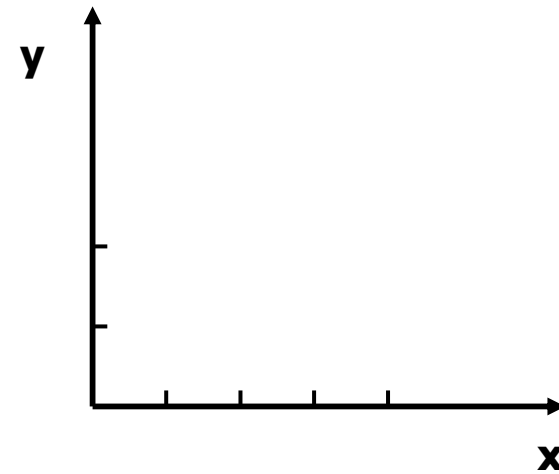
Let R be TAB wrt G . Then, a location from G is reachable iff there exists an equivalence class π of R such that π is reachable in the quotient and π contains a state whose location is in G .

Stable Quotient

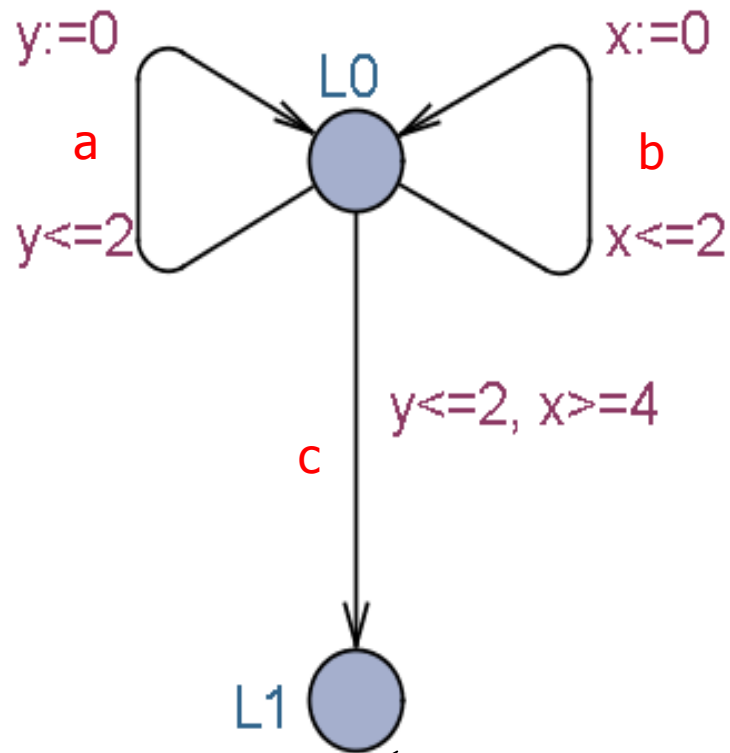
Partitioning



Reachable?

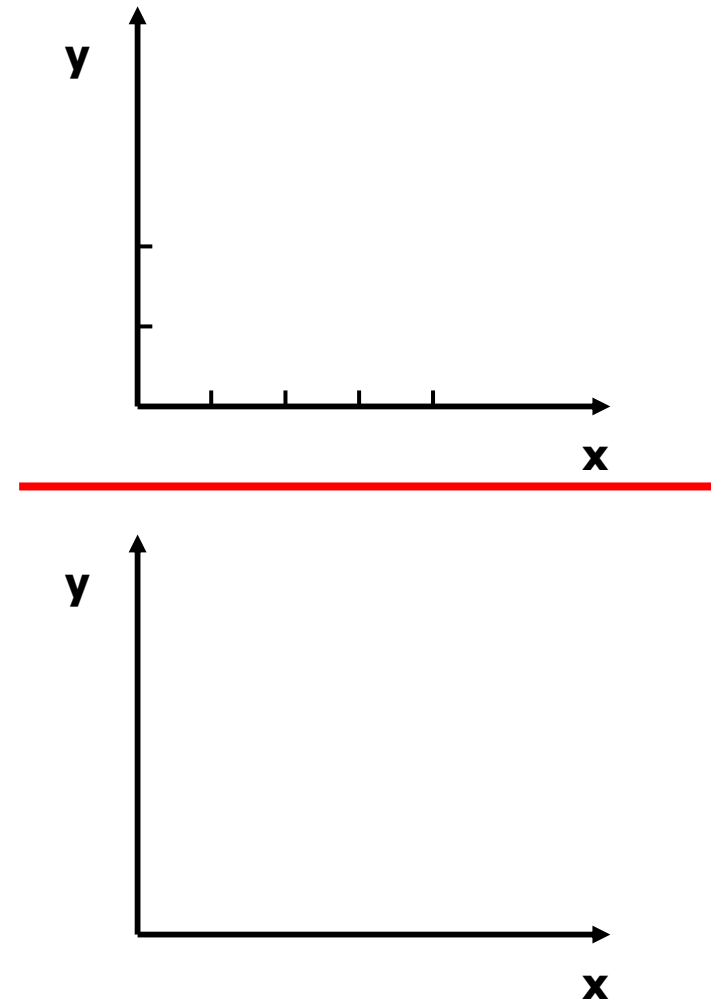


Stable Quotient

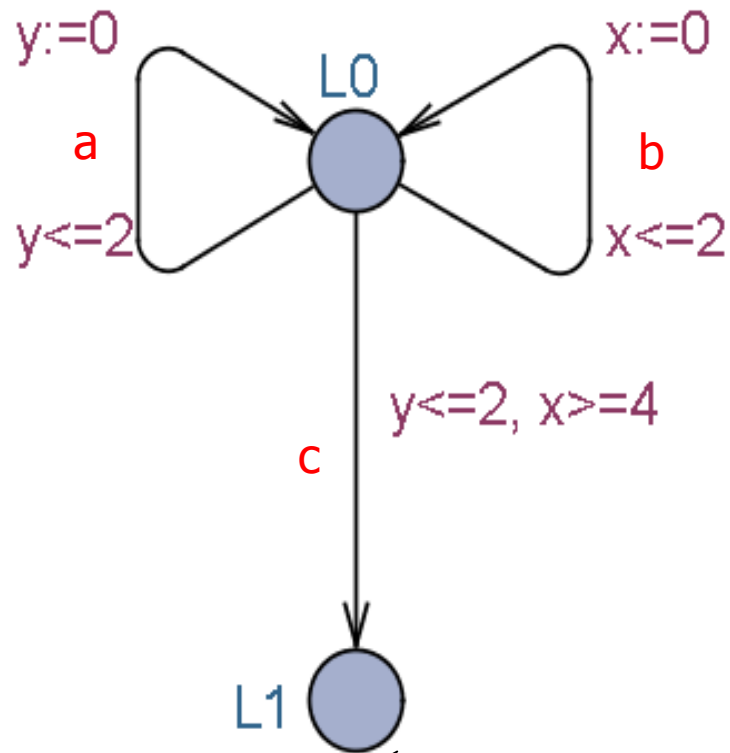


Reachable?

Partitioning

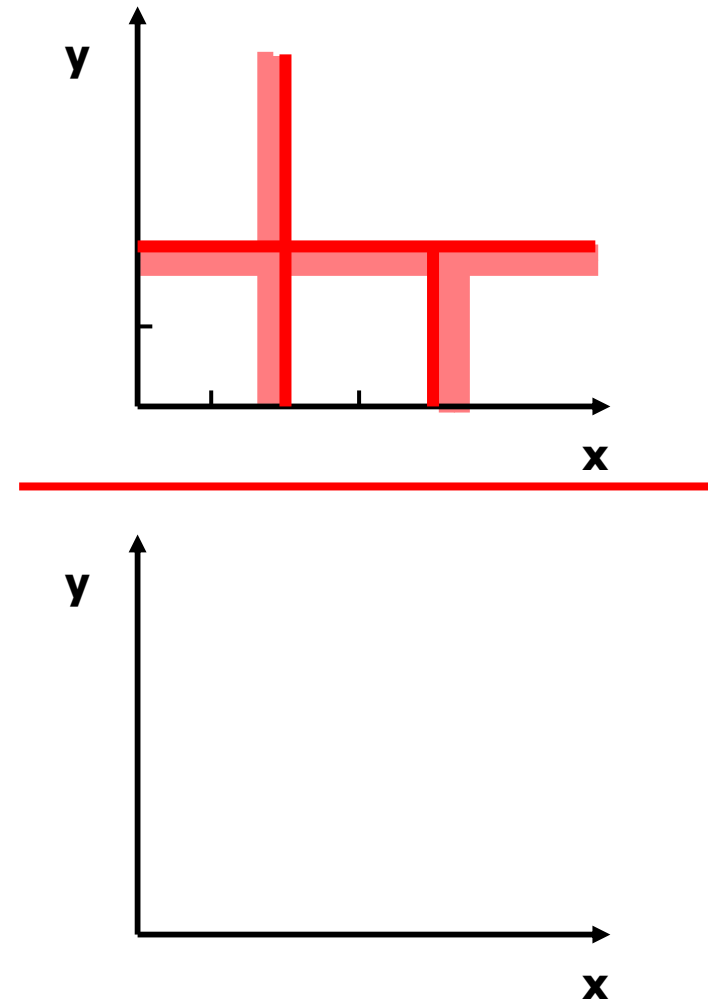


Stable Quotient

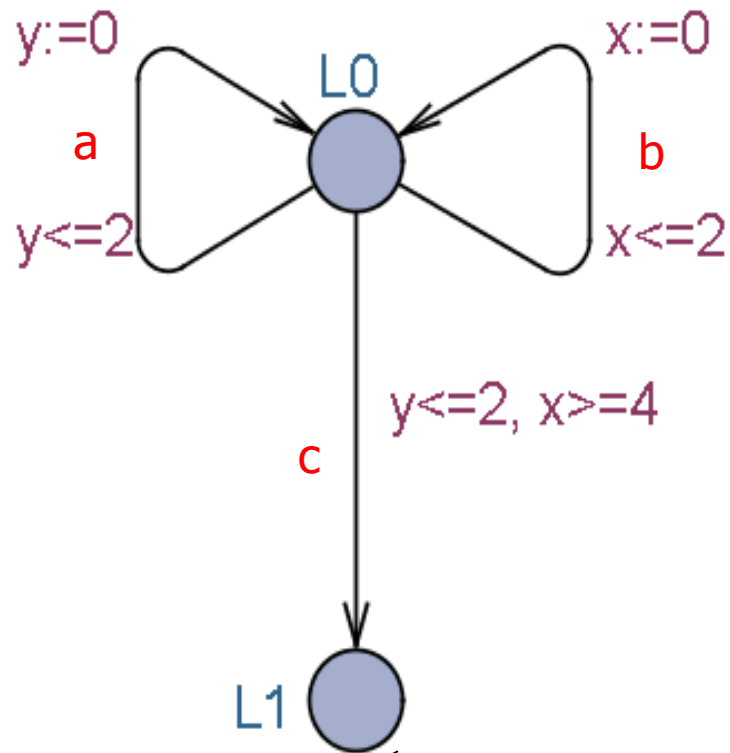


Reachable?

Partitioning

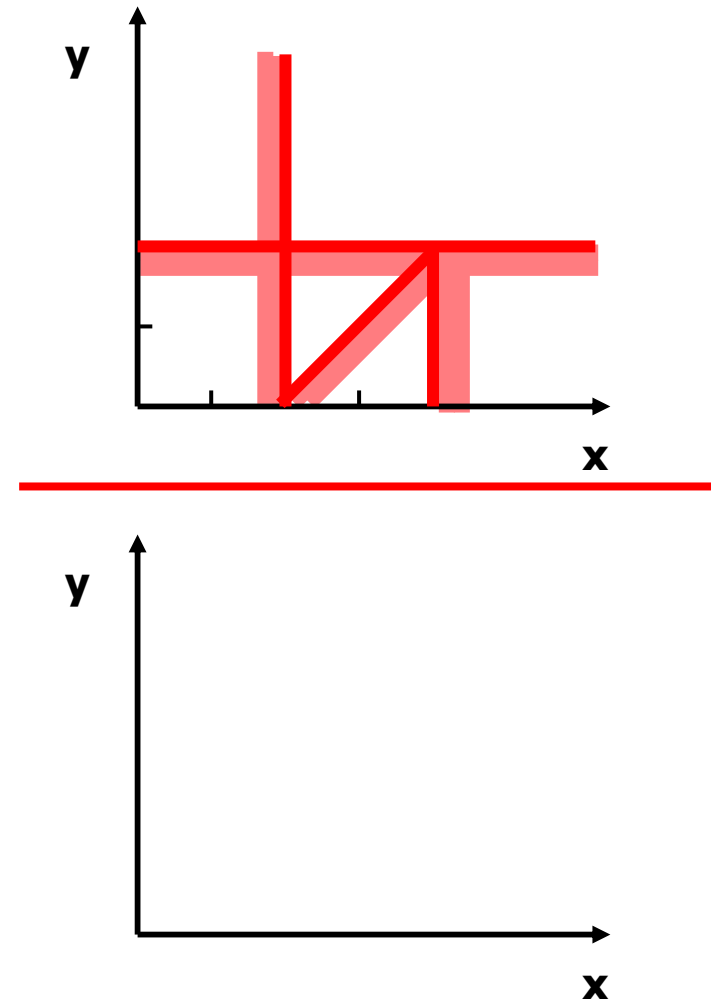


Stable Quotient

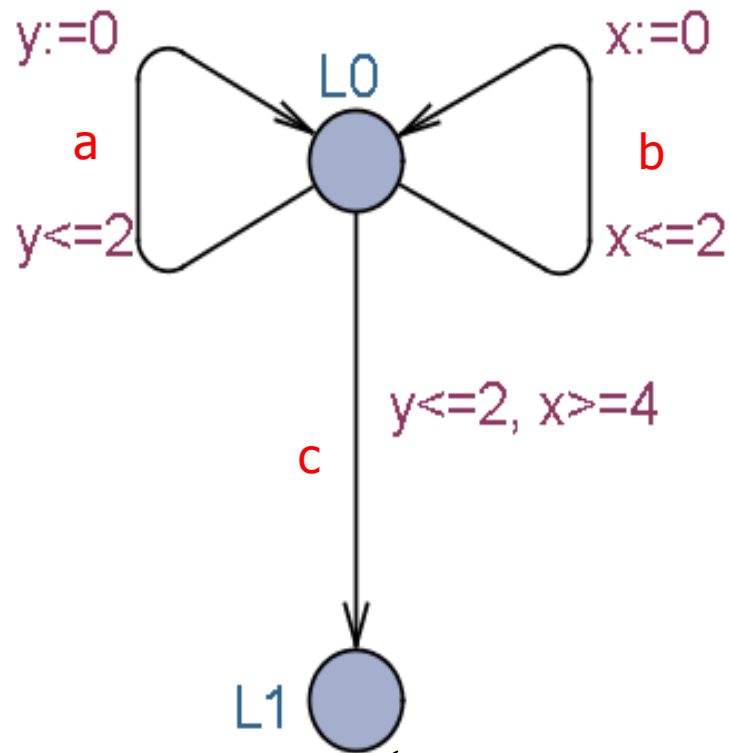


Reachable?

Partitioning

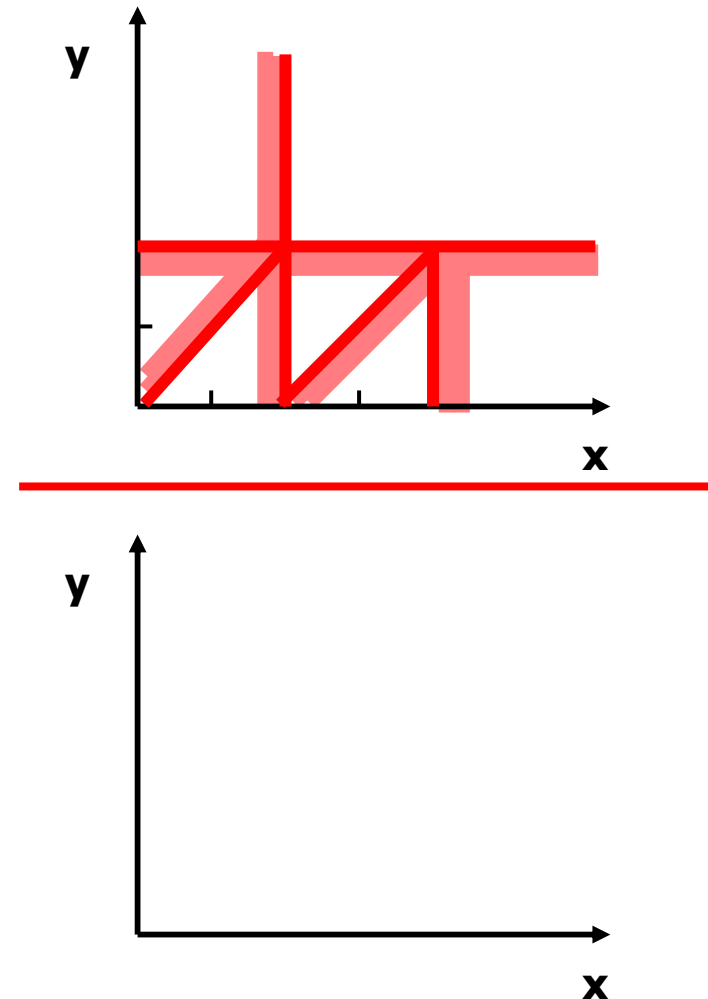


Stable Quotient



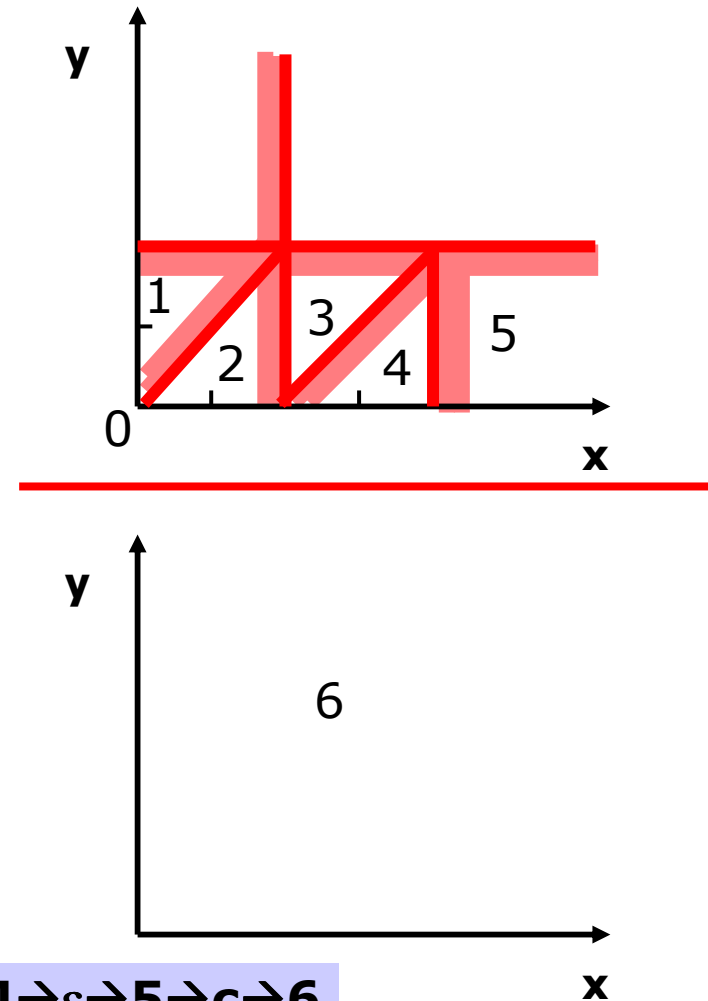
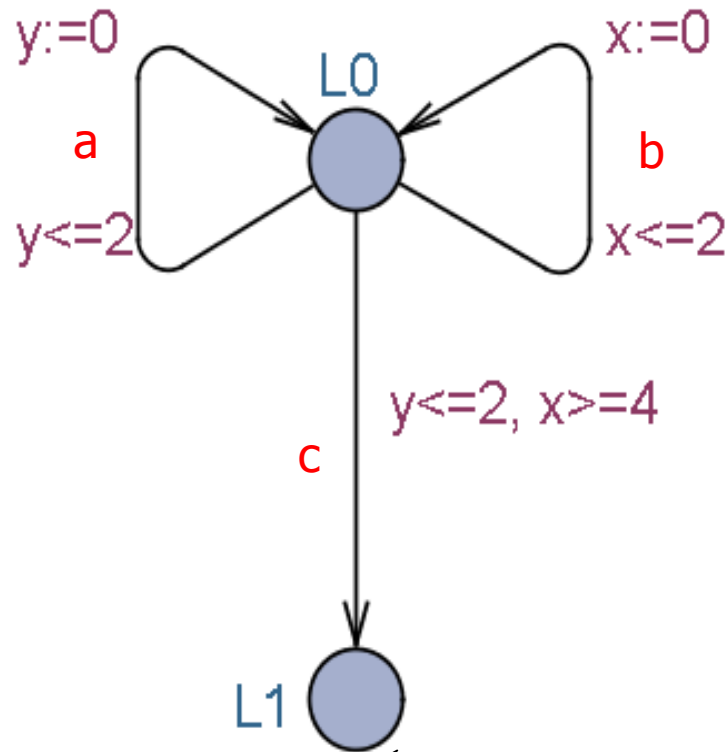
Reachable?

Partitioning



Stable Quotient

Partitioning



Reachable?

$0 \rightarrow \epsilon \rightarrow 1 \rightarrow a \rightarrow 2 \rightarrow \epsilon \rightarrow 3 \rightarrow a \rightarrow 4 \rightarrow \epsilon \rightarrow 5 \rightarrow c \rightarrow 6$

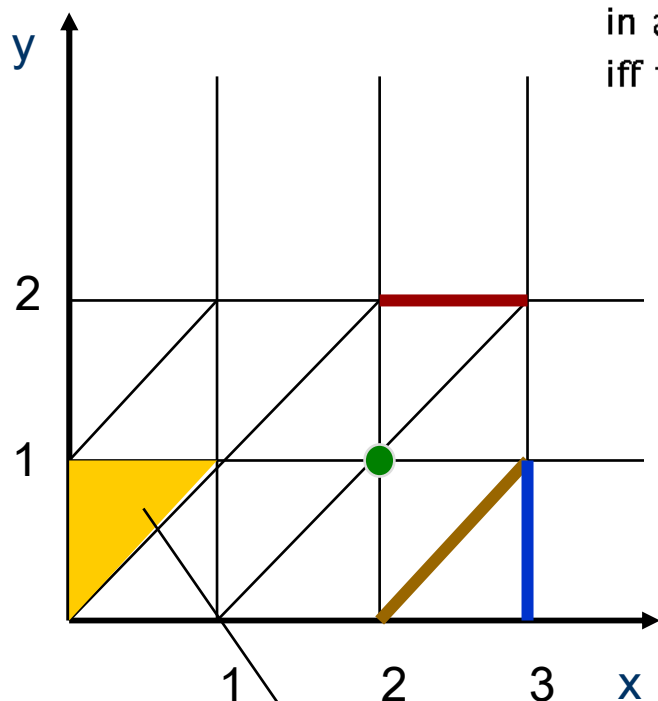
Region Equivalence

For each clock x let c_x be the largest integer with which x is compared in any guard or invariant of A . u and u' are *region equivalent*, $u \cong u'$ iff the following holds:

1. For all $x \in C$, either $\lfloor u(x) \rfloor = \lfloor u'(x) \rfloor$ or $u(x), u'(x) > c_x$;
2. For all $x, y \in C$ with $u(x) \leq c_x$ and $u(y) \leq c_y$,
 $fr(u(x)) \leq fr(u(y))$ iff $fr(u'(x)) \leq fr(u'(y))$;
3. For all $x \in C$ with $u(x) \leq c_x$,
 $fr(u(x)) = 0$ iff $fr(u'(x)) = 0$.

Regions

Finite Partitioning of State Space



For each clock x let c_x be the largest integer with which x is compared in any guard or invariant of A . u and u' are region equivalent, $u \cong u'$ iff the following holds:

1. For all $x \in C$, either $\lfloor u(x) \rfloor = \lfloor u'(x) \rfloor$ or $u(x), u'(x) > c_x$;
2. For all $x, y \in C$ with $u(x) \leq c_x$ and $u(y) \leq c_y$, $fr(u(x)) \leq fr(u(y))$ iff $fr(u'(x)) \leq fr(u'(y))$;
3. For all $x \in C$ with $u(x) \leq c_x$, $fr(u(x)) = 0$ iff $fr(u'(x)) = 0$.

An equivalence class (i.e. a *region*)
 in fact there is only a *finite* number of regions!!

Logical Characterization of Regions

Each region may be represented by specifying

1. for every clock x a constraint from

$$\{x = c \mid c = 0, 1, \dots, c_x\} \cup \{c - 1 < x < c \mid c = 1, \dots, c_x\} \cup \{x > c_x\}$$

2. for every pair of clocks x, y such that $c - 1 < x < c$ and $d - 1 < y < d$ appears in 1., whether $fr(x)$ is $<$, $=$ or $>$ than $fr(y)$.

Theorem

The number of regions is $n! \cdot 2^n \cdot \prod_{x \in C} (2c_x + 2)$.

Stability of Regions

Lemma

1. If $u \cong u'$ then $\forall d. \exists d'. u + d \cong u' + d'$;
2. If $u \cong u'$ then* $\forall g \in \mathcal{B}(X). u \models g \Leftrightarrow u' \models g$;
3. If $u \cong u'$ then $\forall r \subseteq C. u[r] \cong u'[r]$.

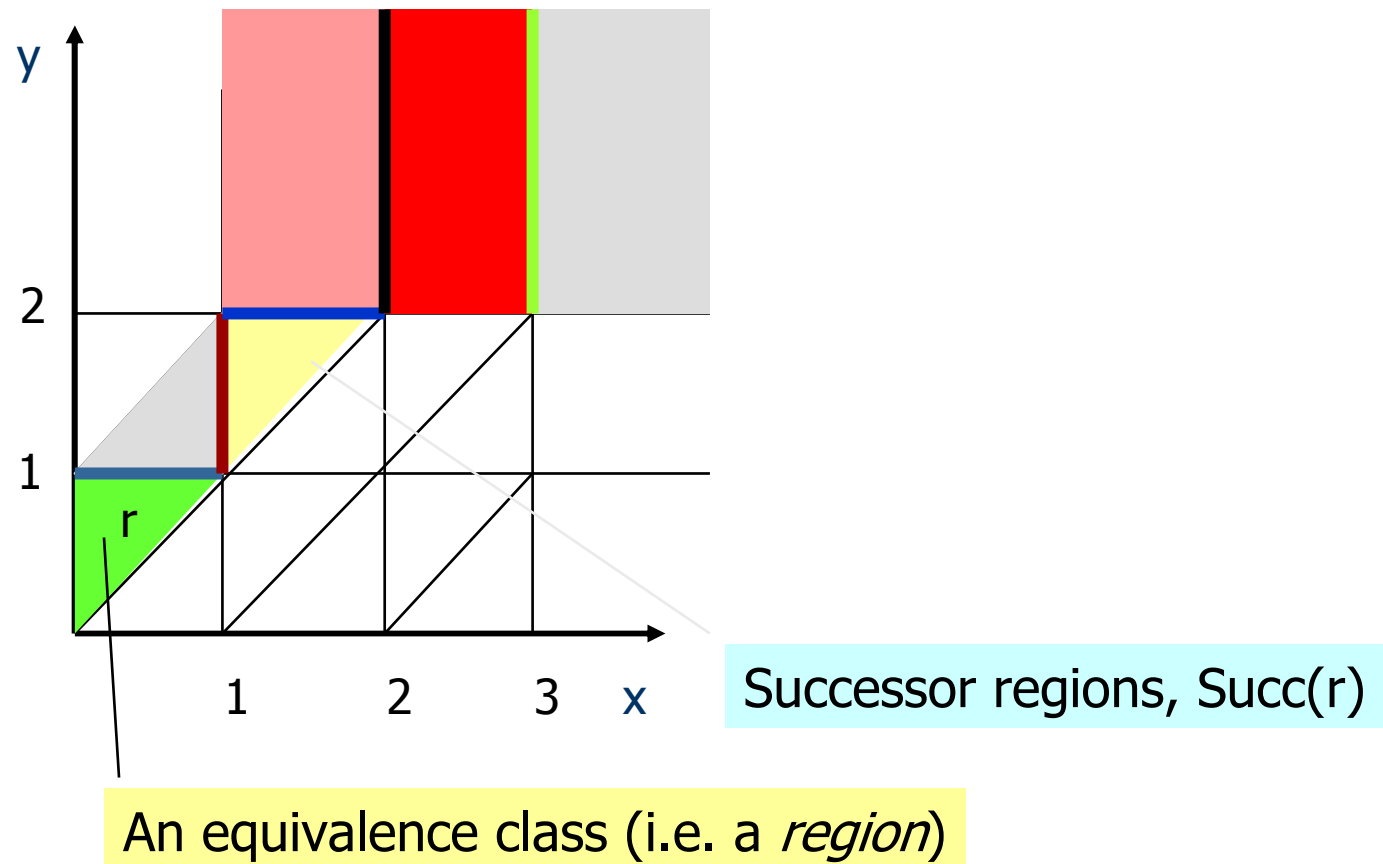
Theorem

Let $(l, u) \cong (l', u')$ iff $l = l'$ and $u \cong u'$. Then \cong is a TAB with respect to any set of goal locations G .

*Here \cong and g should agree on the maximal constants.

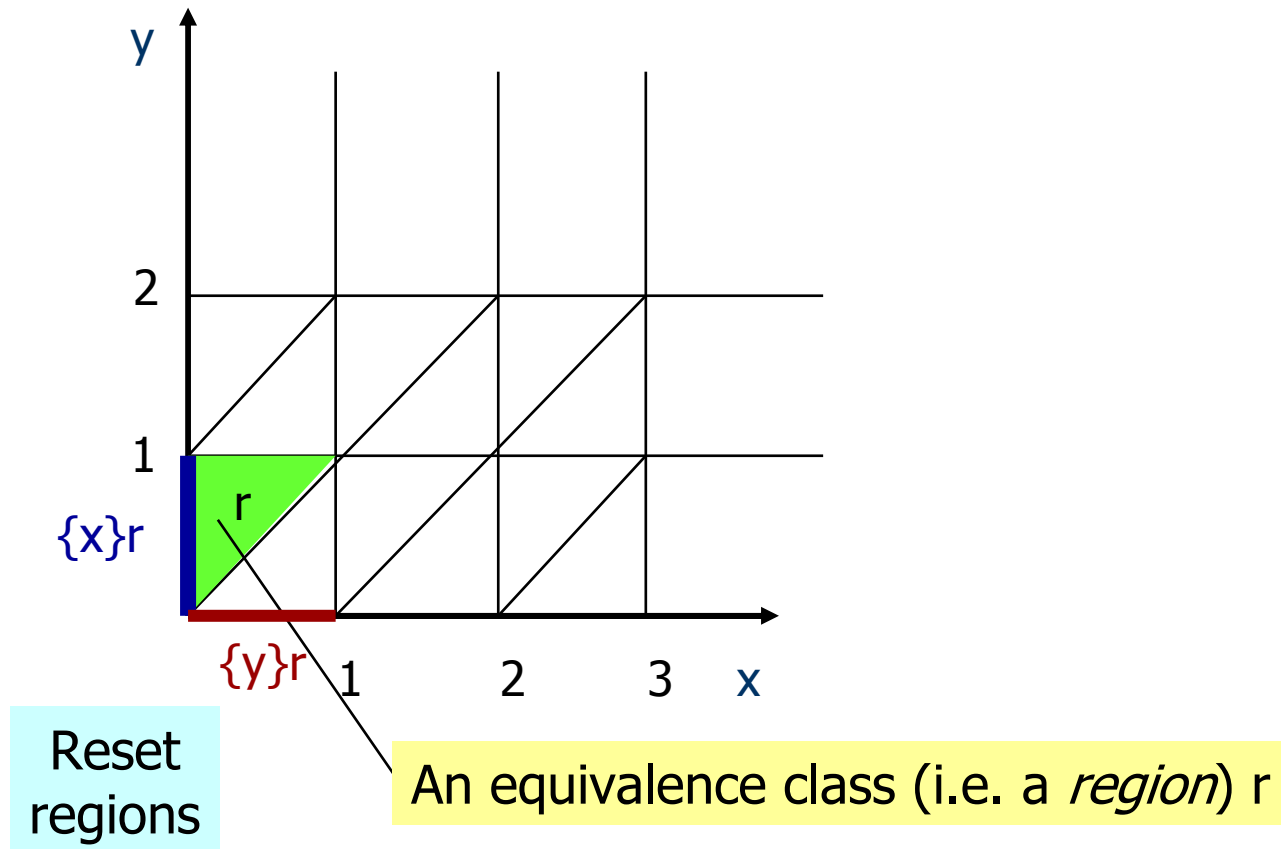
Regions

Successor Operation (wrt delay)

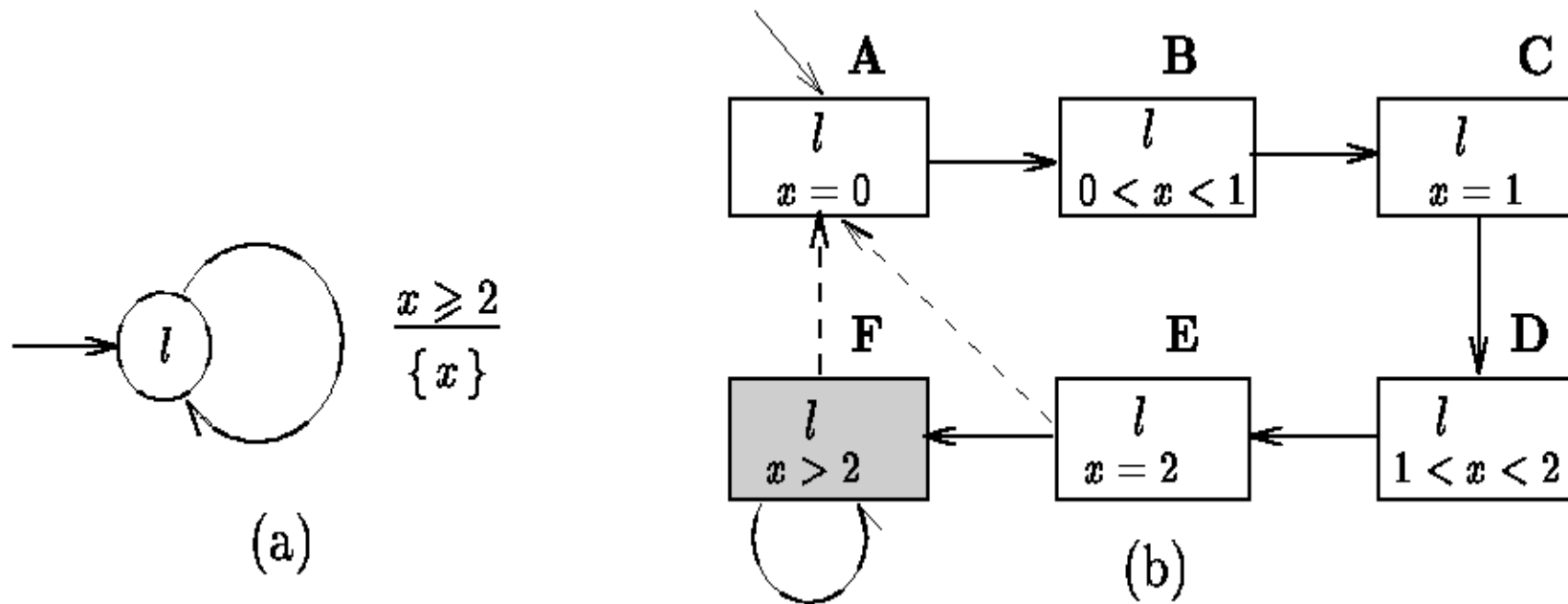


Regions

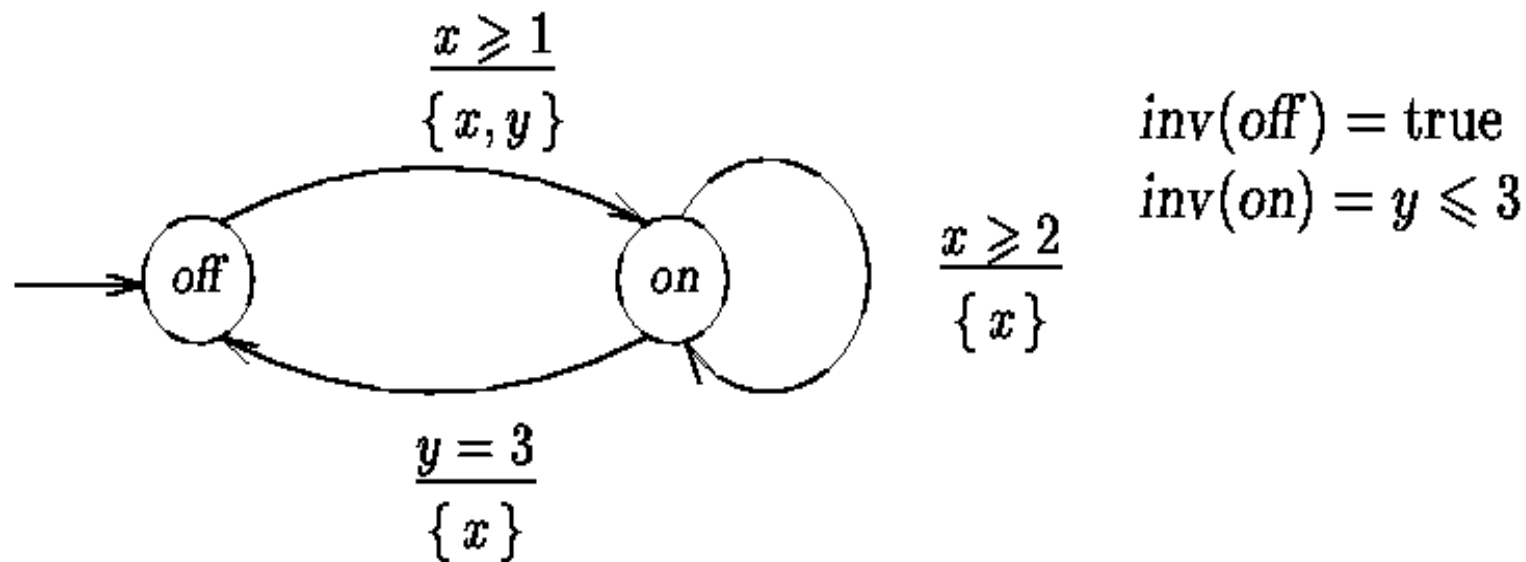
Reset Operation

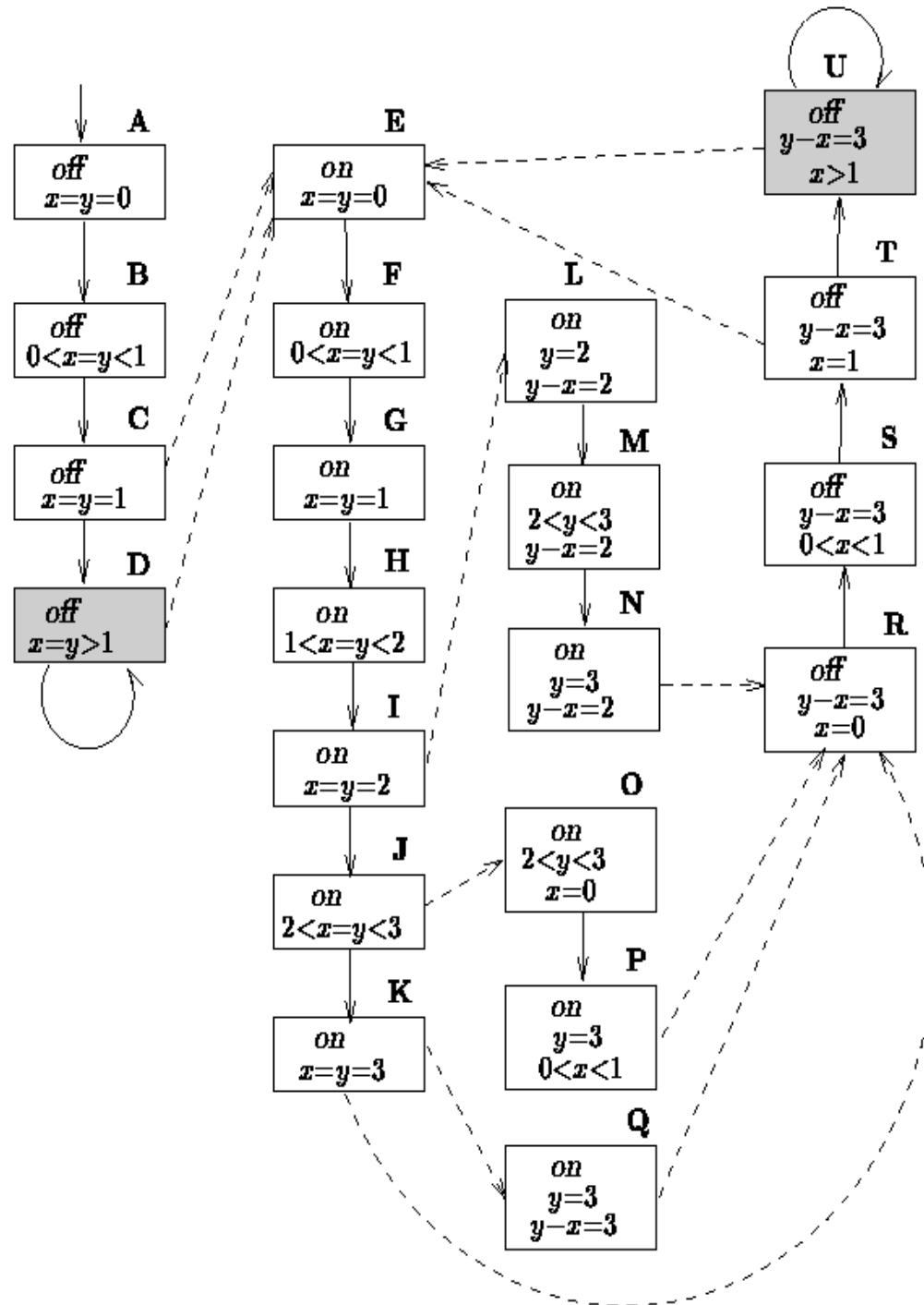


An Example Region Graph



Modified light switch





Reachable part
of region graph

Decidability & Complexity

Theorem

The transition relations $\xrightarrow{\delta}$ and \xrightarrow{a} between regions may be computed effectively using the triplet or the logical characterization of regions.

Theorem

Let A be a timed automaton with location l .
It is decidable whether $l \in Reach(A)$.

Theorem

Let A be a timed automaton with location l .
Deciding $l \in Reach(A)$ is PSPACE-complete.

Fundamental Results

- Reachability 😊

- Model-checking 😊
 - TCTL, L_{nu} , T_{mu} , ...

- Bisimulation, Simulation
 - Timed 😊 ; Untimed 😊

- Trace-inclusion
 - Timed 😞 ; Untimed 😊

Timed Bimulation

Wang'91, Cerans'92



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Timed Bisimulation

Wang'91

R is a timed bisimulation if whenever sRt then the following holds:

$$i) \quad (s \xrightarrow{a} s') \Rightarrow (\exists t'. t \xrightarrow{a} t' \wedge s'Rt')$$

$$ii) \quad (t \xrightarrow{a} t') \Rightarrow (\exists s'. s \xrightarrow{a} s' \wedge s'Rt')$$

for all $a \in \text{Act} \cup \text{Del}$.

$$\text{Del} = \{d : d \in \mathbb{R}_{\geq 0}\}$$

We write $s \approx t$ whenever sRt for some timed bisimulation R .

Timed Simulation

R is a timed simulation if whenever sRt then the following holds:

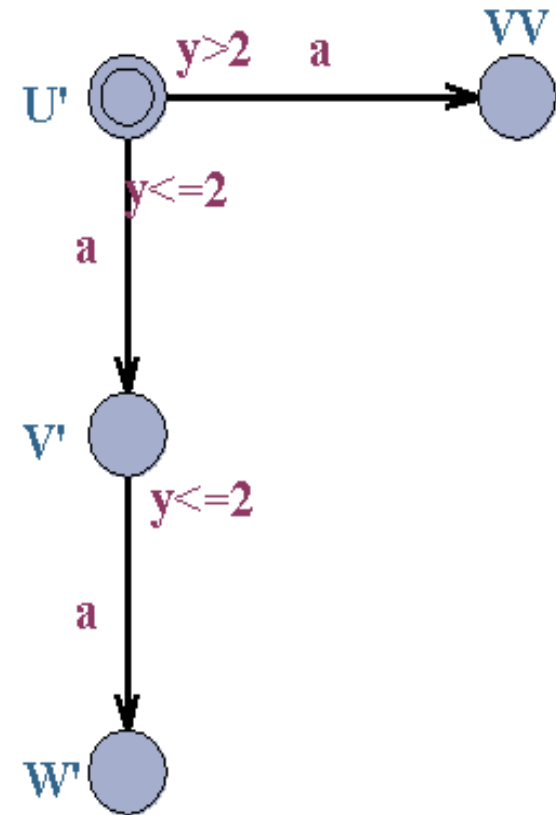
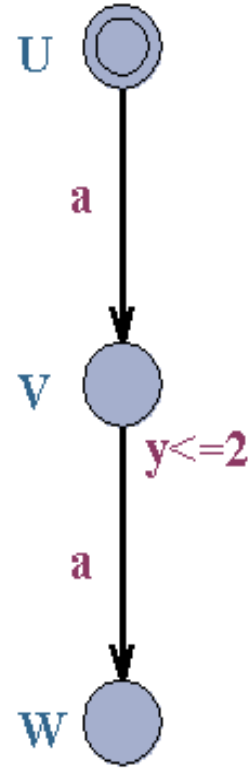
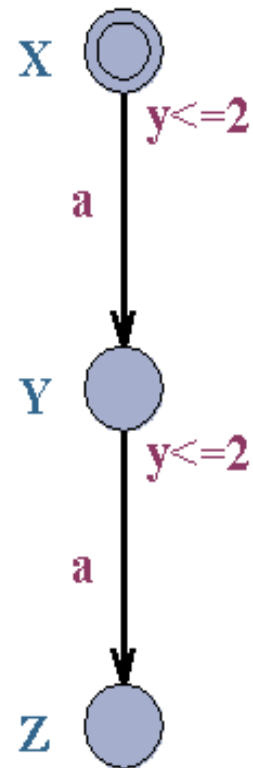
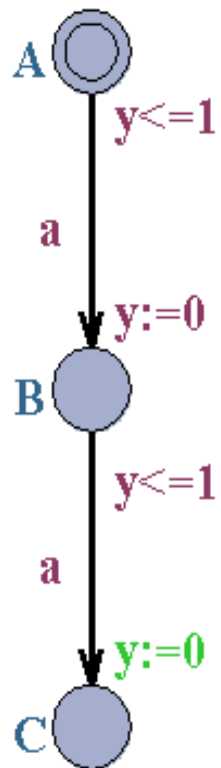
$$i) \quad (s \xrightarrow{a} s') \Rightarrow (\exists t'. t \xrightarrow{a} t' \wedge s'Rt')$$

for all $a \in \text{Act} \cup \text{Del}$.

$$\text{Del} = \{d : d \in R_{\geq 0}\}$$

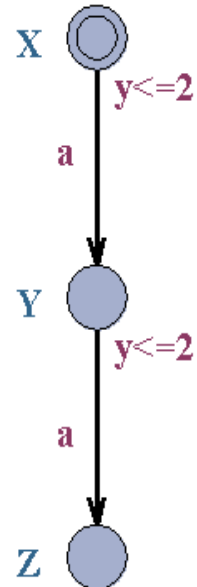
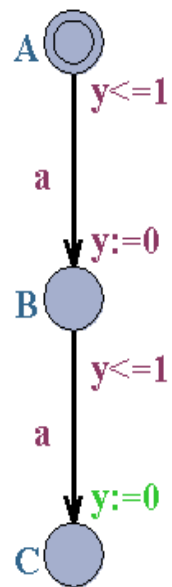
We write $s < t$ iff sRt for some timed simulation R .

Examples

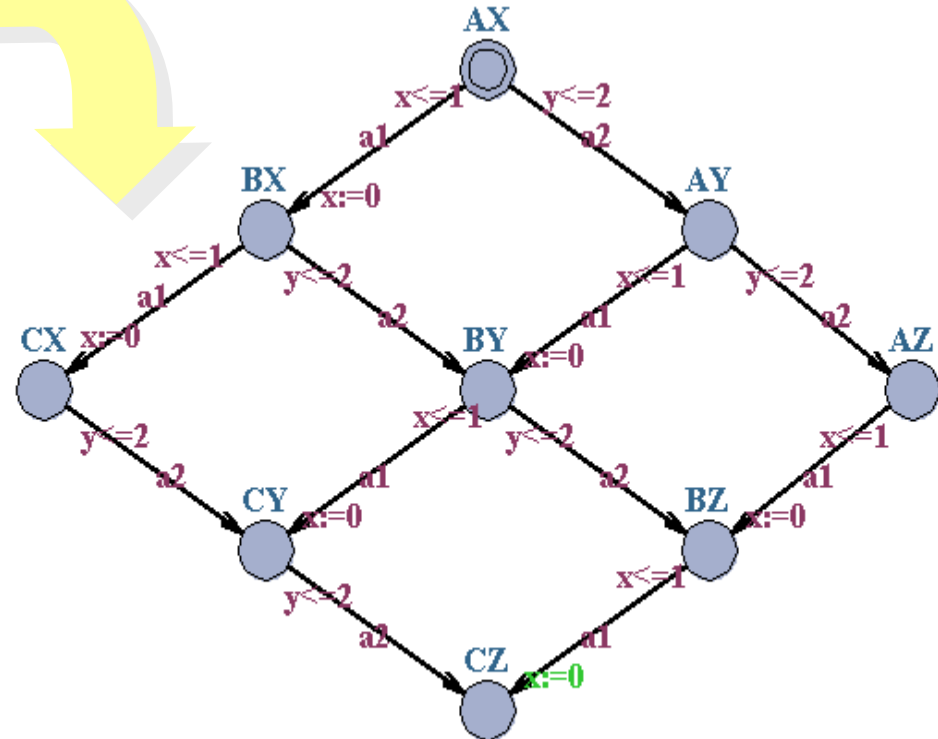
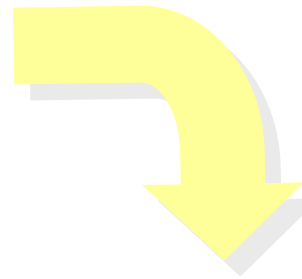


Towards Timed Bisimulation Algorithm

Cerans'92



independent
 "product-construction"



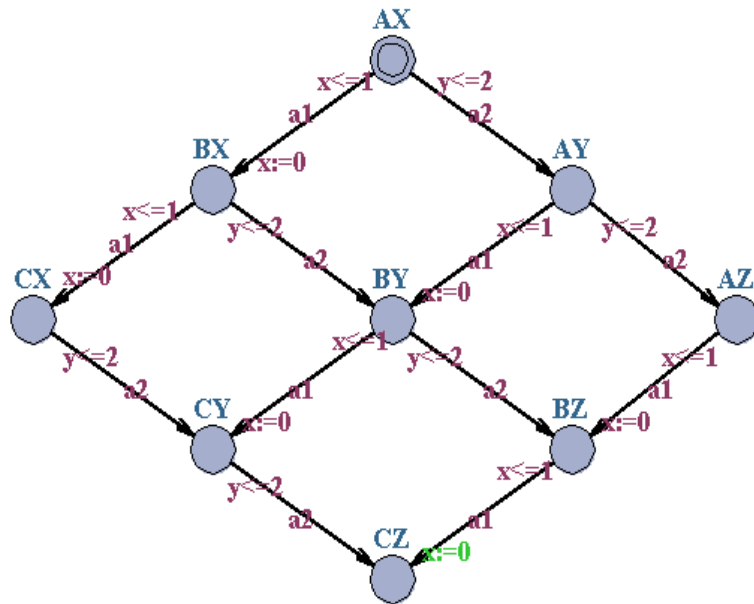
Towards Timed Bisimulation Algorithm

Definition

B is a timed product-bisimulation iff whenever $s \in B$ then the following holds:

- i) if $s \xrightarrow{d} s'$ then $s' \in B$
- ii) if $s \xrightarrow{a_1} s'$ then $s' \xrightarrow{a_2} s''$ s.t. $s'' \in B$
- iii) if $s \xrightarrow{a_2} s'$ then $s' \xrightarrow{a_1} s''$ s.t. $s'' \in B$

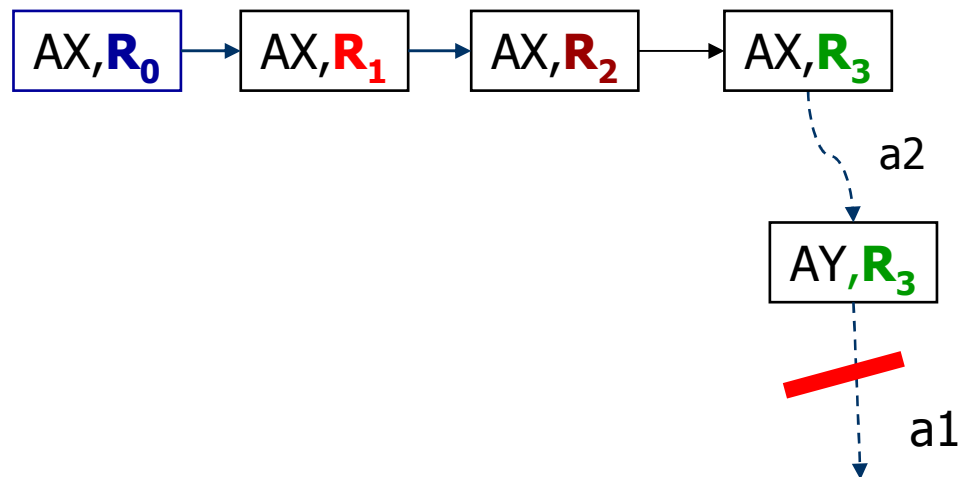
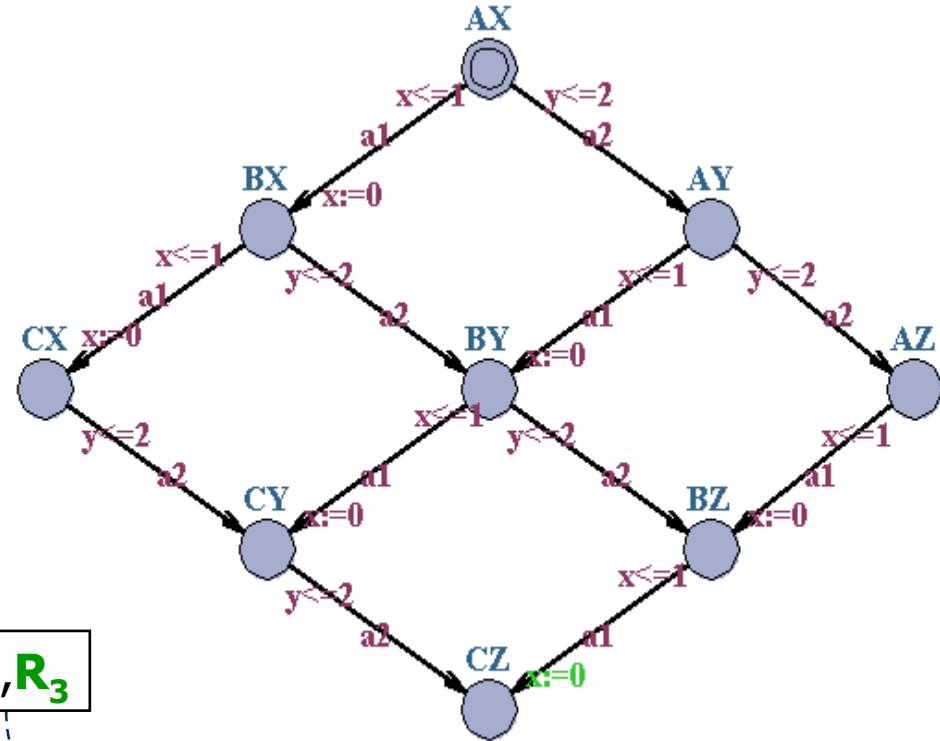
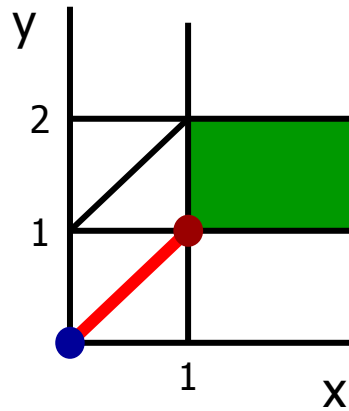
We write $TB(s)$ whenever $s \in B$ for some timed product-bisimulation.



Theorem

$$TB(s) \iff (S_1 \approx S_2)$$

Timed Bisimulation Algorithm = Checking for TB-ness using Regions



Timed Trace Inclusion

Undecidability



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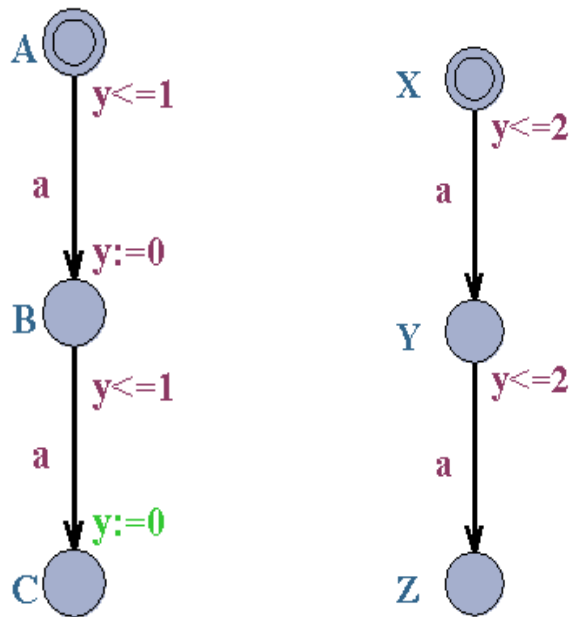
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Timed Trace Languages

■ Timed trace

$$(t_1, a_1), (t_2, a_2), \dots, (t_k, a_k)$$

where a_i is an action and $t_i \in \mathbf{R}$, with t_i 's non-decreasing.



$$L(A) = \{ (t_1, a), (t_2, a) : \\ t_1 \cdot 1 \in \mathbb{E} \\ t_1 \cdot t_2 \cdot t_1 + 1 \}$$

$$L(X) = \{ (t_1, a), (t_2, a) : \\ t_1 \cdot t_2 \cdot 2 \}$$

Timed Trace Languages

■ Timed trace

$$(t_1, a_1), (t_2, a_2), \dots, (t_k, a_k)$$

where a_i is an action and $t_i \in \mathbf{R}$, with t_i 's strictly increasing.

■ PROPOSITIONS

- Given a timed automaton A it is **UNDECIDABLE** whether the set of timed traces of A is the **UNIVERSAL** set.
- Given two timed automata A and B it is **UNDECIDABLE** whether the set of timed traces of A is **INCLUDED** in the set of timed traces of B.

Two-counter Machine

- $M = (\{b_0, b_1, \dots, b_k\} , C, D)$

where b_i 's are instructions
 C and D are counters ranging
 over \mathbf{N} .
 Initially both C and D are 0.

- Instructions ($i < k$):
 - Increment b_j : $C := C + 1$; goto b_l
 - Decrement b_j : if $C \neq 0$
 then $C := C - 1$; goto b_l
 else goto b_m
- b_k represents termination

Two-counter Machine

- $M = (\{b_0, b_1, \dots, b_k\} , C, D)$
- Configuration of M:
 (b_i , c , d)
 where c and d are values of C and D .
- Computation of M is a “valid” sequence of configurations starting with $(b_0, 0, 0)$ and ending with $(b_k, _, _)$.
- **PROPOSITION**
 Deciding whether a two-counter machine has a (halting) computation is **UNDECIDABLE**.

Timed Trace Language for Two-counter Machine

- Let M be a two-counter machine. We define $L(M)$ to be the set of timed traces over

$$\Sigma = \{b_0, \dots, b_k, c, d\}$$

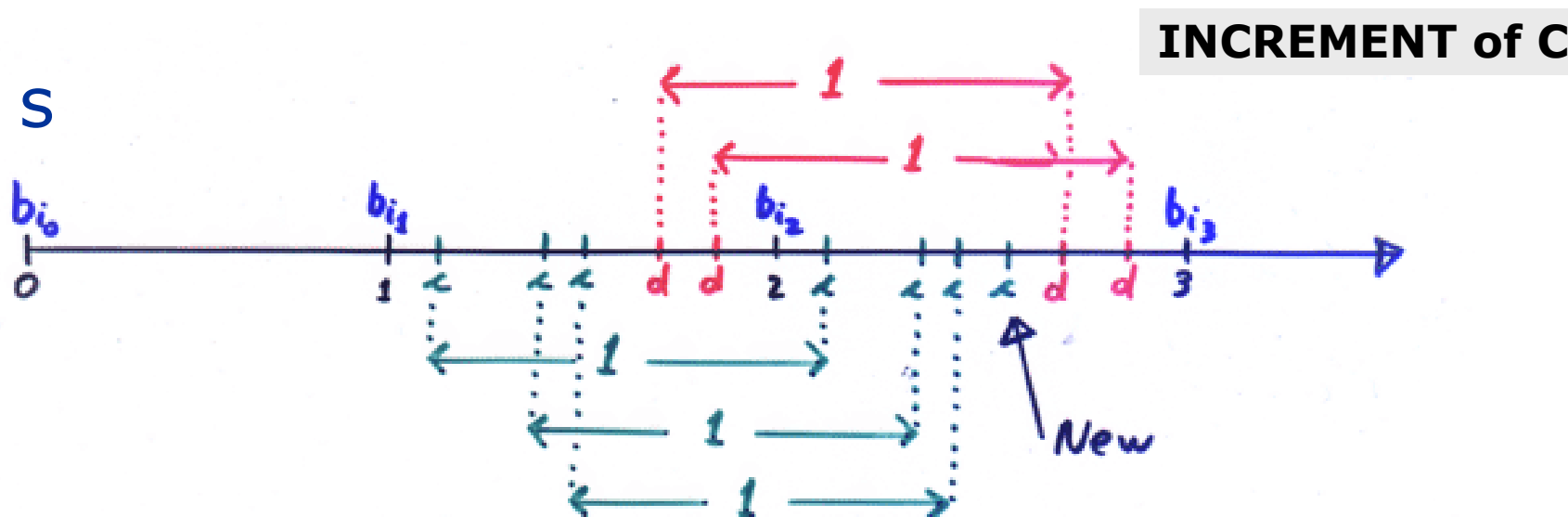
such that whenever

$$(b_{i_0}, c_0, d_0)(b_{i_1}, c_1, d_1) \dots (b_{i_n}, c_n, d_n)$$

is a computation of M then the timed trace s is in $L(M)$ where:

- $\text{Untime}(s) = b_{i_0}c^{c_0}d^{d_0}b_{i_1}c^{c_1}d^{d_1} \dots b_{i_n}c^{c_n}d^{d_n}$
- $\text{Time}(b_j) = j$
- “proper matching of c ’s and d ’s”

Proper Matching

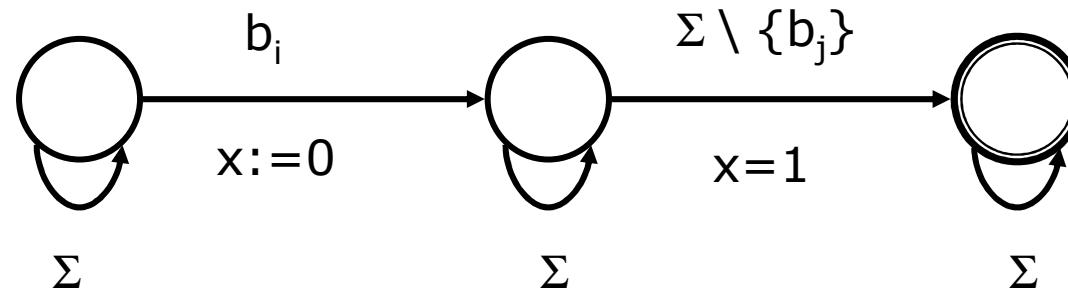


- Clearly M has a (halting) computation iff $L(M) \neq \emptyset$
- One can show that $L(M)^C$ can be captured by a Timed Automaton (a union of several small ones).

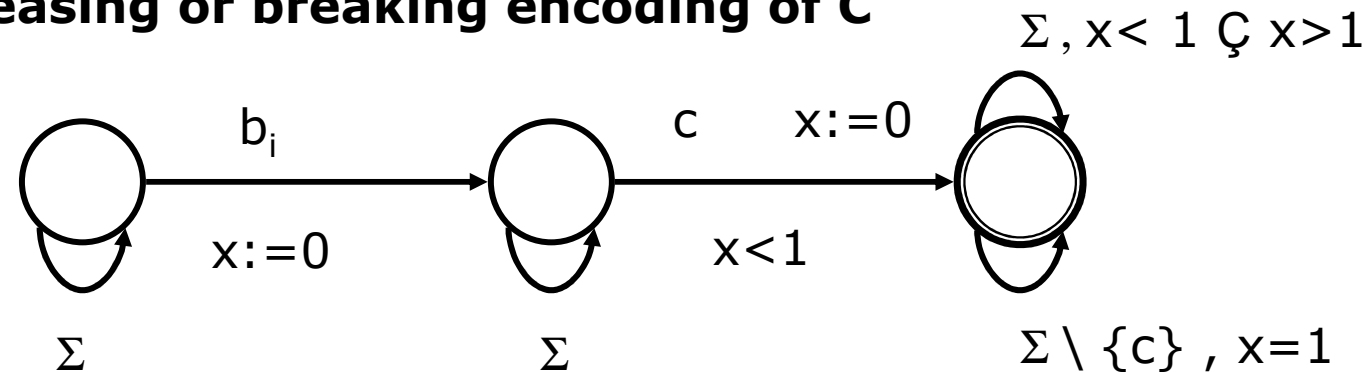
Example Automata

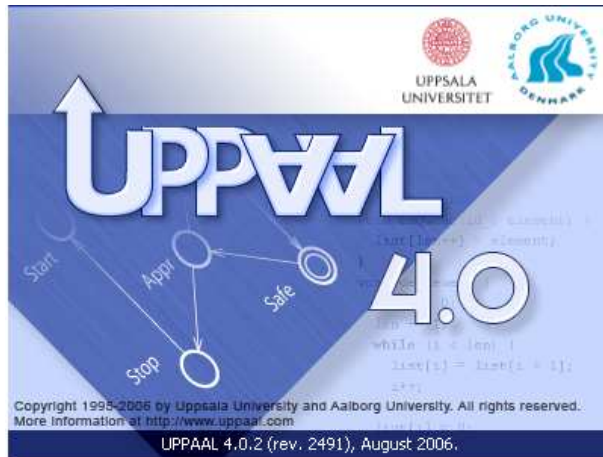
Violation of $b_i : D := D + 1 ; \text{goto } b_j$

1) Jumping to another instruction than b_j :



2) Decreasing or breaking encoding of C





The UPPAAL Verification Engine

Kim Guldstrand Larsen



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Basic Research
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Overview

- Train Crossing
 - Full Modeling & Specification Formalism
 - Schedulability Analysis

- UPPAAL Verification Engine
 - Symbolic On-the-fly Exploration
 - Zones & DBMs
 - CDDs

- Verification Options
 - Over- / Under Approximations
 - Storage Strategies

Timed Automata in UPPAAL

Train Crossing



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UPPAAL

Tool Environment for
modeling, simulation,
verification, optimization &
synthesis
of real-time systems

Graphical Design Tool

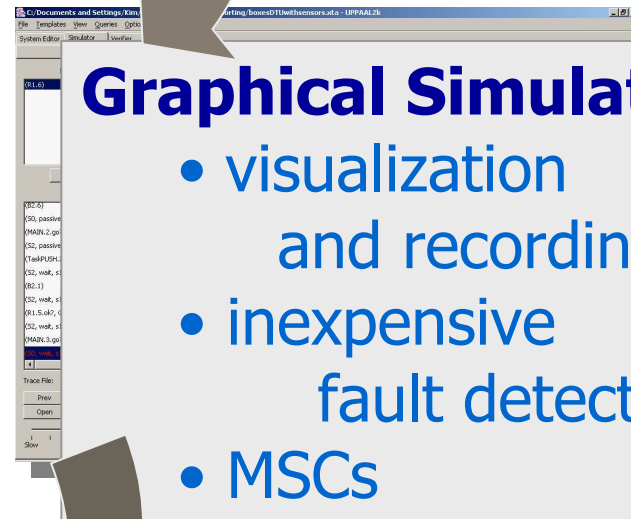
- timed automata
- clocks
- communication
- datatypes & functions
- cost variables
- uncontr. behaviour

Graphical Simulator

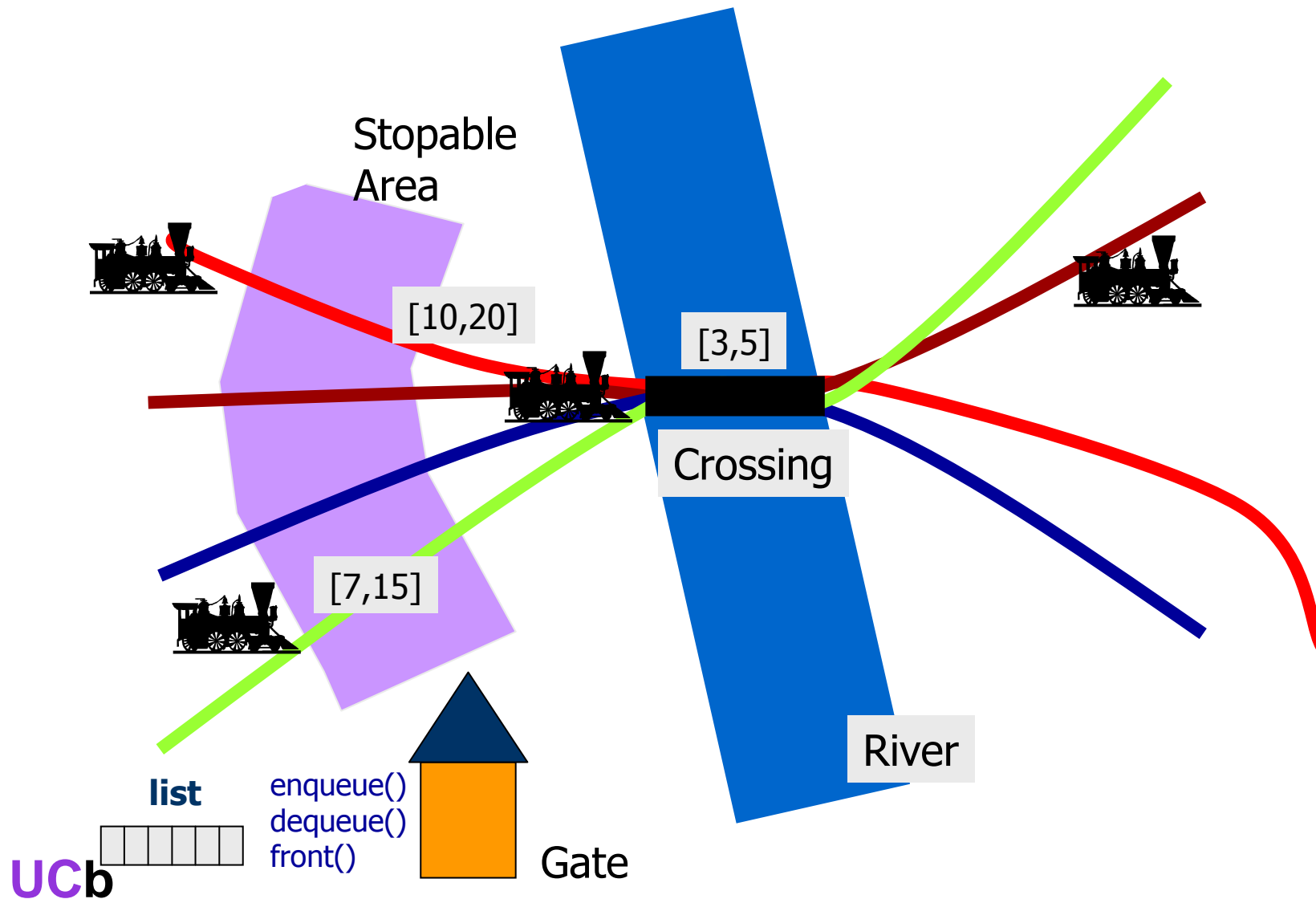
- visualization and recording
- inexpensive fault detect.
- MSCs

Verifier

- exhaustive & automatic checking of requirements
- diagnostic traces
- optimal scheduling
- controller synthesis

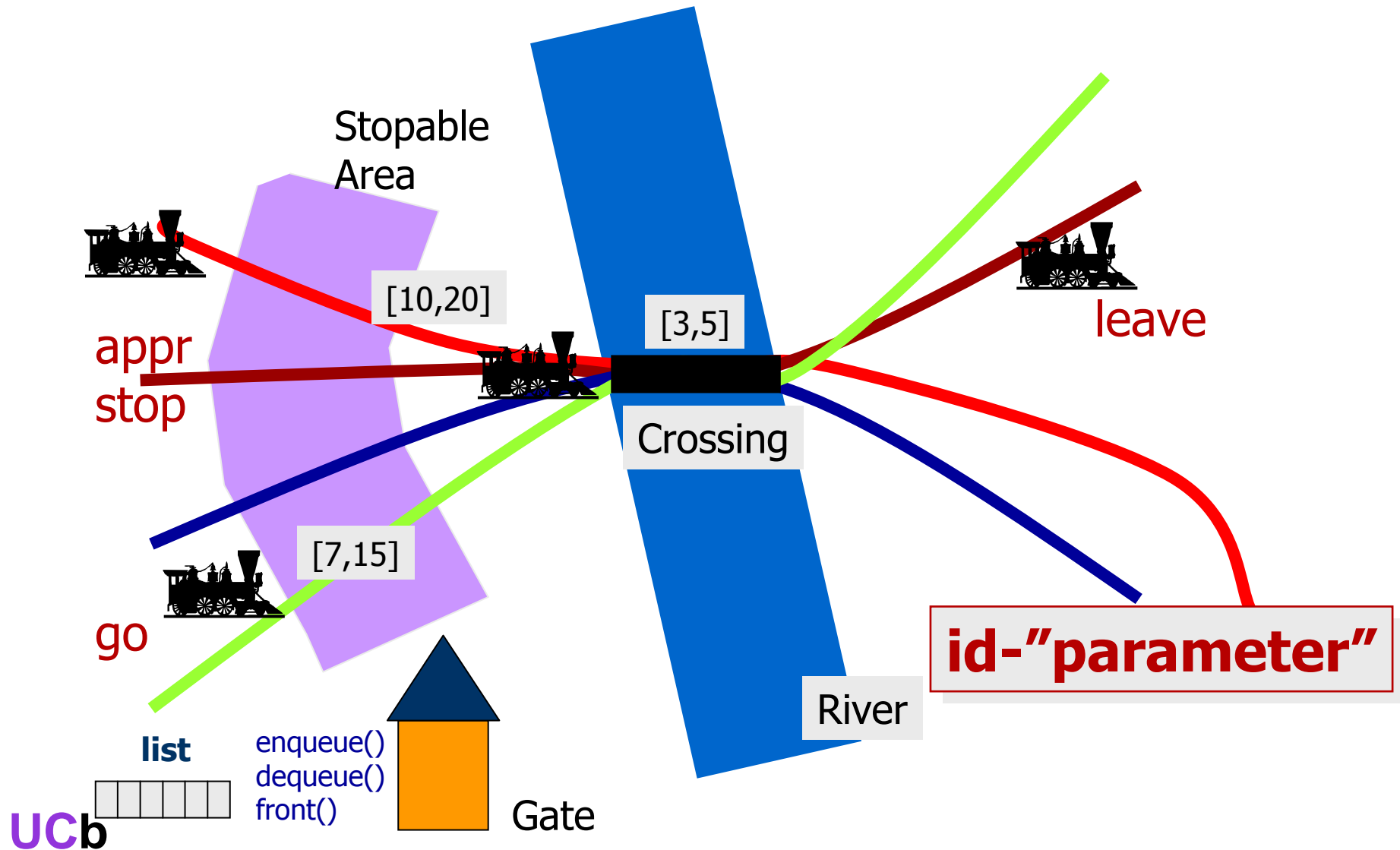


Train Crossing



Train Crossing

Communication via channels!



Queries : Specification Language



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Logical Specifications

- Validation Properties

- Possibly: $E \leftrightarrow P$

- Safety Properties

- Invariant: $A[] P$

- Pos. Inv.: $E[] P$

- Liveness Properties

- Eventually: $A \leftrightarrow P$

- Leadsto: $P \rightarrow Q$

- Bounded Liveness

- Leads to within: $P \rightarrow_{.t} Q$

The expressions P and Q must be type safe, side effect free, and evaluate to a boolean.

Only references to integer variables, constants, clocks, **and locations** are allowed (and arrays of these).

Logical Specifications

- Validation Properties

- Possibly: $E \leftrightarrow P$

- Safety Properties

- Invariant: $A[] P$

- Pos. Inv.: $E[] P$

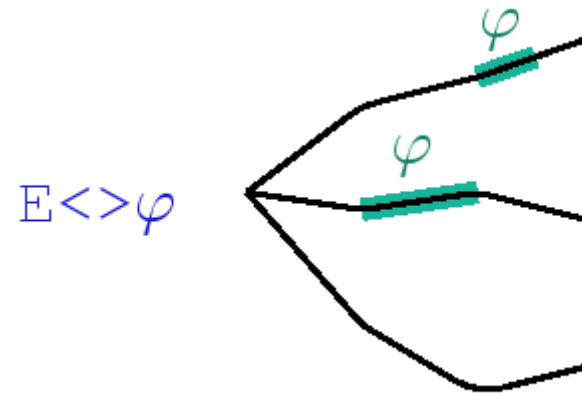
- Liveness Properties

- Eventually: $A \langle \rangle P$

- Leadsto: $P \rightarrow Q$

- Bounded Liveness

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Logical Specifications

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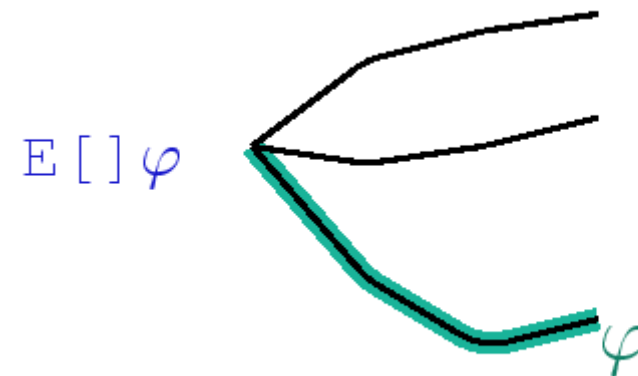
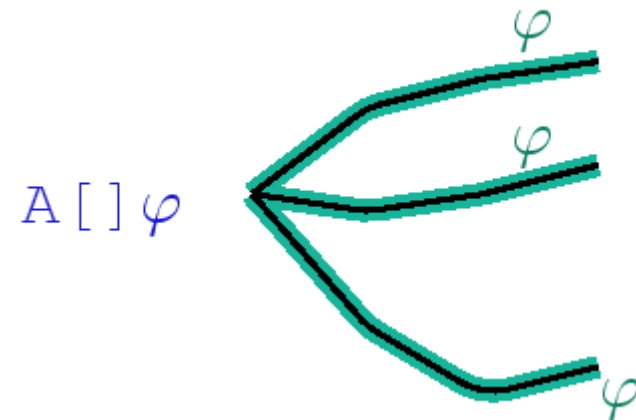
- Liveness Properties

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Logical Specifications

- Validation Properties

- Possibly: $E \langle \rangle P$

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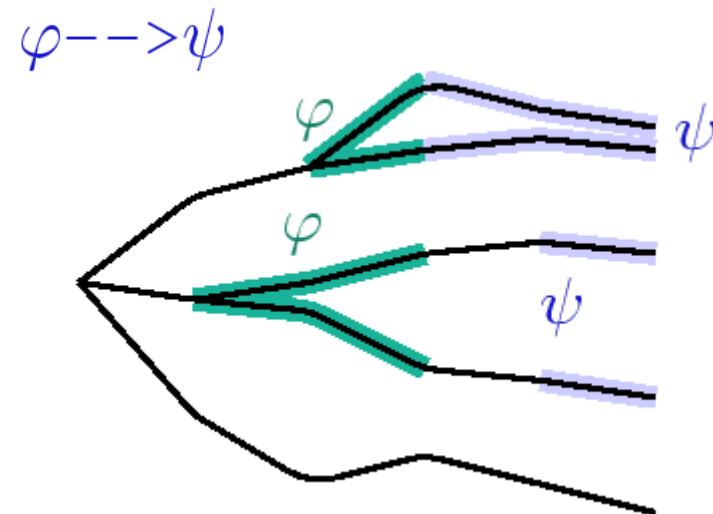
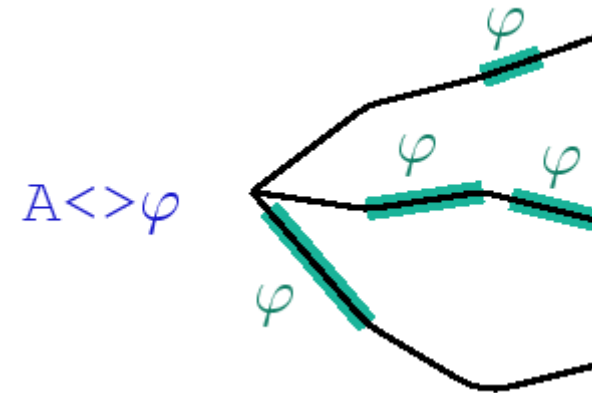
- Liveness Properties

- Eventually: $A \langle \rangle P$

- Leadsto: $P \rightarrow Q$

- Bounded Liveness

- Leads to within: $P \rightarrow_{.t} Q$



Logical Specifications

- Validation Properties

- Possibly: $E \langle \rangle P$

- Safety Properties

- Invariant: $A [] P$

- Pos. Inv.: $E [] P$

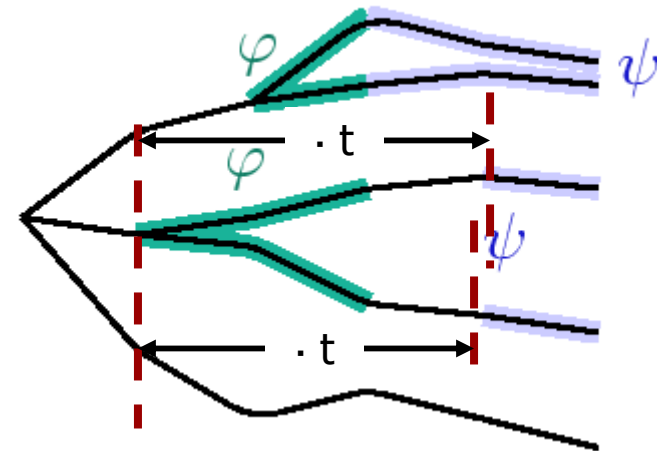
- Liveness Properties

- Eventually: $A \langle \rangle P$

- Leadsto: $P \rightarrow Q$

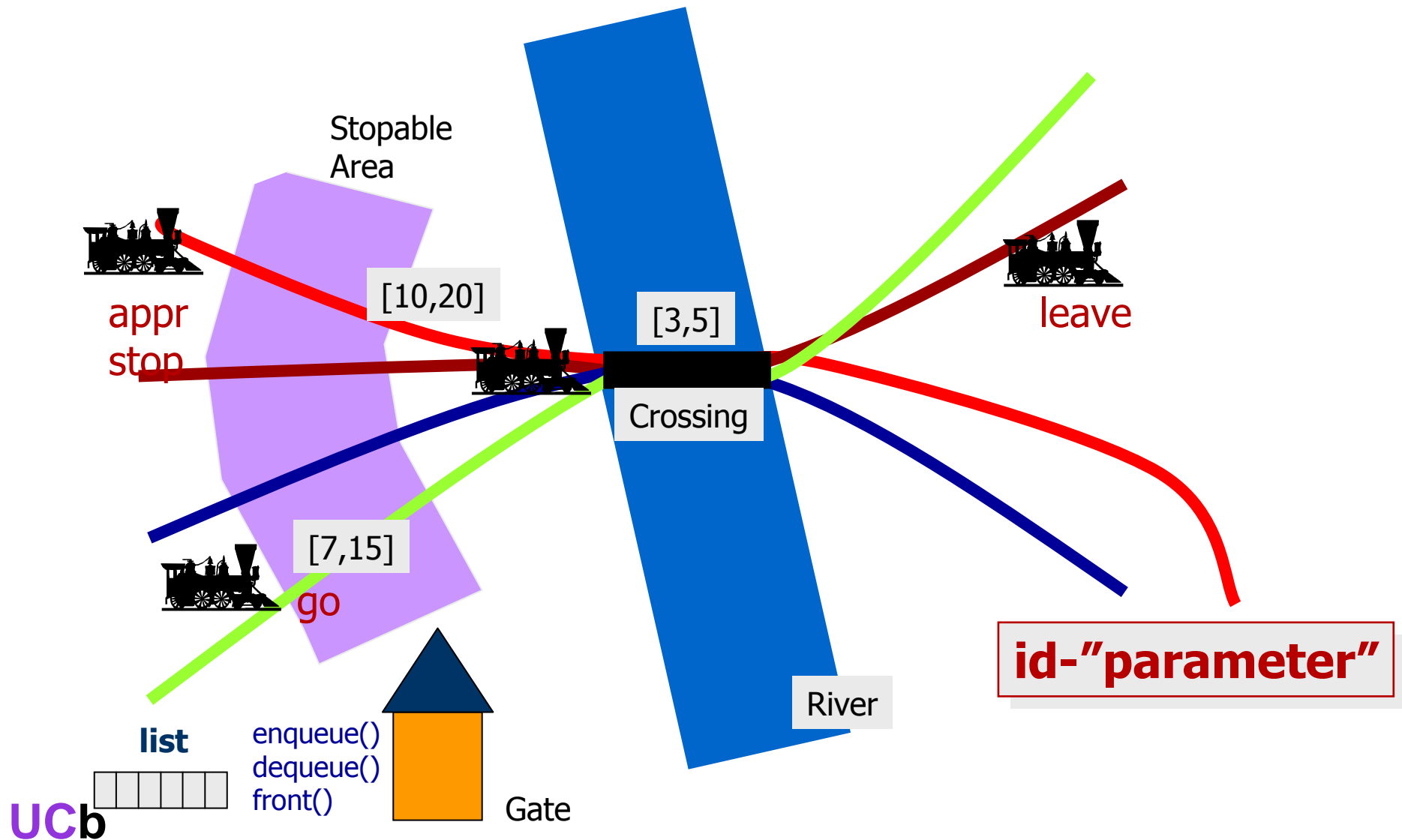
- Bounded Liveness

- Leads to within: $P \rightarrow_{.t} Q$



Train Crossing

Communication via channels!



Case-Studies: Controllers

- Gearbox Controller [TACAS'98]
- Bang & Olufsen Power Controller [RTPS'99, FTRTFT'2k]
- SIDMAR Steel Production Plant [RTCSA'99, DSVV'2k]
- Real-Time RCX Control-Programs [ECRTS'2k]
- Experimental Batch Plant (2000)
- RCX Production Cell (2000)
- **Terma, Verification of Memory Management for Radar (2001)**
- Scheduling Lacquer Production (2005)
- Memory Arbiter Synthesis and Verification for a Radar Memory Interface Card [NJC'05]

Case Studies: Protocols

- Philips Audio Protocol [HS'95, CAV'95, RTSS'95, CAV'96]
- Collision-Avoidance Protocol [SPIN'95]
- Bounded Retransmission Protocol [TACAS'97]
- **Bang & Olufsen Audio/Video Protocol** [RTSS'97]
- TDMA Protocol [PRFTS'97]
- Lip-Synchronization Protocol [FMICS'97]
- Multimedia Streams [DSVIS'98]
- ATM ABR Protocol [CAV'99]
- **Leader Election for Mobile Ad Hoc Networks**
[Charme05]
- ABB Fieldbus Protocol [ECRTS'2k]
- IEEE 1394 Firewire Root Contention (2000)
- Distributed Agreement Protocol [Formats05]

Zones & DBMs



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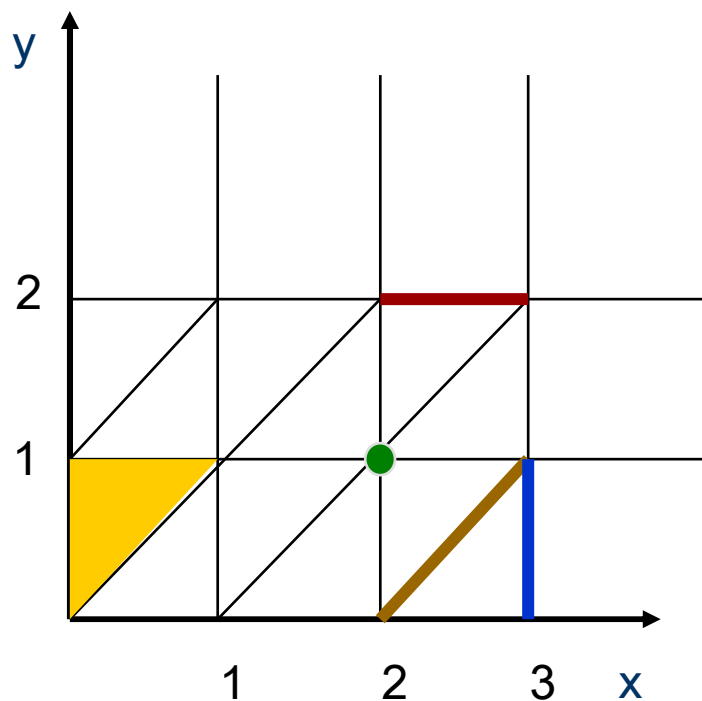
Basic Research
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Regions

Finite Partitioning of State Space



Time Abstracted Bisimulation

Equivalence classes (i.e. a *region*)

in fact there is only a *finite* number of regions!!

Theorem

The number of regions is $n! \cdot 2^n \cdot \prod_{x \in C} (2c_x + 2)$.

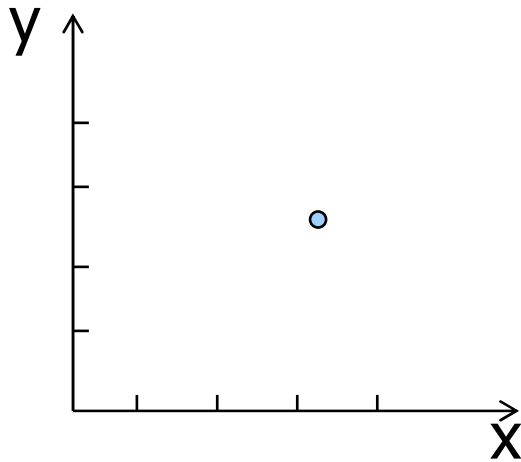
$$0 < x < 1 \wedge 0 < y < 1 \wedge y - x > 0$$

Zones

From infinite to finite

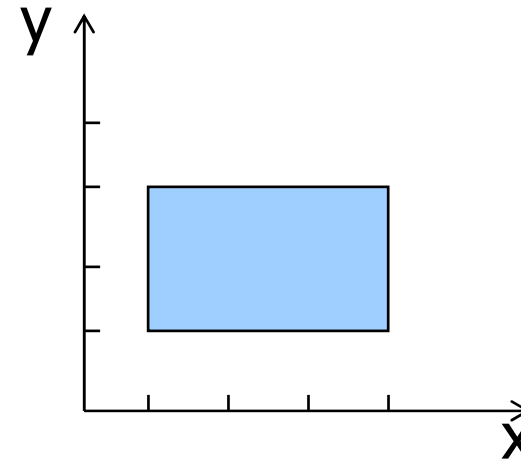
State

$(n, x=3.2, y=2.5)$



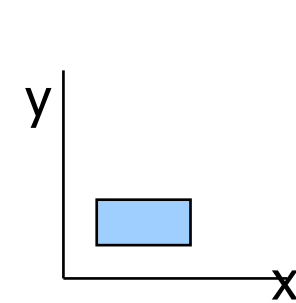
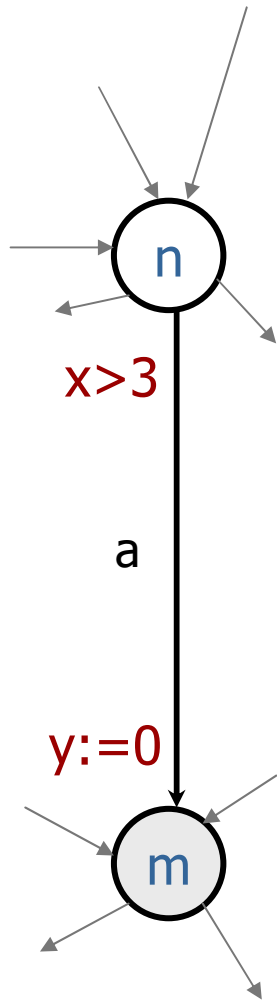
Symbolic state (set)

$(n, 1 \cdot x \cdot 4, 1 \cdot y \cdot 3)$

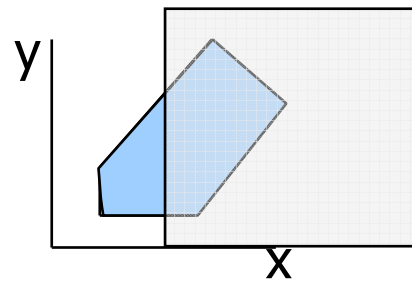
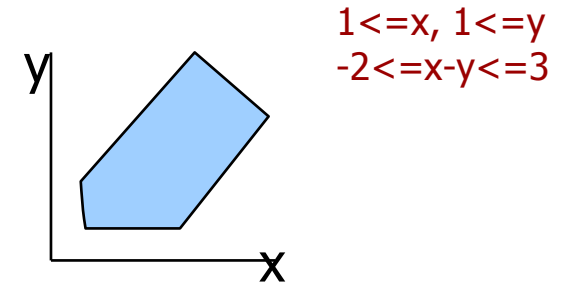


Zone:
 conjunction of
 $x-y \leq n, x \leq n$

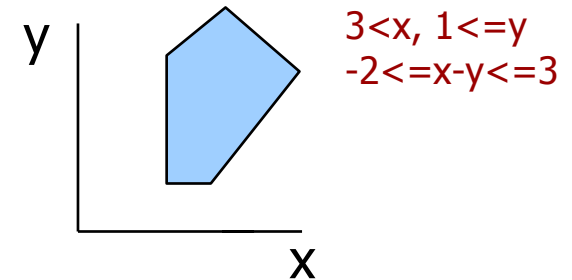
Symbolic Transitions



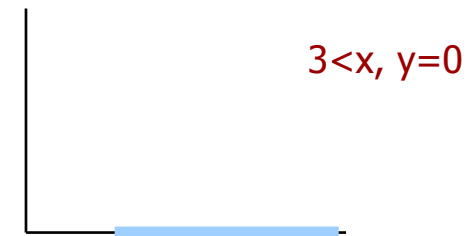
delays to



conjuncts to

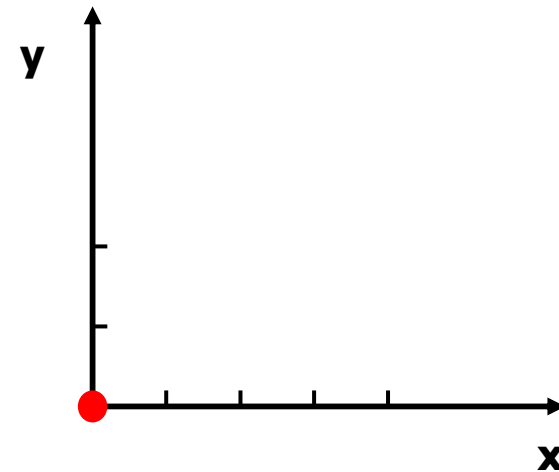
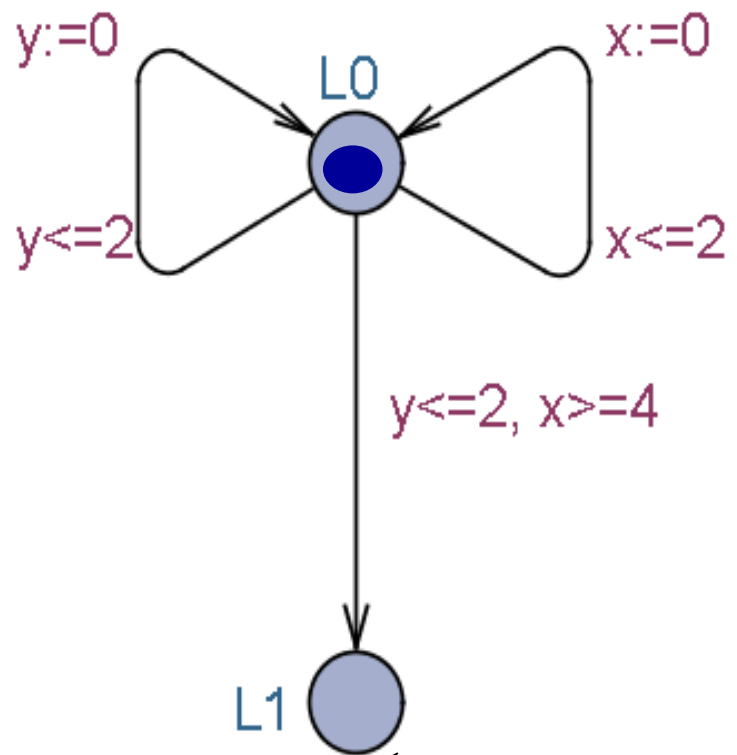


projects to



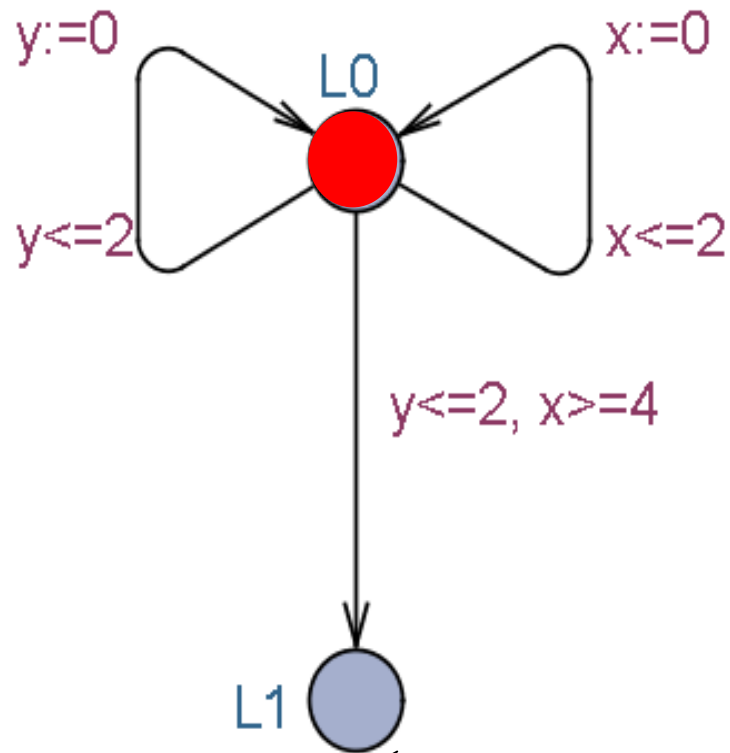
Thus $(n, 1 \leq x \leq 4, 1 \leq y \leq 3) = a \Rightarrow (m, 3 < x, y = 0)$

Symbolic Exploration

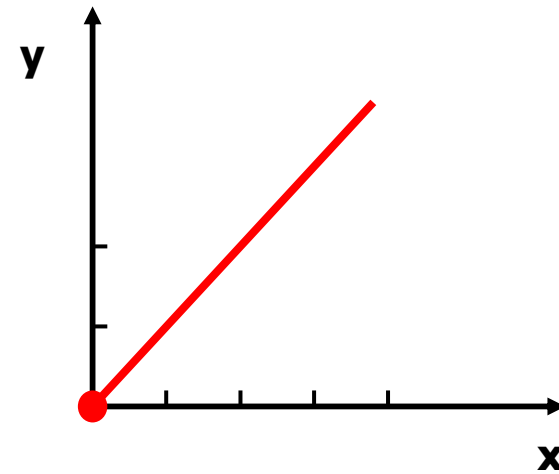


Reachable?

Symbolic Exploration

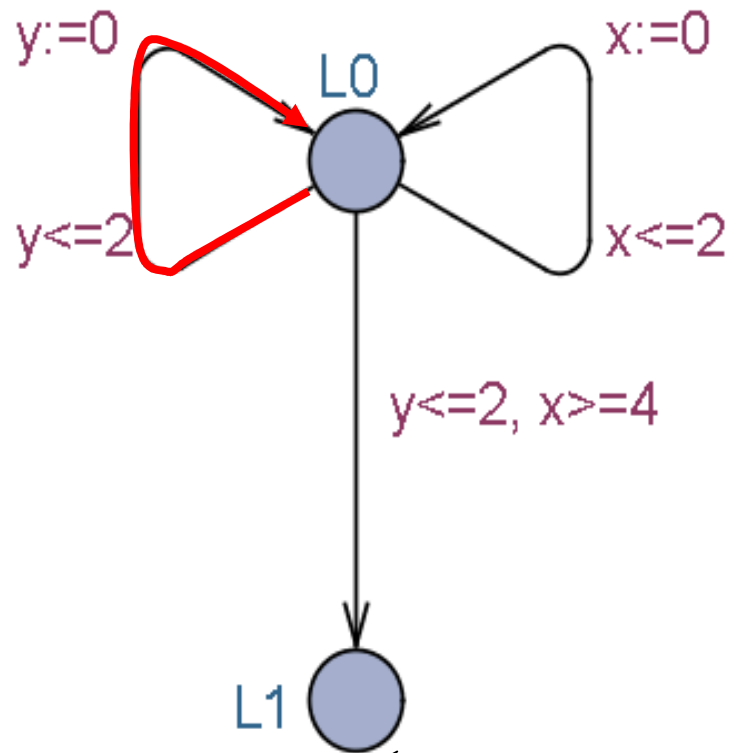


Reachable?

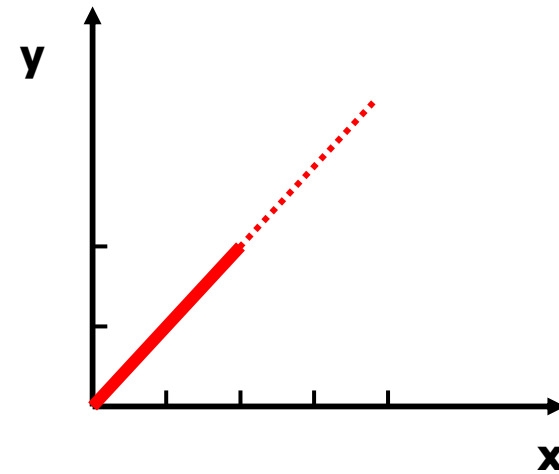


Delay

Symbolic Exploration

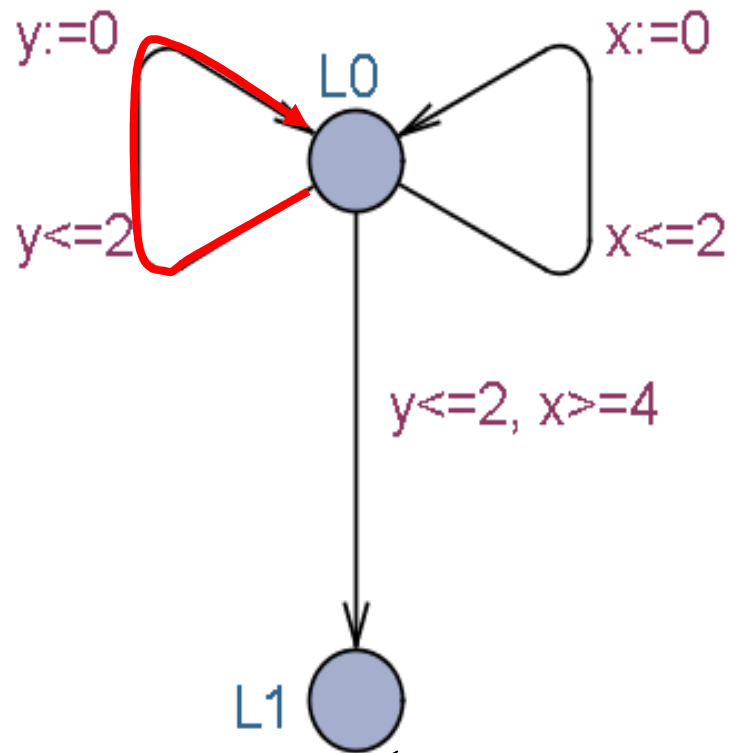


Reachable?

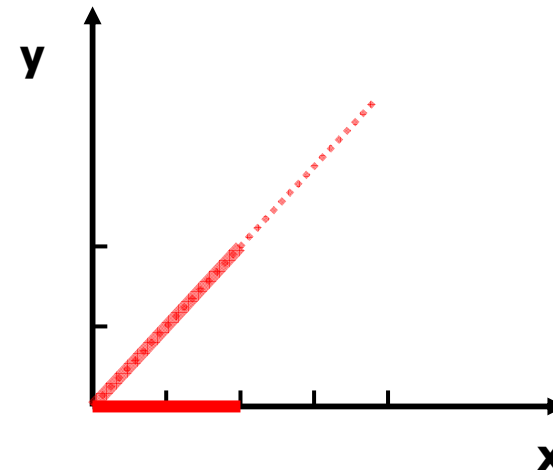


Left

Symbolic Exploration

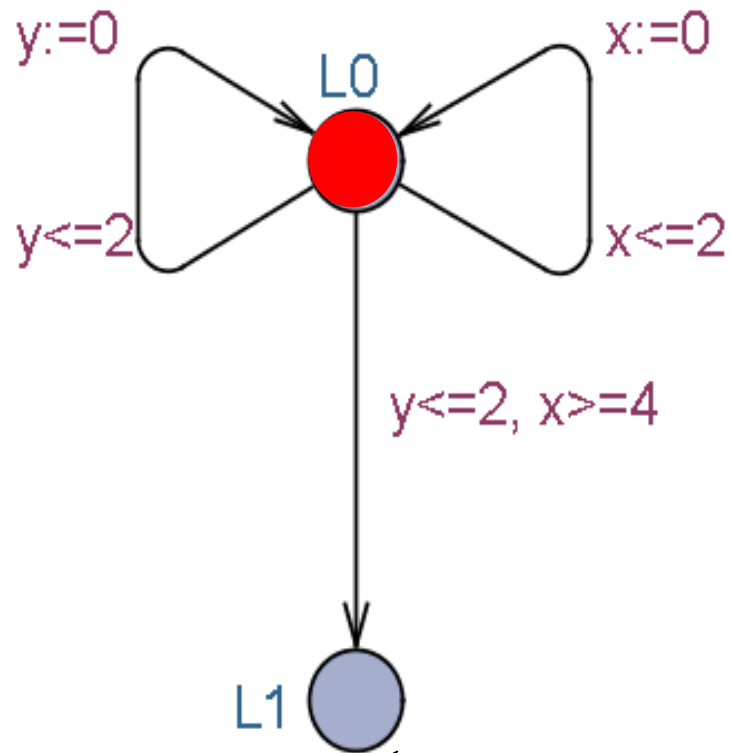


Reachable?

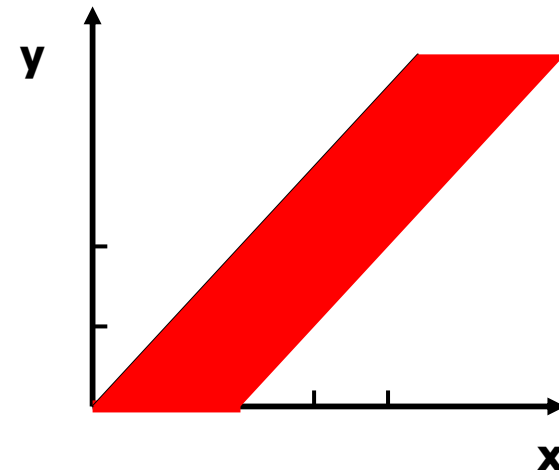


Left

Symbolic Exploration

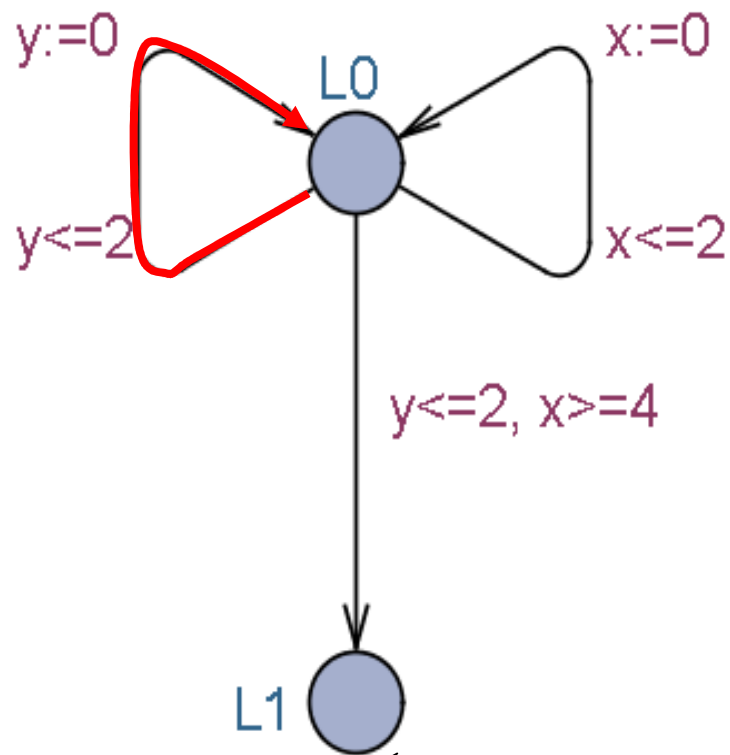


Reachable?

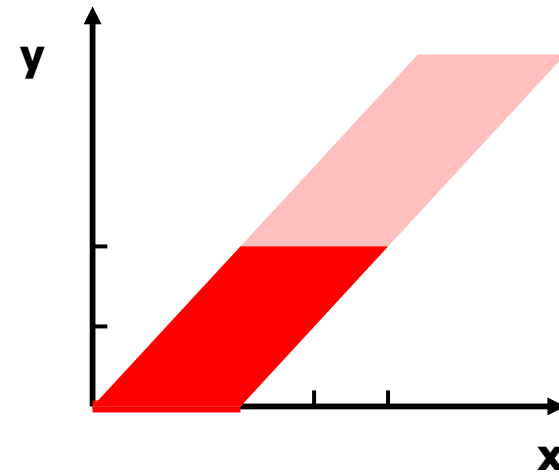


Delay

Symbolic Exploration

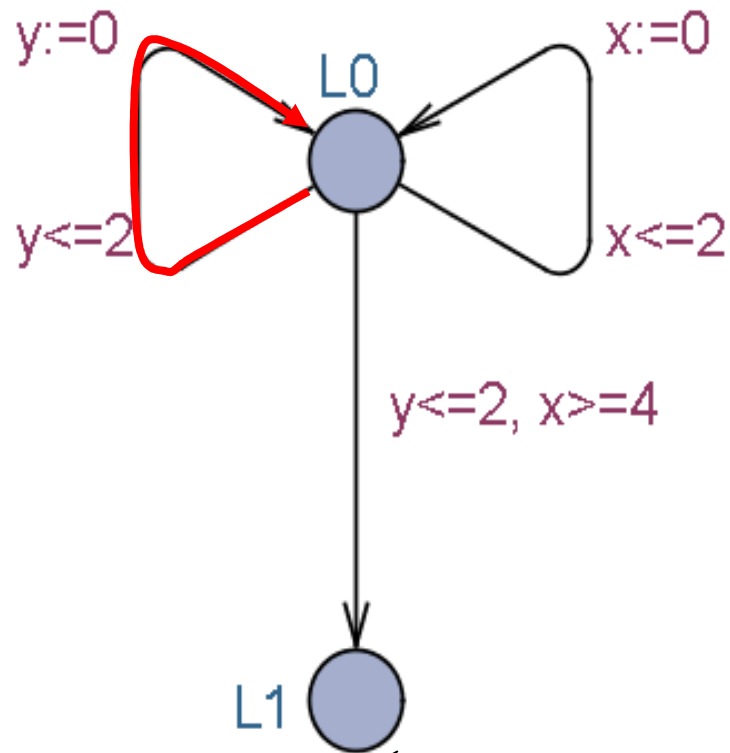


Reachable?

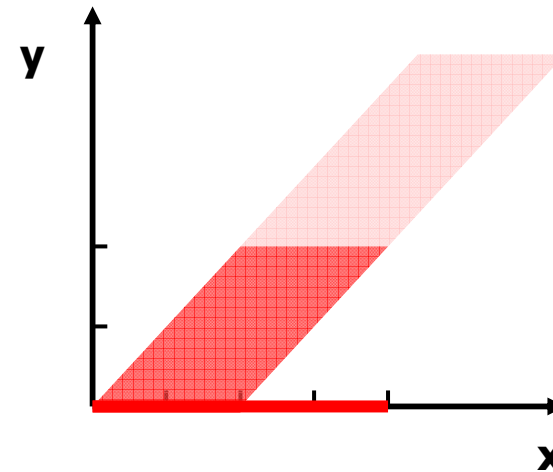


Left

Symbolic Exploration

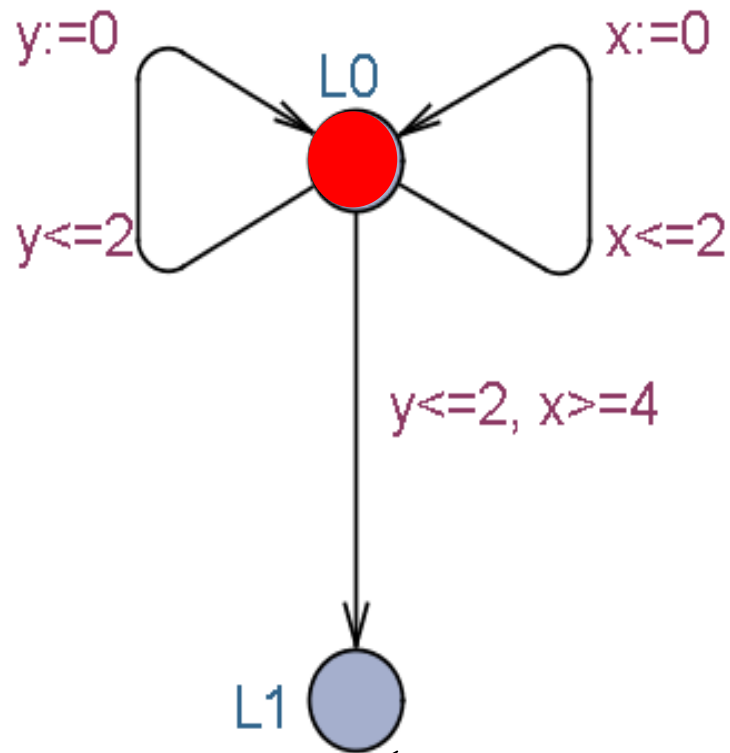


Reachable?

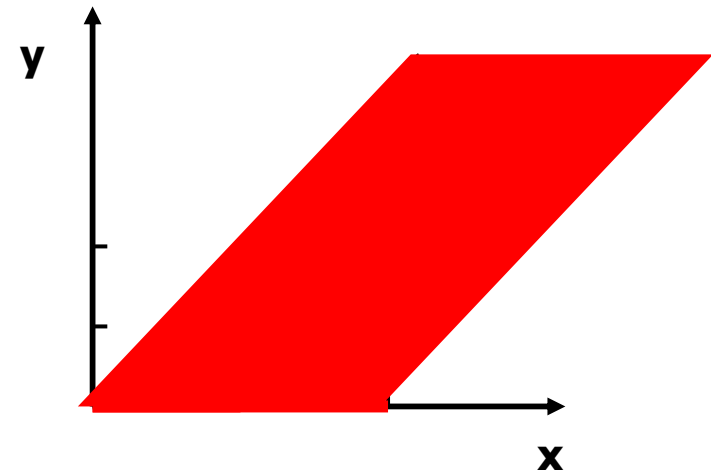


Left

Symbolic Exploration

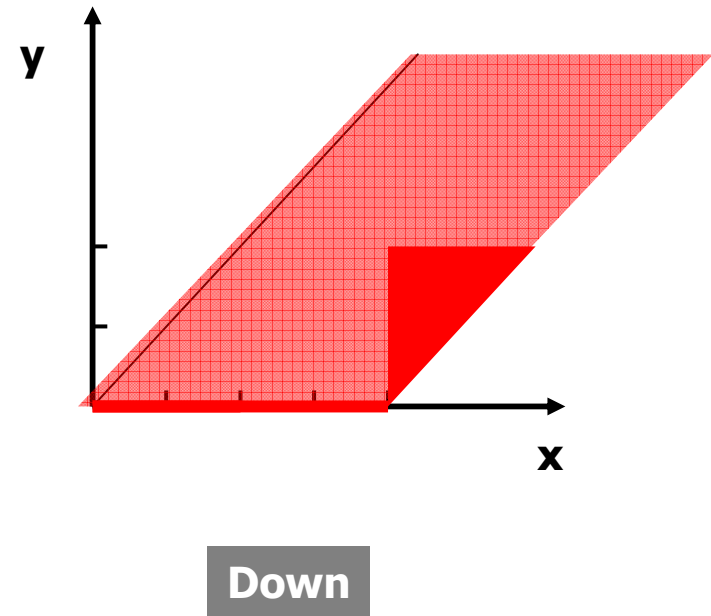
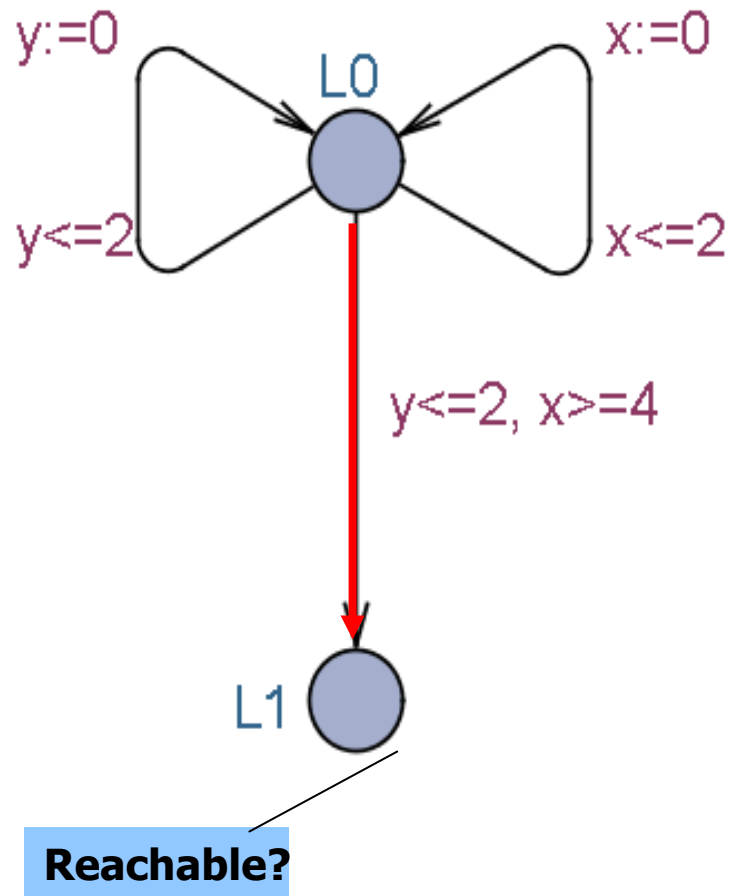


Reachable?



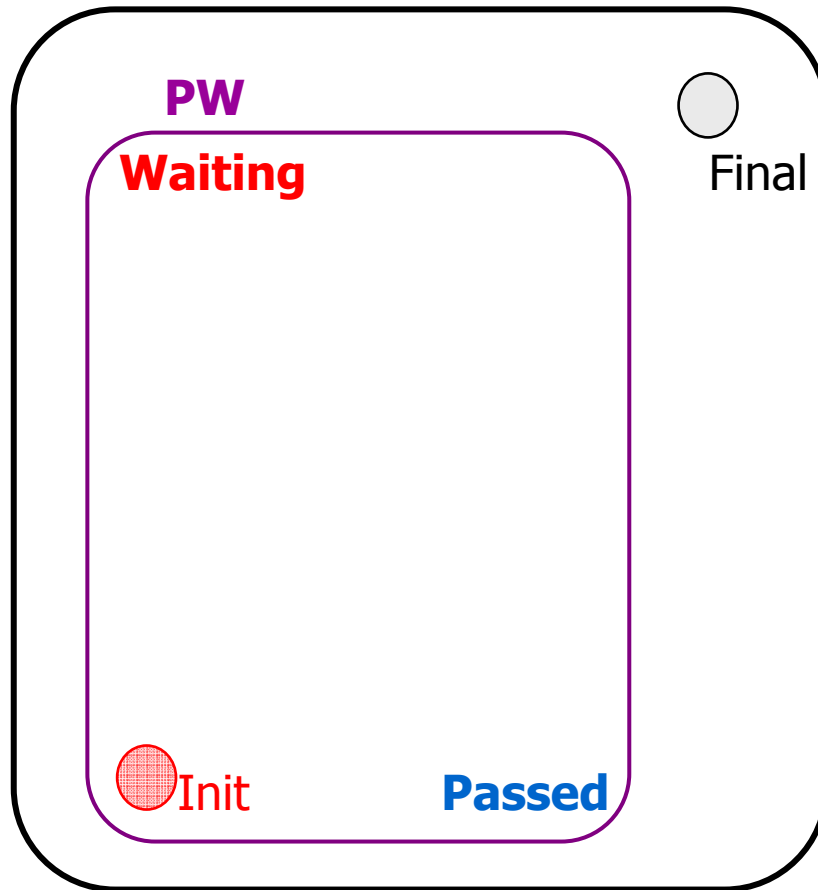
Delay

Symbolic Exploration



Forward Reachability

Init -> **Final** ?



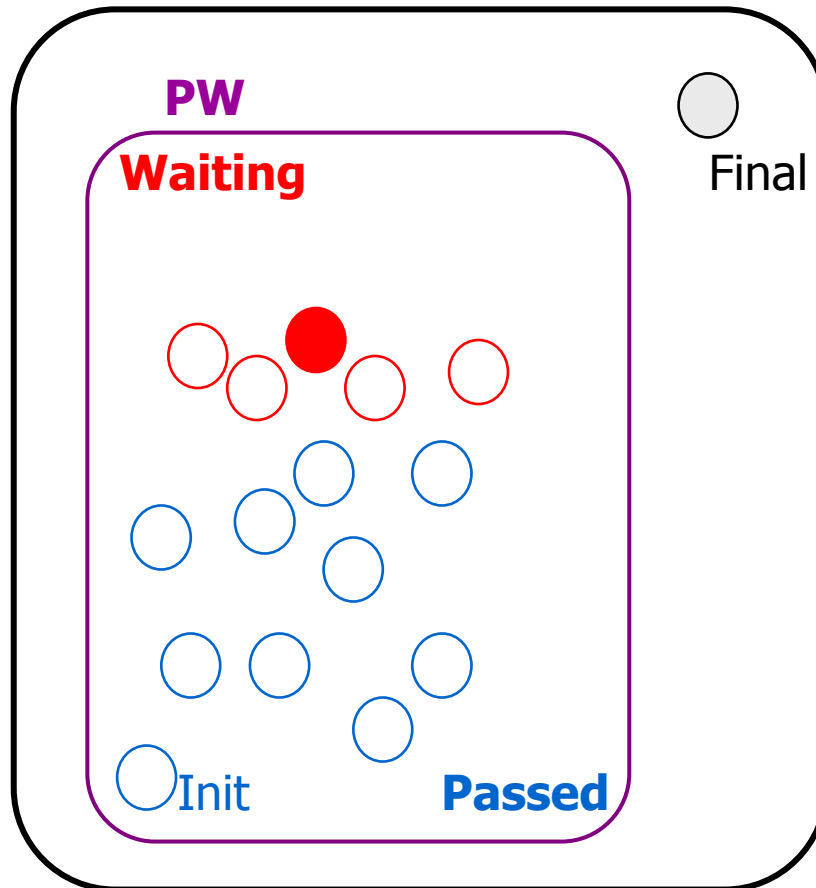
INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

UNTIL **Waiting** = \emptyset
return false

Forward Reachability

Init -> **Final** ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT
 pick **(n,Z)** in **Waiting**

UNTIL **Waiting** = \emptyset
 return false

Forward Reachability

Init -> **Final** ?



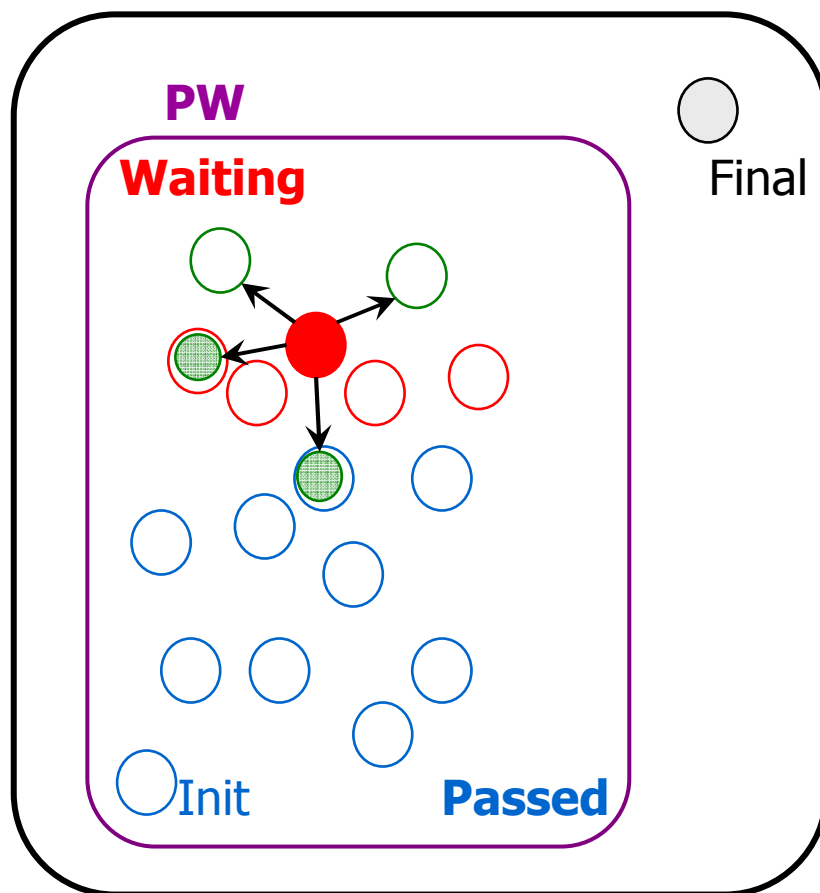
INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT
 pick **(n,Z)** in **Waiting**
 if $(n,Z) = \text{Final}$ **return true**

UNTIL **Waiting** = \emptyset
return false

Forward Reachability

Init -> Final ?



INITIAL Passed := \emptyset ;
 Waiting := $\{(n_0, Z_0)\}$

REPEAT

pick (n, Z) in **Waiting**

if $(n, Z) = \text{Final}$ return true

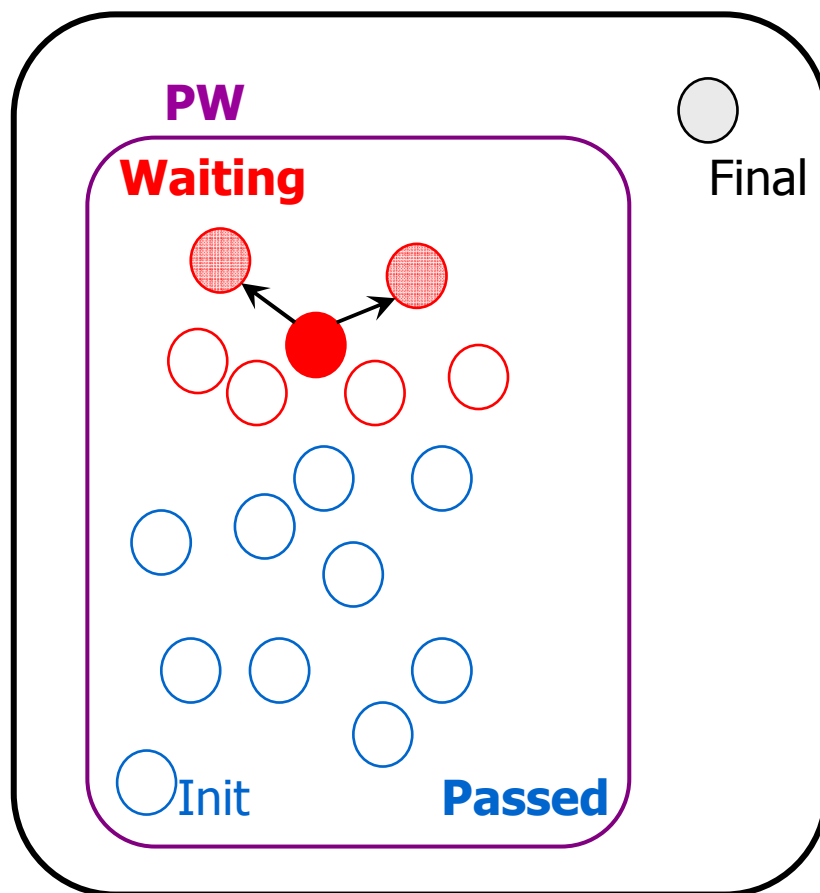
for all $(n, Z) \rightarrow (n', Z')$:

if for some (n', Z'') $Z' \subseteq Z''$ continue

UNTIL **Waiting** = \emptyset
 return false

Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

pick (n, Z) in **Waiting**

if $(n, Z) = \text{Final}$ **return true**

for all $(n, Z) \rightarrow (n', Z')$:

if for some (n', Z'') $Z' \subseteq Z''$ **continue**

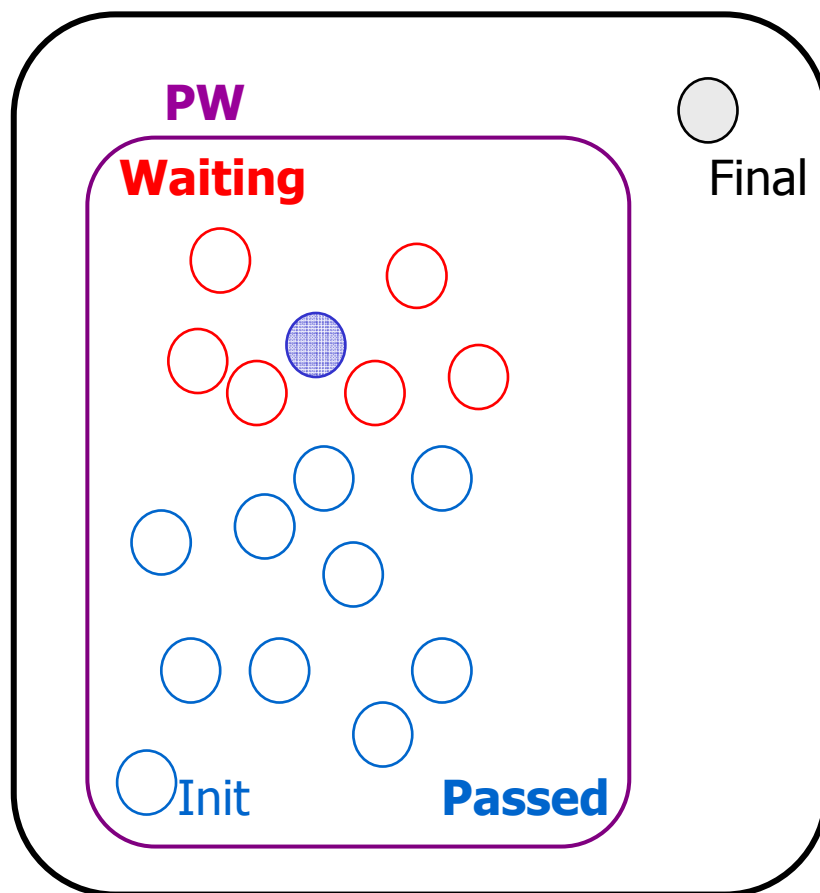
else add (n', Z') to **Waiting**

UNTIL **Waiting** = \emptyset

return false

Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

pick (n, Z) in **Waiting**

if $(n, Z) = \text{Final}$ **return true**

for all $(n, Z) \rightarrow (n', Z')$:

if for some (n', Z'') $Z' \subseteq Z''$ **continue**

else add (n', Z') to **Waiting**

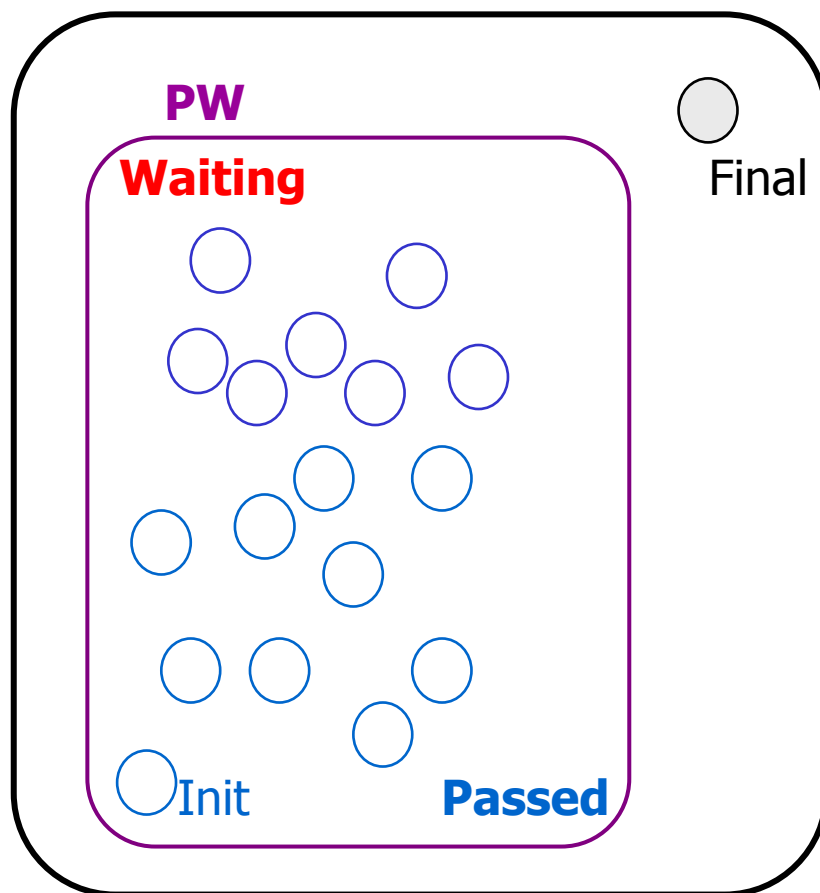
move (n, Z) to **Passed**

UNTIL **Waiting** = \emptyset

return false

Forward Reachability

Init -> Final ?



INITIAL **Passed** := \emptyset ;
Waiting := $\{(n_0, Z_0)\}$

REPEAT

pick **(n,Z)** in **Waiting**

if $(n,Z) = \text{Final}$ **return true**

for all $(n,Z) \rightarrow (n',Z')$:

if for some (n',Z'') $Z' \subseteq Z''$ **continue**

else add (n',Z') to **Waiting**

move (n,Z) to **Passed**

UNTIL **Waiting** = \emptyset

return false

Canonical Datastructures for Zones

Difference Bounded Matrices

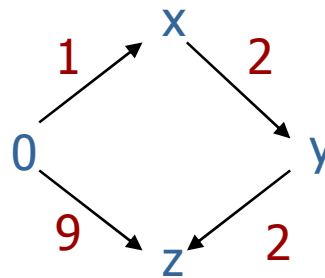
Bellman 1958, Dill 1989

Inclusion

D1

$$\begin{aligned} x &\leq 1 \\ y - x &\leq 2 \\ z - y &\leq 2 \\ z &\leq 9 \end{aligned}$$

Graph

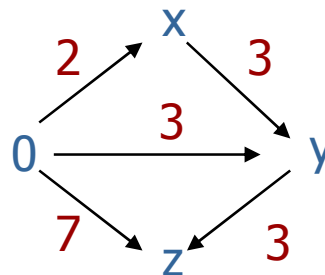


? \subseteq ?

D2

$$\begin{aligned} x &\leq 2 \\ y - x &\leq 3 \\ y &\leq 3 \\ z - y &\leq 3 \\ z &\leq 7 \end{aligned}$$

Graph



Canonical Datastructures for Zones

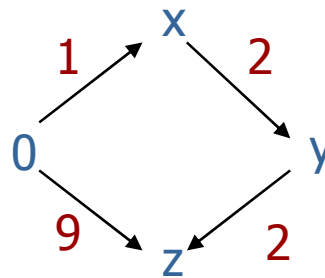
Difference Bounded Matrices

Inclusion

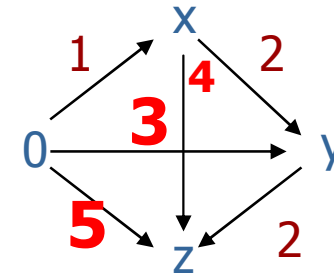
D1

$$\begin{aligned} x &\leq 1 \\ y - x &\leq 2 \\ z - y &\leq 2 \\ z &\leq 9 \end{aligned}$$

Graph



Shortest
Path
Closure

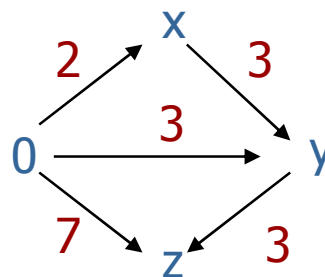


? \subseteq ?

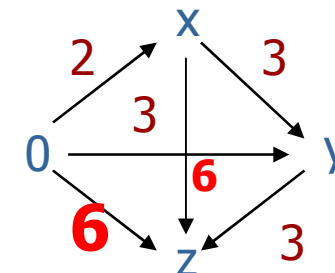
D2

$$\begin{aligned} x &\leq 2 \\ y - x &\leq 3 \\ y &\leq 3 \\ z - y &\leq 3 \\ z &\leq 7 \end{aligned}$$

Graph



Shortest
Path
Closure



Canonical Datastructures for Zones

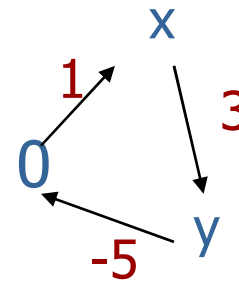
Difference Bounded Matrices

Emptiness

D

$$\begin{array}{l} x \leq 1 \\ y \geq 5 \\ y - x \leq 3 \end{array}$$

Graph



Negative Cycle
iff
empty solution set

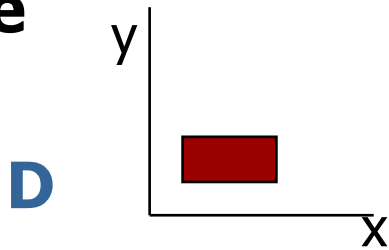
Compact

UCb

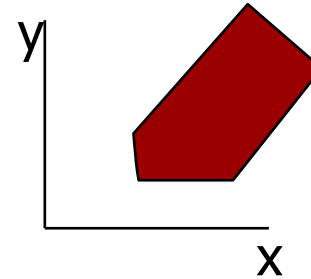
Canonical Datastructures for Zones

Difference Bounded Matrices

Future

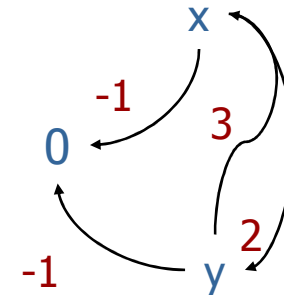
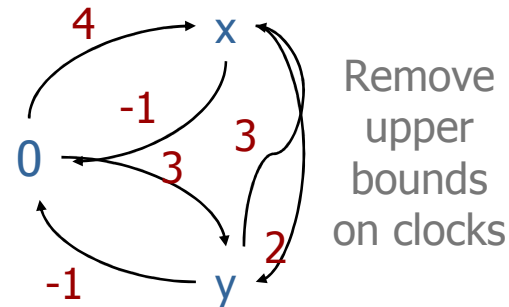
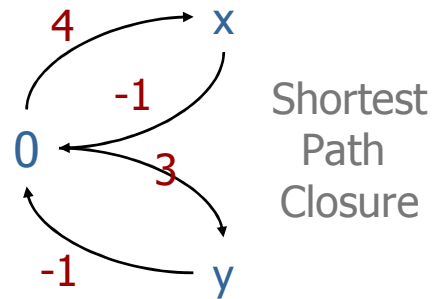


$$\begin{aligned} 1 \leq x \leq 4 \\ 1 \leq y \leq 3 \end{aligned}$$



Future **D**

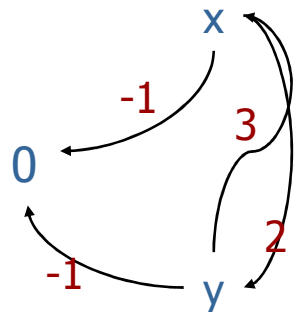
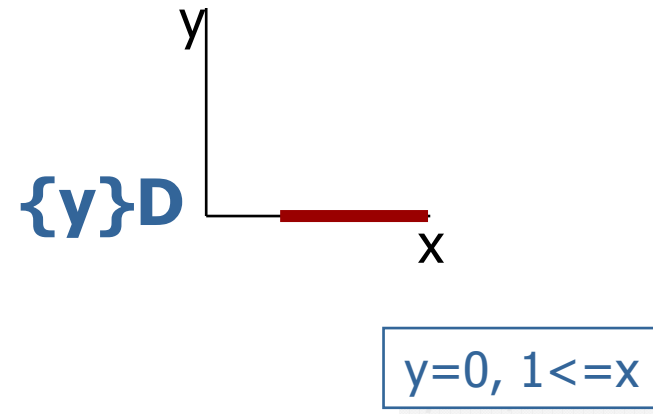
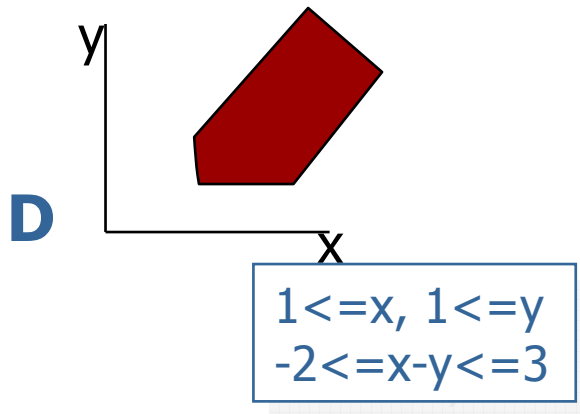
$$\begin{aligned} 1 \leq x, 1 \leq y \\ -2 \leq x - y \leq 3 \end{aligned}$$



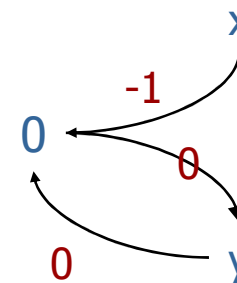
Canonical Datastructures for Zones

Difference Bounded Matrices

Reset



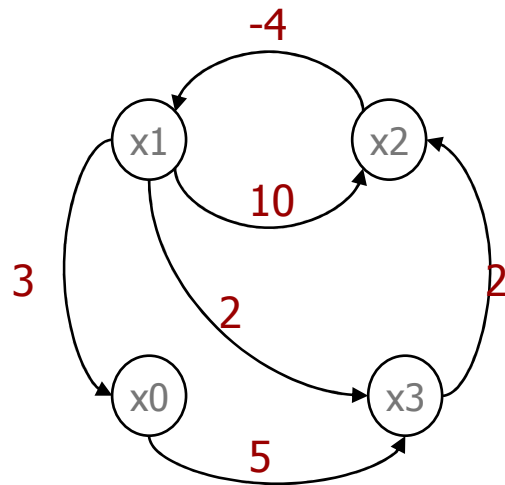
Remove all bounds involving y and set y to 0



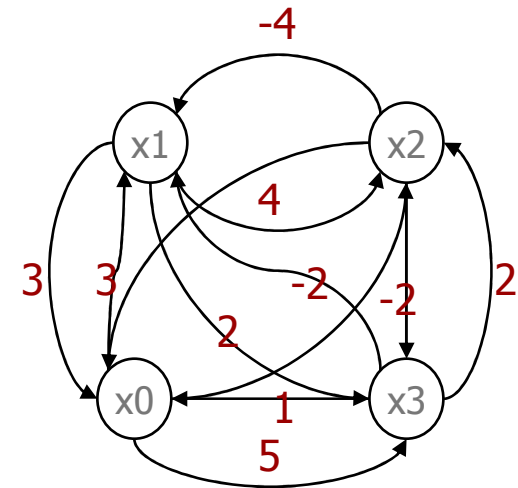
Canonical Datastructures for Zones

Difference Bounded Matrices

$x_1 - x_2 \leq 4$
 $x_2 - x_1 \leq 10$
 $x_3 - x_1 \leq 2$
 $x_2 - x_3 \leq 2$
 $x_0 - x_1 \leq 3$
 $x_3 - x_0 \leq 5$



Shortest Path Closure
 $O(n^3)$

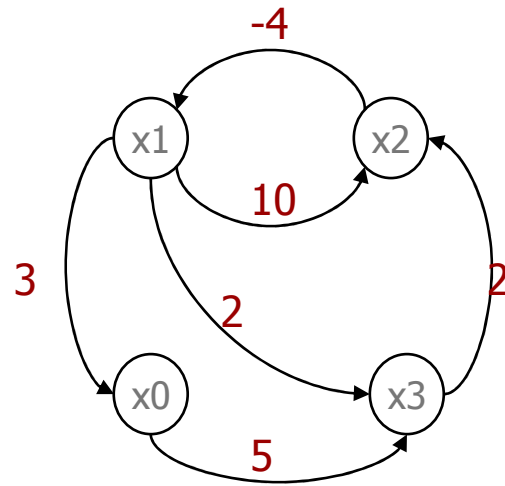


Canonical Datastructures for Zones

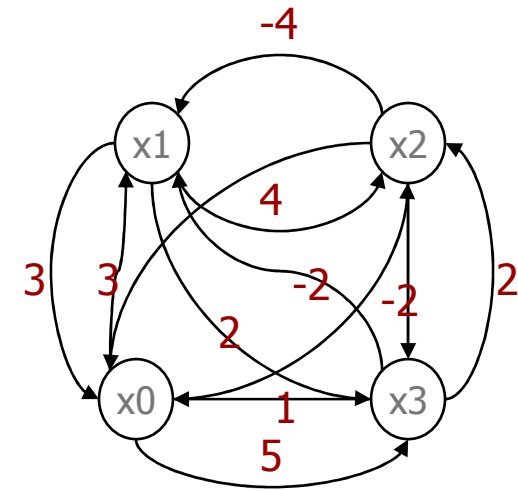
Minimal Constraint Form

RTSS 1997

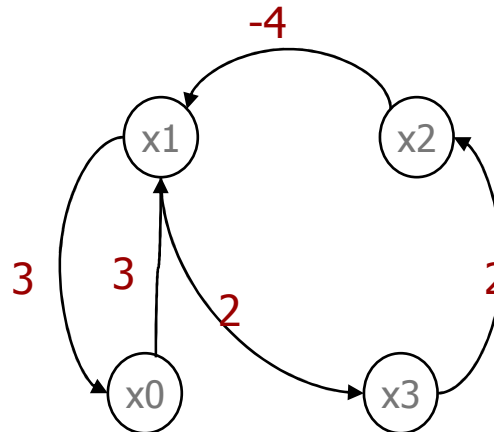
$x_1 - x_2 \leq 4$
 $x_2 - x_1 \leq 10$
 $x_3 - x_1 \leq 2$
 $x_2 - x_3 \leq 2$
 $x_0 - x_1 \leq 3$
 $x_3 - x_0 \leq 5$



Shortest Path Closure
 $O(n^3)$

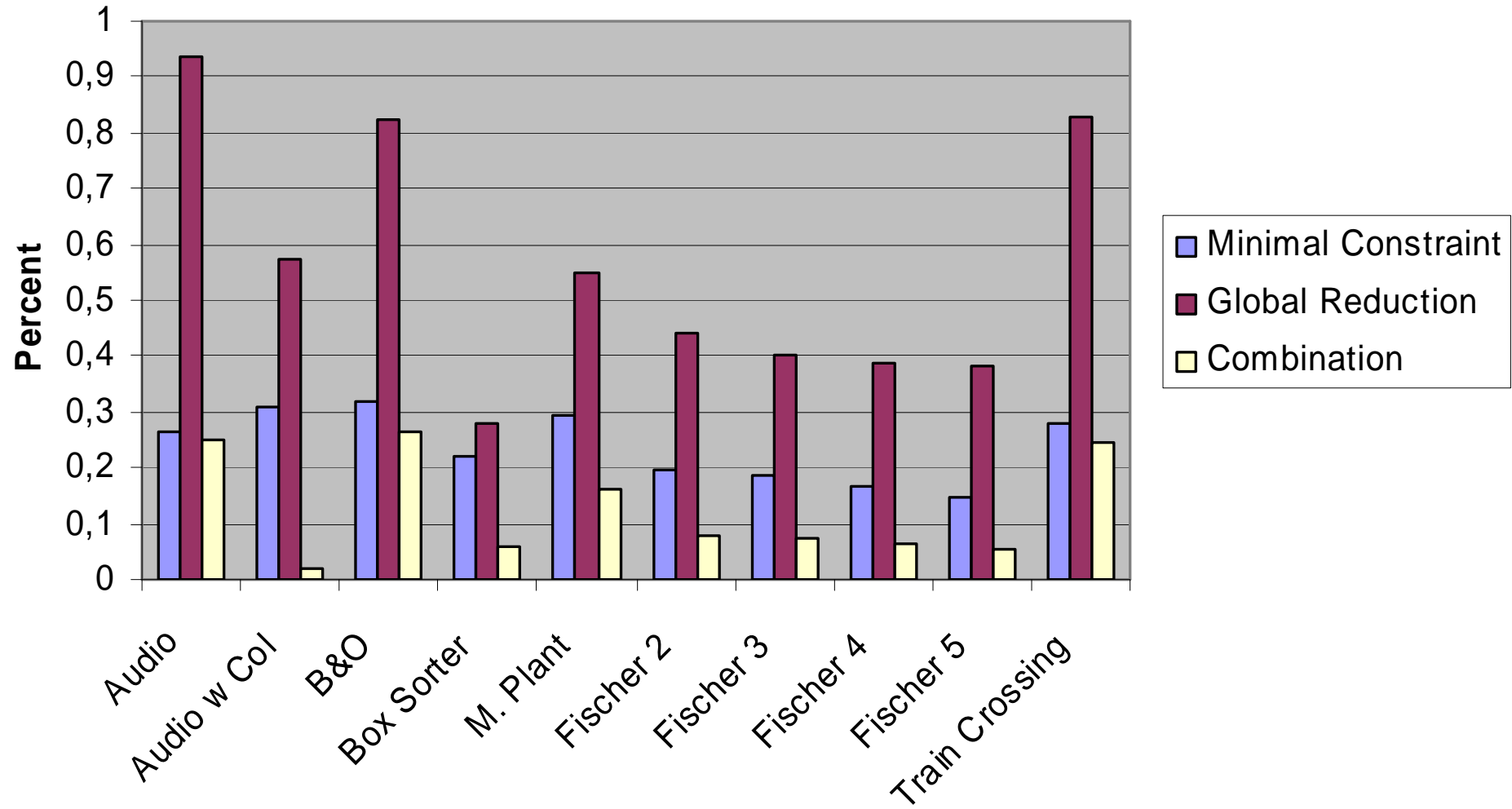


Shortest Path Reduction
 $O(n^3)$

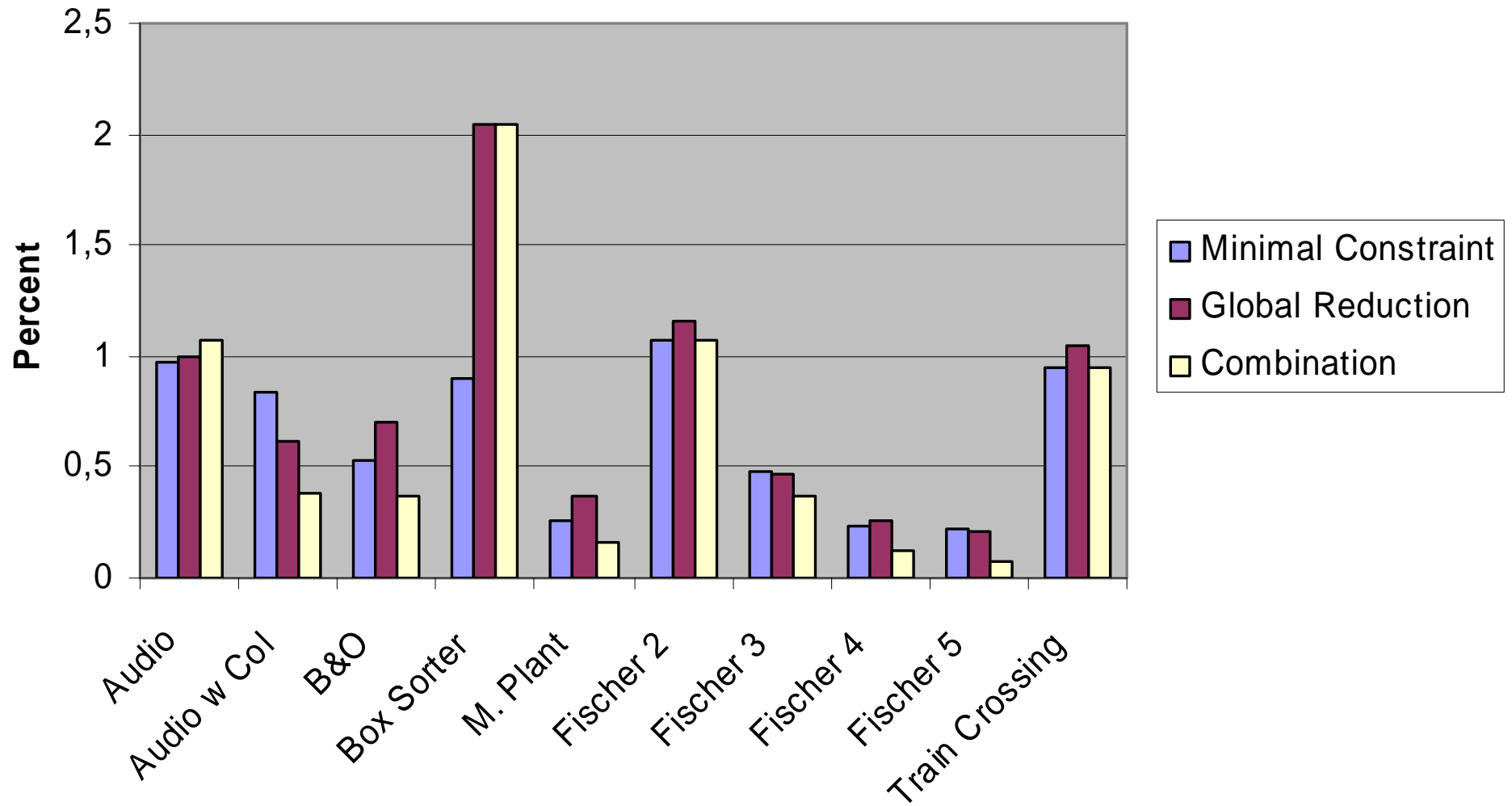


Space worst $O(n^2)$
practice $O(n)$

SPACE PERFORMANCE



TIME PERFORMANCE



Clock Difference Diagrams



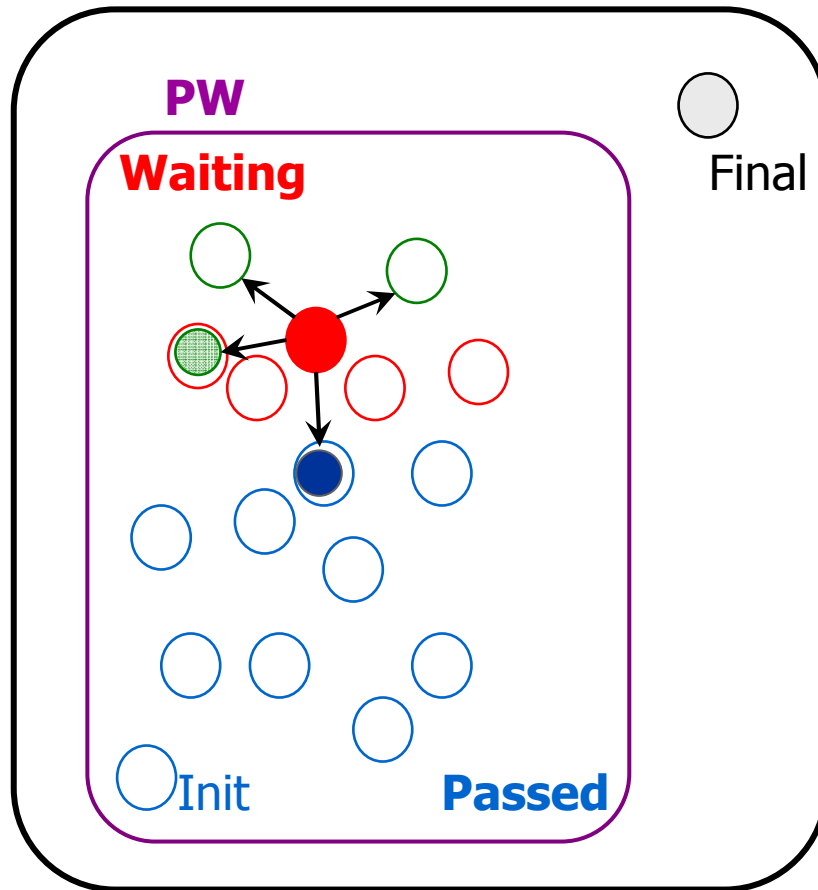
BRICS
Basic Research
in Computer Science



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

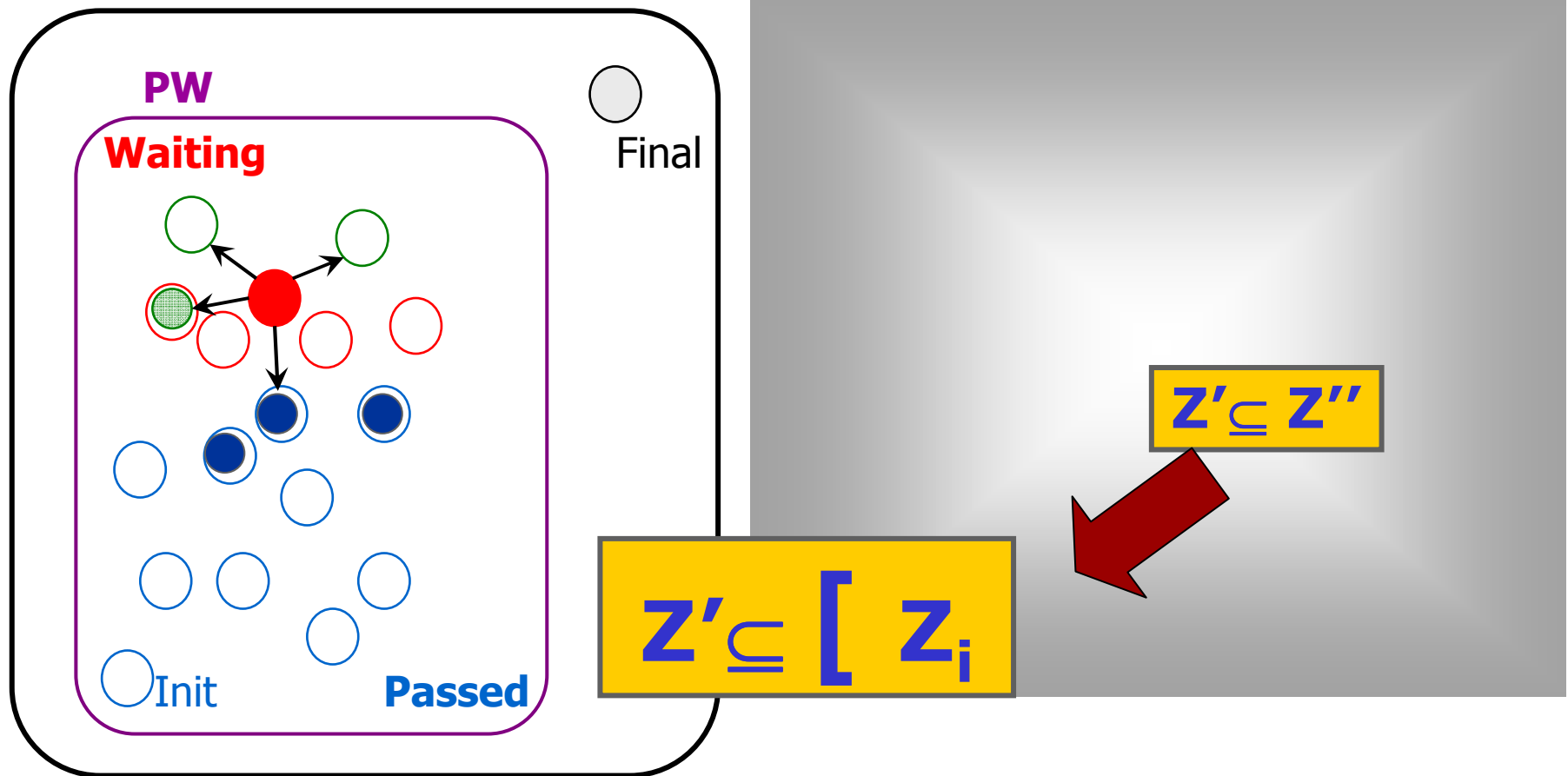
Earlier Termination

Init -> Final ?



Earlier Termination

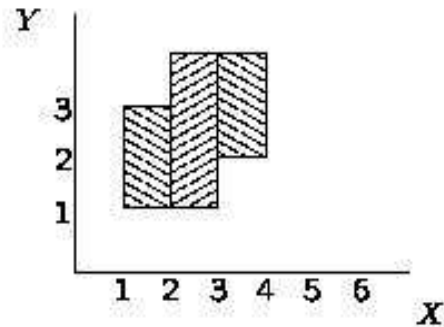
Init -> Final ?



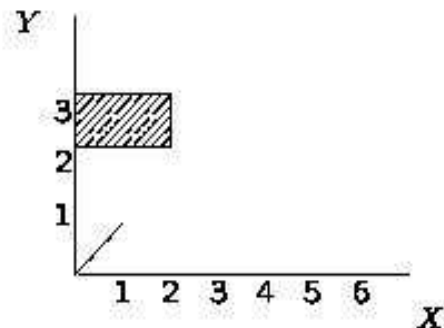
Clock Difference Diagrams

= Binary Decision Diagrams + Difference Bounded Matrices CAV99

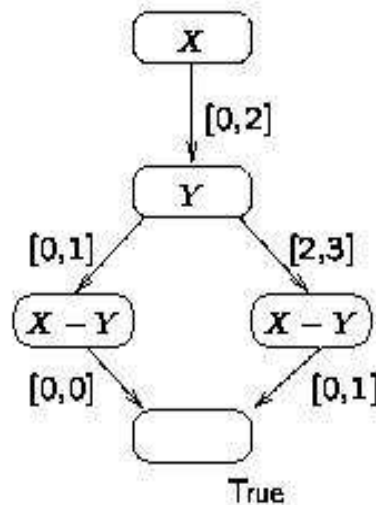
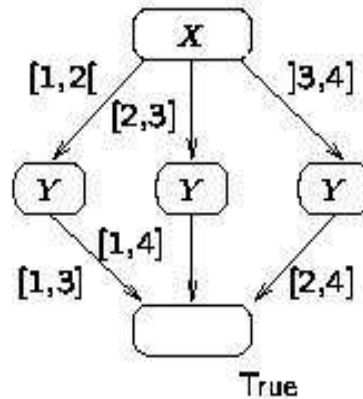
CDD-representations



(b)

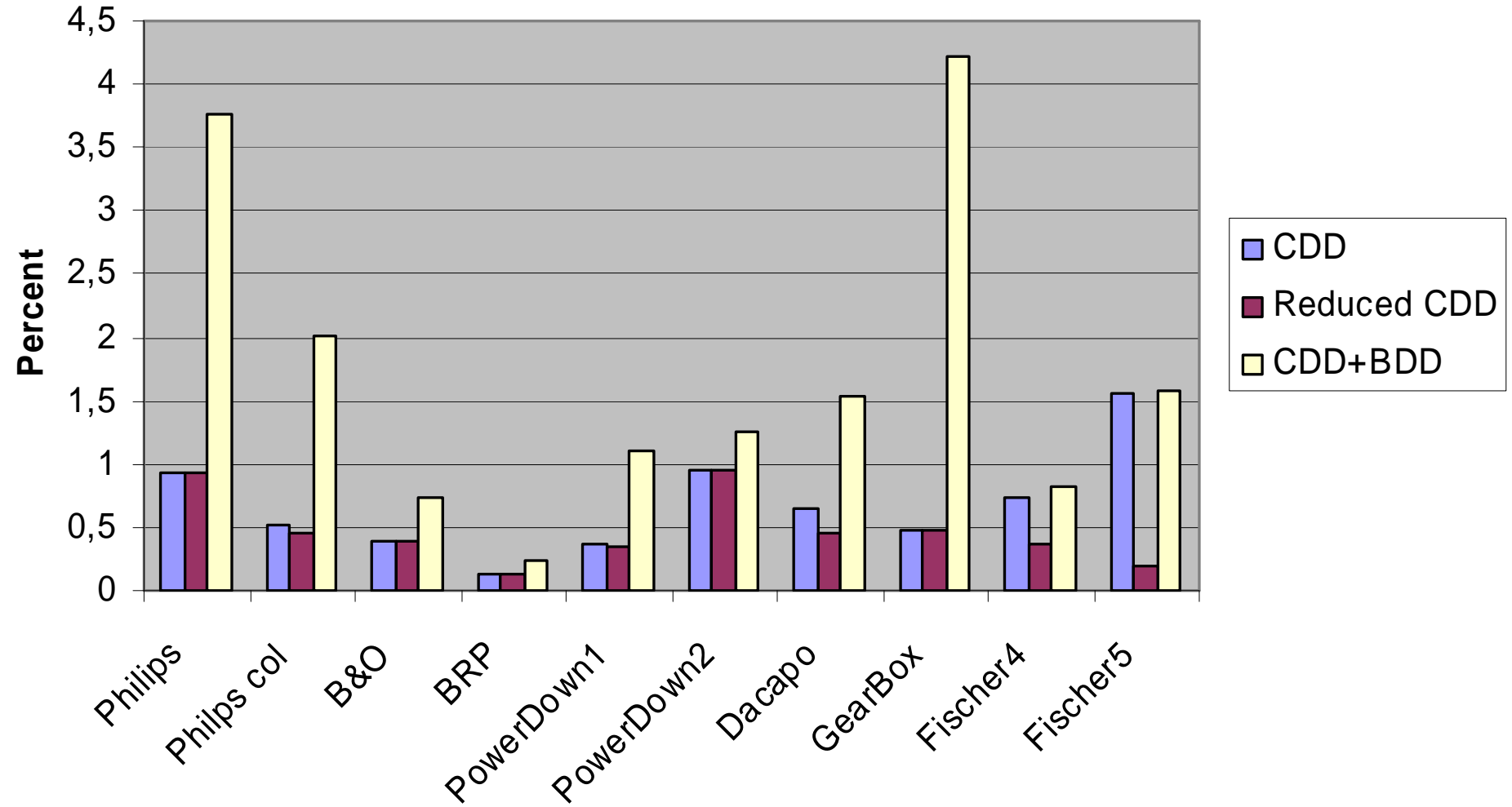


(c)

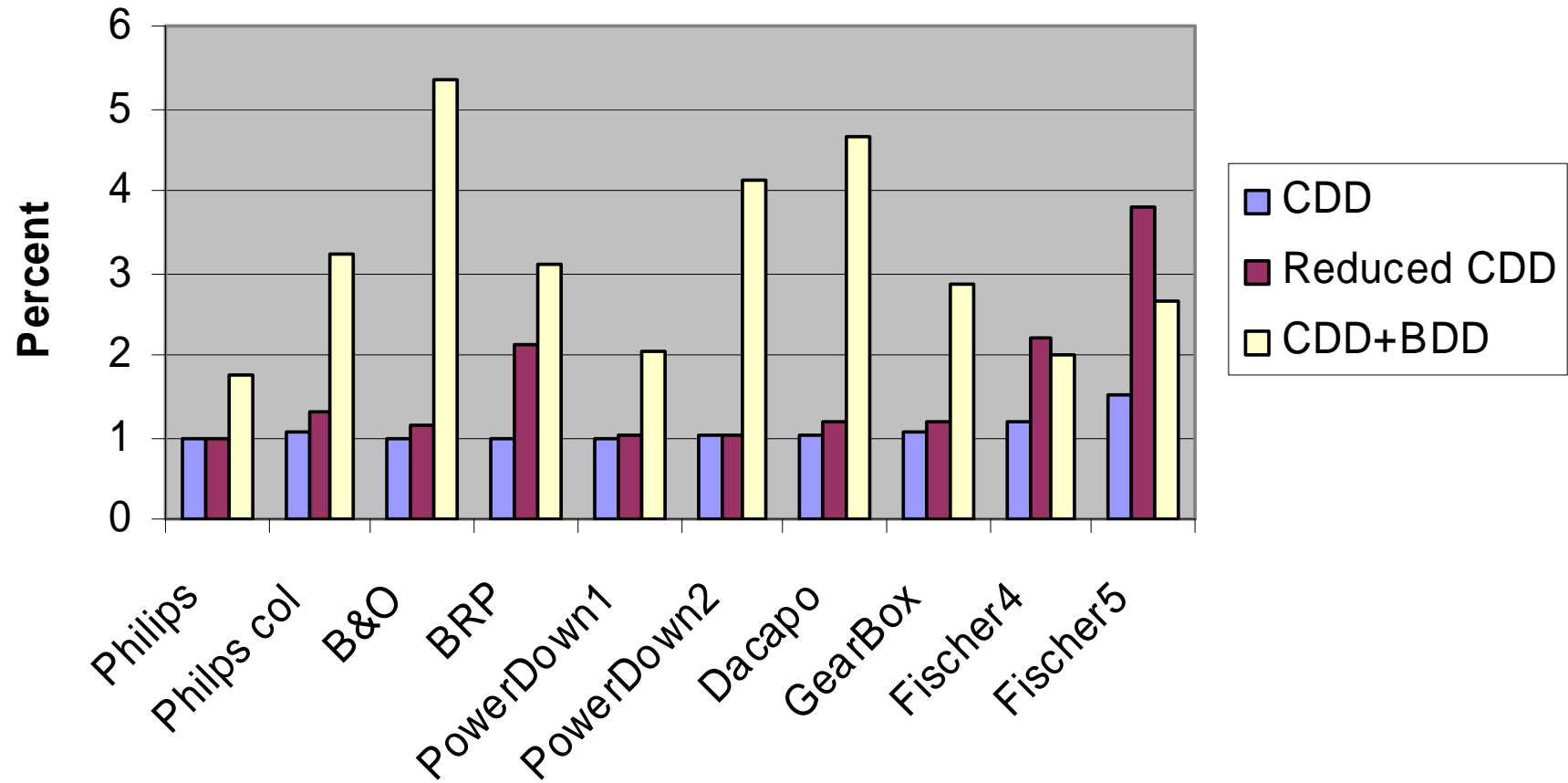


- Nodes labeled with differences
- Maximal sharing of substructures (also across different CDDs)
- Maximal intervals
- Linear-time algorithms for set-theoretic operations.

SPACE PERFORMANCE



TIME PERFORMANCE



Related & Future Work

- DDD: Andersen et al.
- NDD: Asarin, Bozga, Kerbrat, Maler, Pnueli, Rasse.
- IDD: Strehl, Thiele.

- No efficient algorithm for FUTURE and RESET operation on CDD.
- No canonical form.

- An efficient, fully symbolic engine for TA is still missing!!

Additional “secrets”

- Sharing among symbolic states
 - location vector / discrete values / zones
- Distributed implementation of UPPAAL
- Symmetry Reduction
- Sweep Line Method
- Guiding wrt Heuristic Value
 - User-supplied / Auto-generated
- Slicing wrt “C” Code

Open Problems

- Fully symbolic exploration of TA (both discrete and continuous part) ?
- Canonical form for CDD's ?
- Partial Order Reduction ?
- Compositional Backwards Reachability ?
- Bounded Model Checking for TA ?
- Exploitation of multi-core processors ?
- ...

Datastructures for Zones

- Difference Bounded

UPPAAL DBM Library
The library used to manipulate DBMs in UPPAAL

Main Page | Download | Ruby Binding | Help | Contact us

Welcome!

DBMs [dill89, rokicki93, lpw:fct95, bengtsson02] are efficient data structures to represent clock constraints in timed automata [ad90]. They are used in UPPAAL [lpy97, by04, bd04] as the core data structure to represent time. The library features all the common operations such as up (delay, or future), down (past), general updates, different extrapolation functions, etc.. on DBMs and federations. The library also supports subtractions. The API is in C and C++. The C++ part uses active clocks and hides memory management.

References

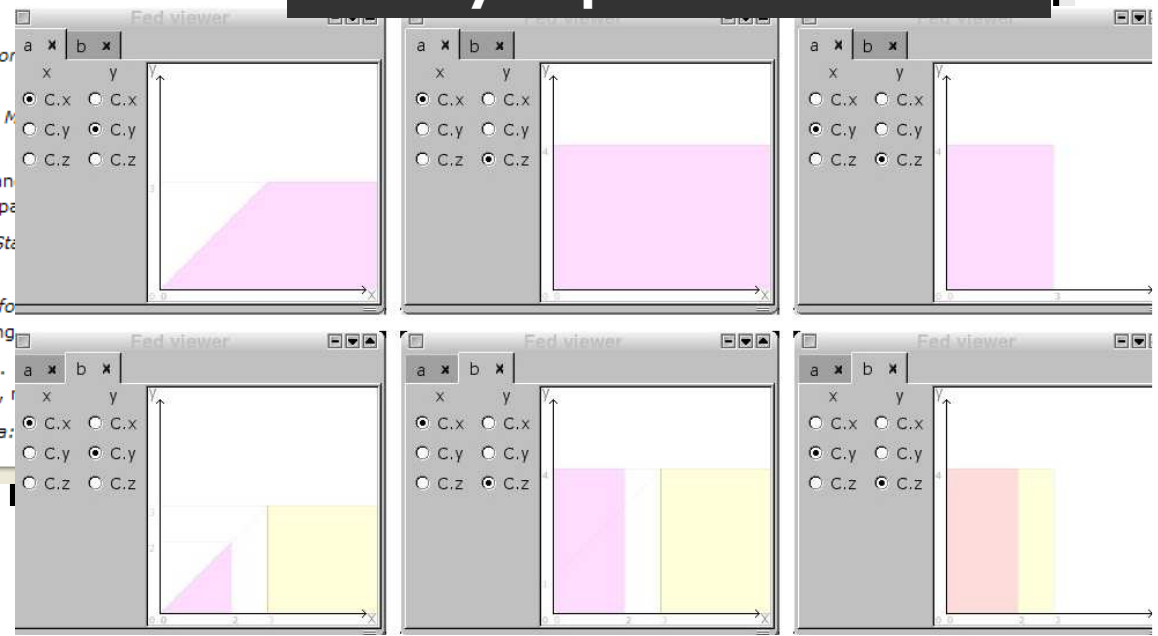
- [dill89] David L. Dill. *Timing Assumptions and Verification*. Springer Berlin 1989, pp 197-212.
- [rokicki93] Tomas Gerhard Rokicki. *Representing and Manipulating Difference Bound Matrices*. University 1993.
- [lpw:fct95] Kim G. Larsen, Paul Pettersson, and Wang Yi. *Fundamentals of Computation Theory 1995*, LNCS 965 pp 1-12.
- [bengtsson02] Johan Bengtsson. *Clocks, DBM, and State Space Reduction*. University 2002.
- [ad90] Rajeev Alur and David L. Dill. *Automata for Clock Constraints*. Colloquium on Algorithms, Languages, and Programming.
- [lpy97] Kim G. Larsen, Paul Pettersson, and Wang Yi. *Software Tools for Technology Transfer*, October 1997, pp 1-12.
- [by04] Johan Bengtsson and Wang Yi. *Timed Automata: Symbolic Model Checking*. LNCS 2004.

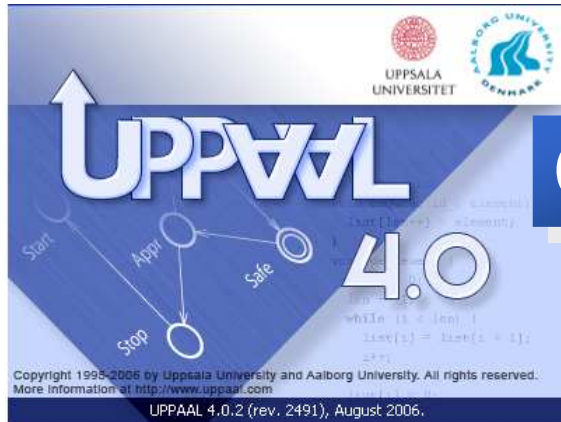


Alexandre David

Elegant RUBY bindings for easy implementations

[SPIN03]





CORA



Optimal & Real Time Scheduling

— *using* —
Model Checking Technology



BRICS
Basic Research
in Computer Science



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

Overview

- Timed Automata and Scheduling
- Priced Timed Automata
- Optimal Scheduling
- Optimal Scheduling wrt Multiple Objectives
- Optimal Infinite Scheduling

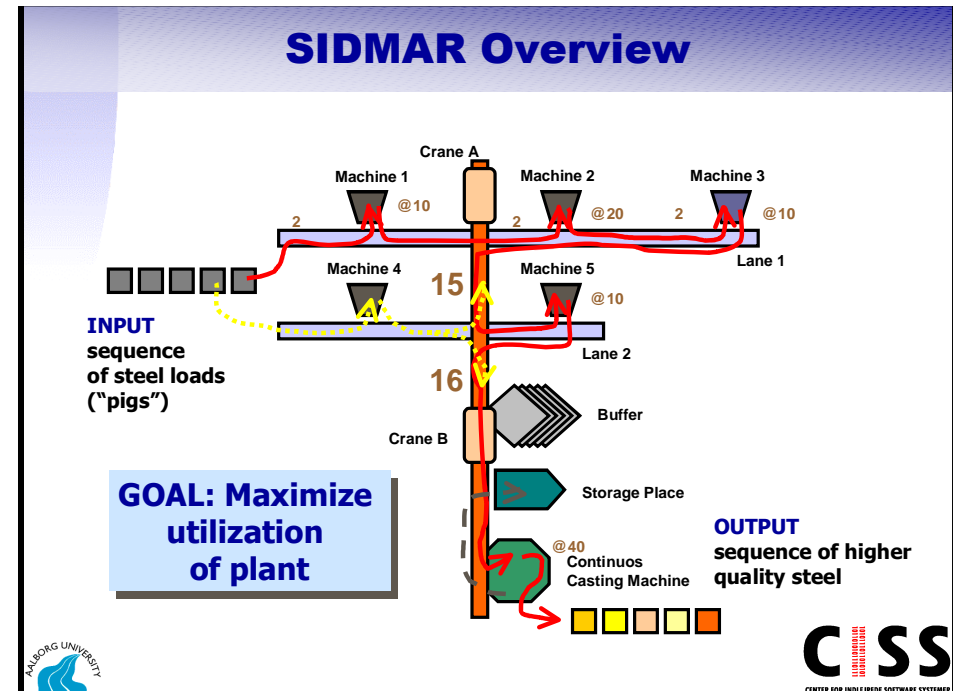
April 2002 – June 2005 IST-2001-35304

■ **Academic partners:** ■ **Industrial Partners**

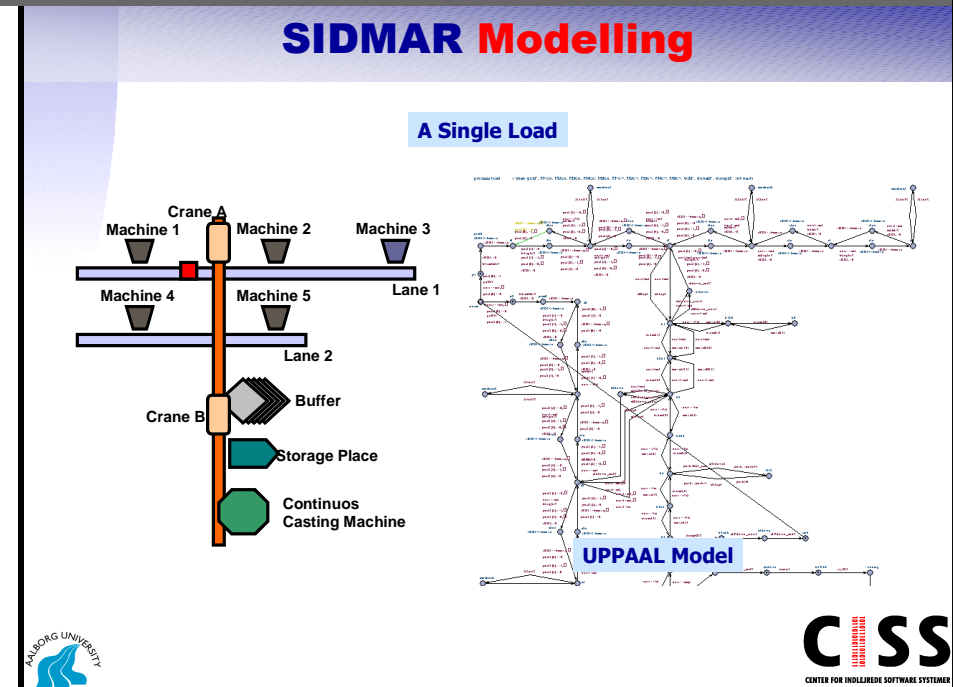
- Nijmegen
 - Aalborg
 - Dortmund
 - Grenoble
 - Marseille
 - Twente
 - Weizmann
- Axxom
 - Bosch
 - Cybernetix
 - Terma

OBJECTIVES

- powerful, unifying mathematical modelling
- efficient computerized problem-solving tools
- distributed real-time systems
- time-dependent behaviour and dynamic resource allocation
- TIMED AUTOMATA**

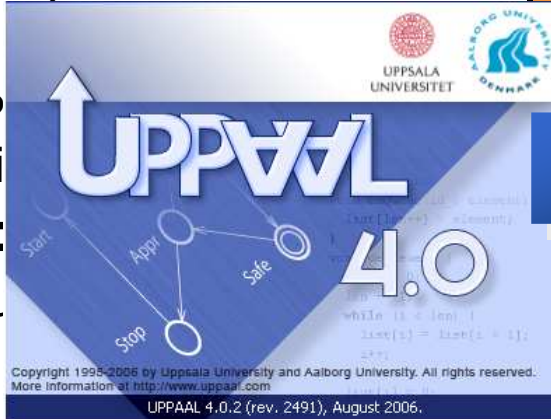
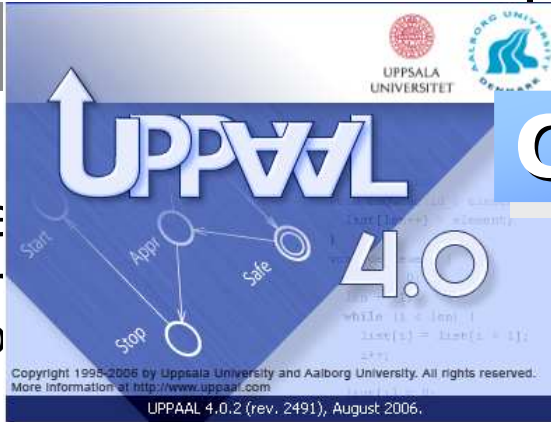


LTR Project VHS (Verification of Hybrid systems)



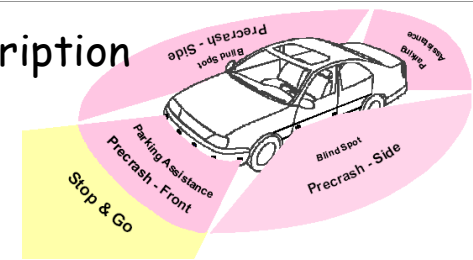
Case S

- Cyberne
 - Smart
 - Perso
- Terma:
 - Memory Interface
- Bosch:
 - Car P
 - Sensi
- AXXOM:
 - Lacqu
- Benchmarks



CPS: Informal description

- CPS obtains and makes available for other systems information about environment of a car. This information may be used for:
 - Parking assistance
 - Stop & go
 - Etc
- Based on Short Range Radar (SRR) technology



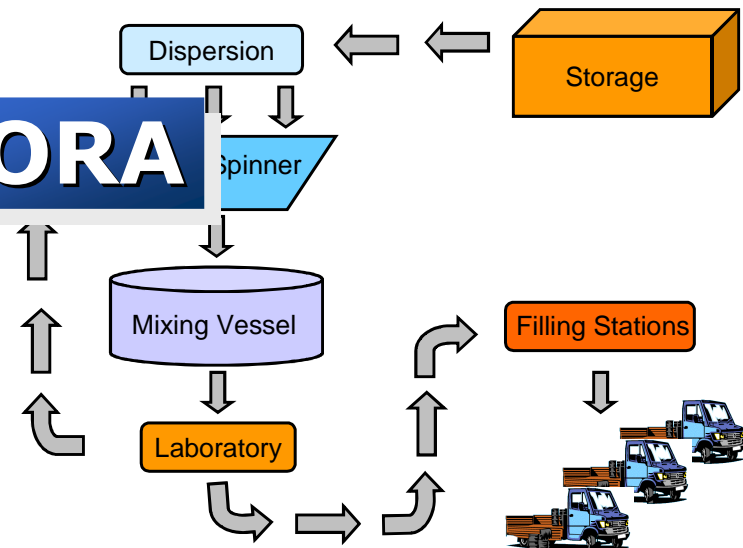
- The CPS considered in this case study
 - One sensor group only (currently 2 sensors)
 - Only the front sensors and corresponding controllers
 - Application: pre-crash detection, parking assistance, stop & go

Brinksma

Car Periphery Supervision System: Case Study 3

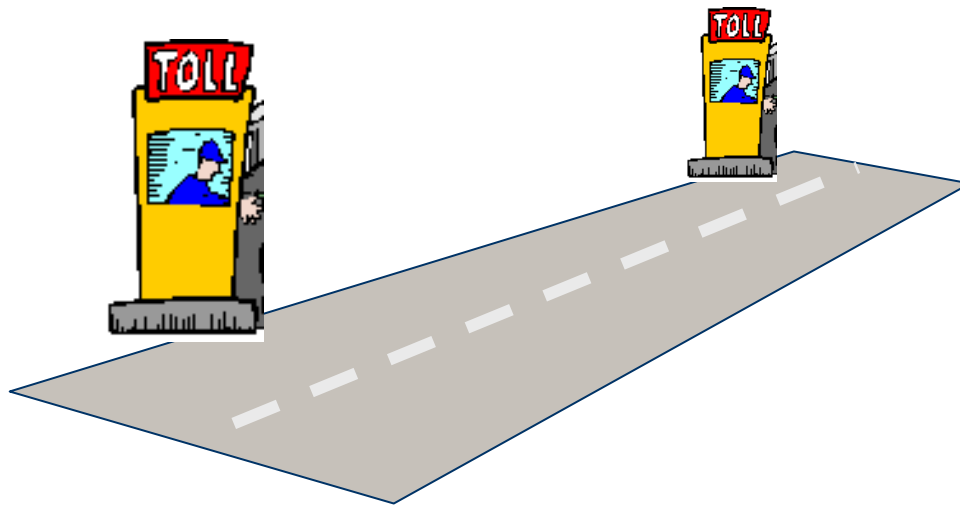
2

Product flow of a Product



Real Time Scheduling

- Only 1 "Pass"
- Cheat is possible
 (drive close to car with "Pass")



UNSAFE



5



10



Pass



20



25

SAFE

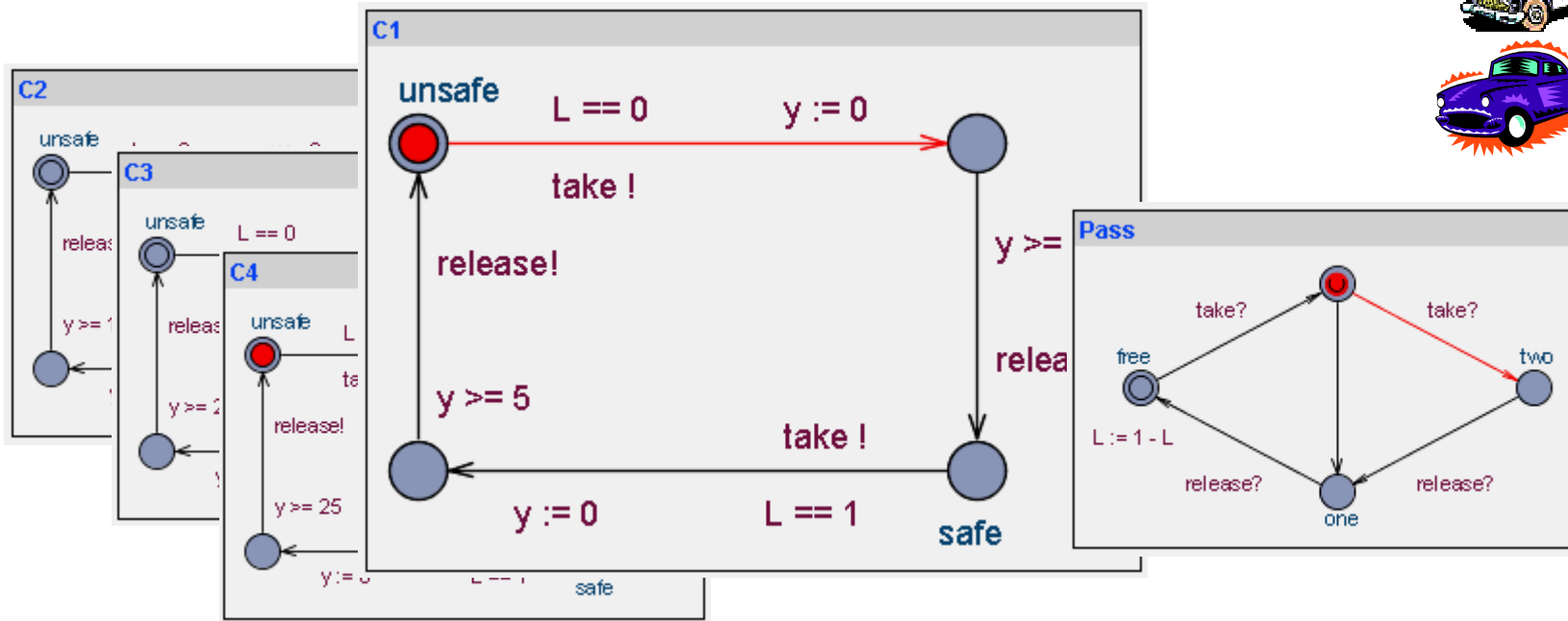
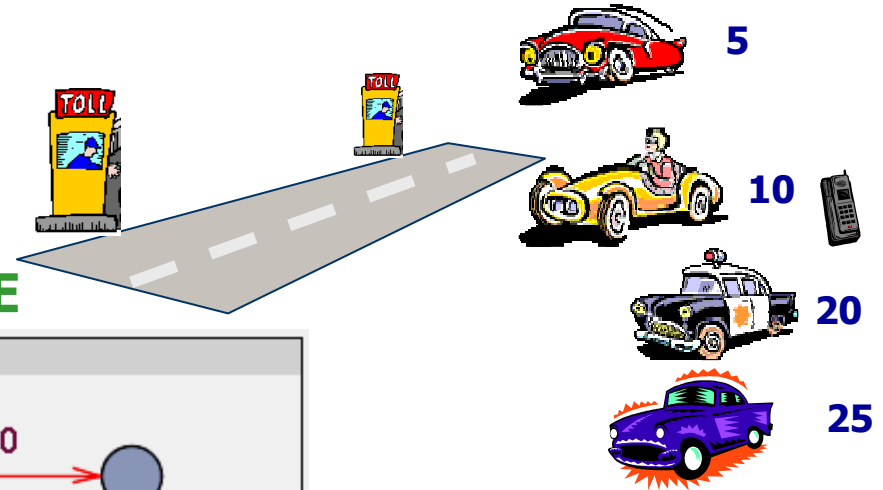
**CAN THEY MAKE IT TO SAFE
 WITHIN 70 MINUTES ???**

Real Time Scheduling

Solve Scheduling Problem using UPPAAL

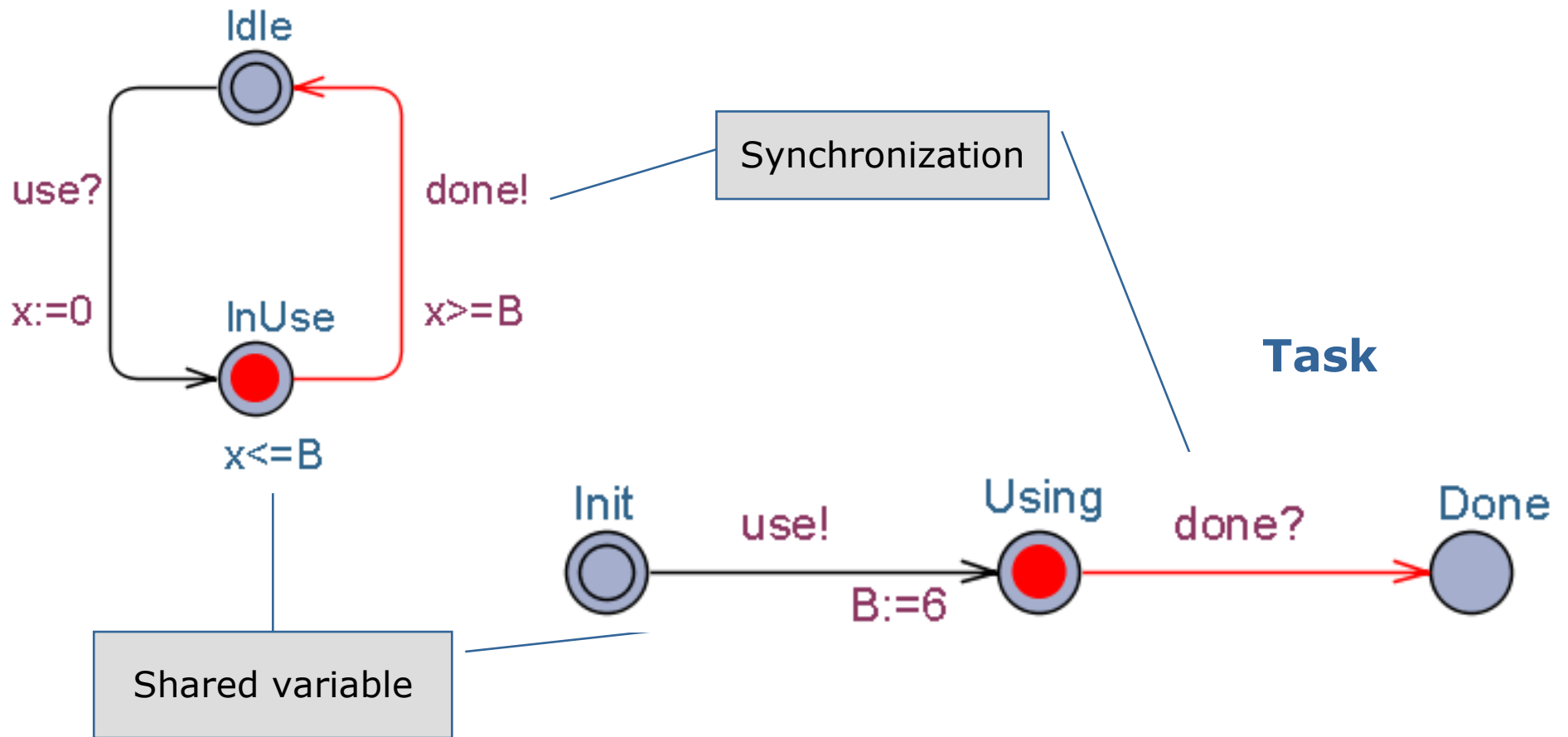
UNSAFE

SAFE



Resources & Tasks

Resource



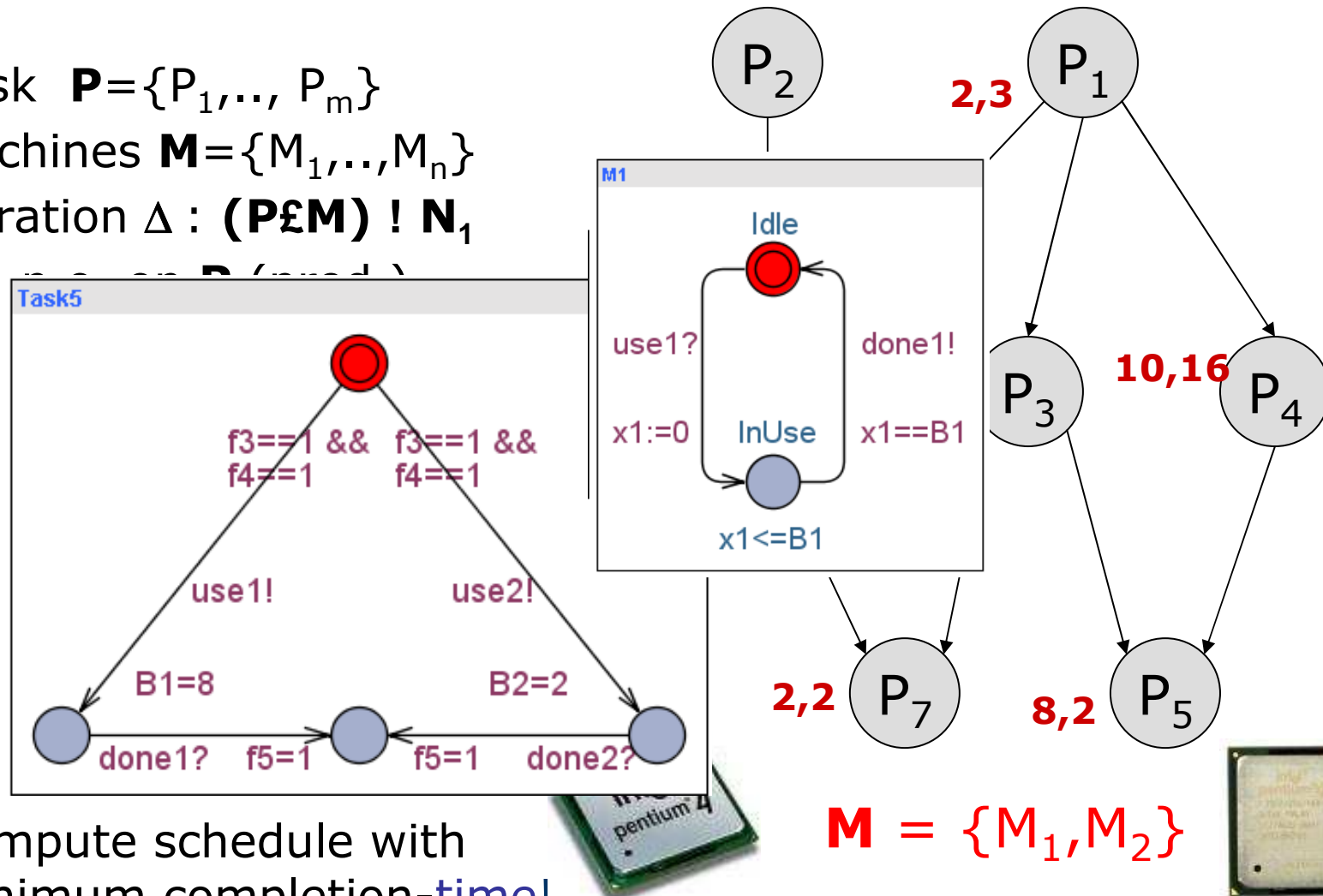
Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$
- Duration $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbf{N}_1$
- $< : \mathbf{P} \times \mathbf{P} \rightarrow \mathbf{N}_1$ (precedence)

- A task graph is a directed acyclic graph
- Each node represents a task
- Task P_i precedes task P_j if a directed edge exists from P_i to P_j

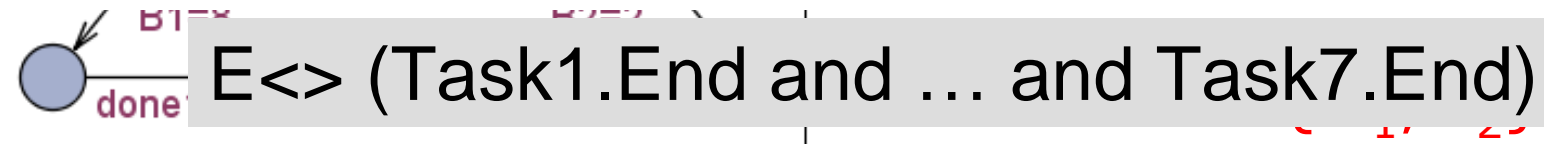
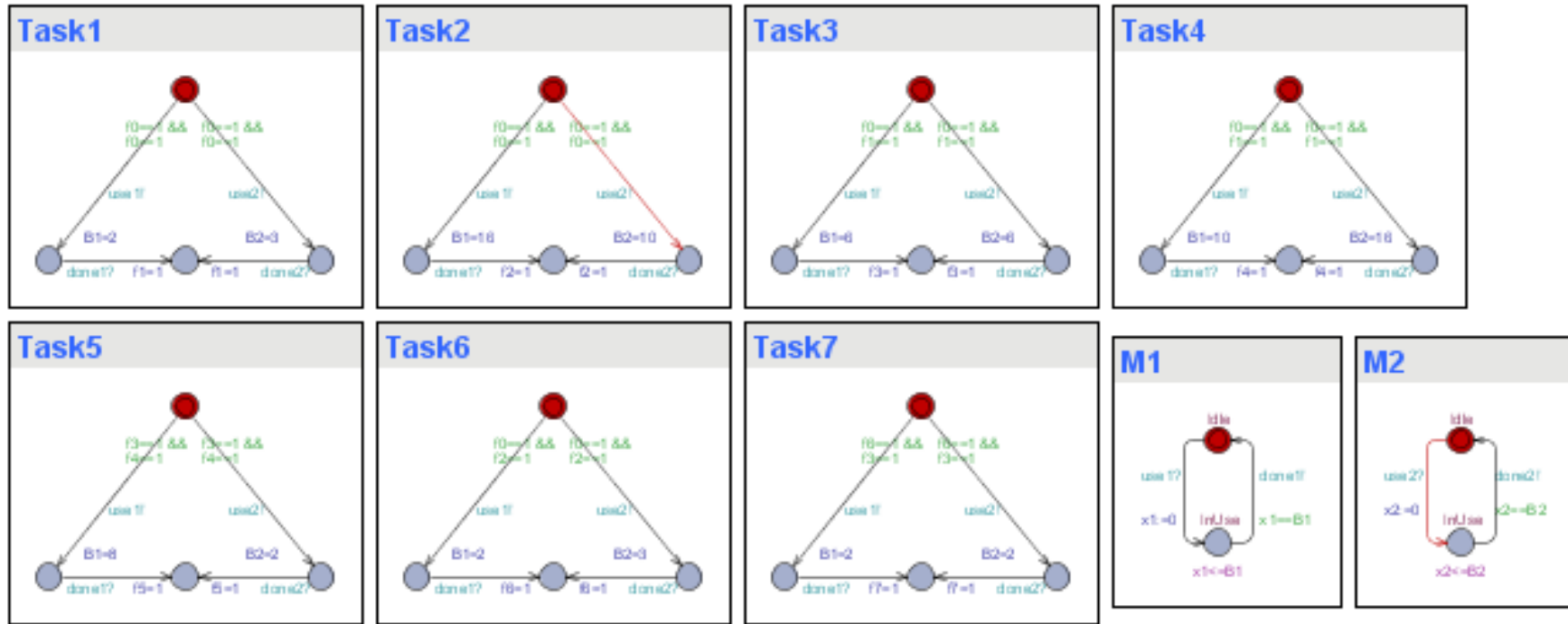
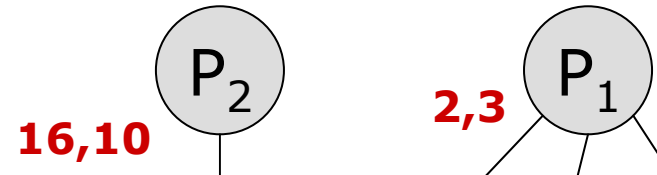
- Compute schedule with minimum completion-time!



Task Graph Scheduling

Optimal Static Task Scheduling

- Task $\mathbf{P} = \{P_1, \dots, P_m\}$
- Machines $\mathbf{M} = \{M_1, \dots, M_n\}$



Experimental Results

name	#tasks	#chains	# machines	optimal	TA
001	437	125	4	1178	1182
000	452	43	20	537	537
018	730	175	10	700	704
074	1007	66	12	891	894
021	1145	88	20	605	612
228	1187	293	8	1570	1574
071	1193	124	20	629	634
271	1348	127	12	1163	1164
237	1566	152	12	1340	1342
231	1664	101	16	t.o.	1137
235	1782	218	16	t.o.	1150
233	1980	207	19	1118	1121
294	2014	141	17	1257	1261
295	2168	965	18	1318	1322
292	2333	318	3	8009	8009
298	2399	303	10	2471	2473



Symbolic A*
 Brand-&-Bound
 60 sec

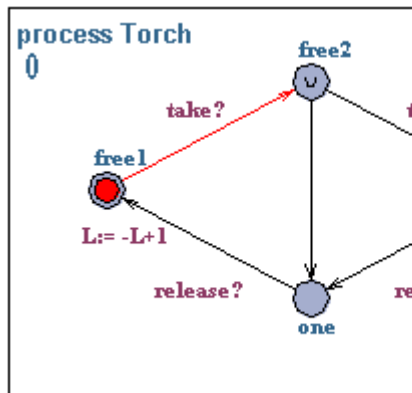
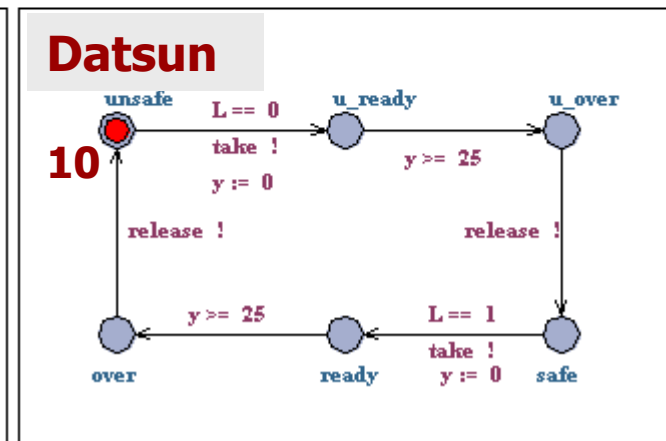
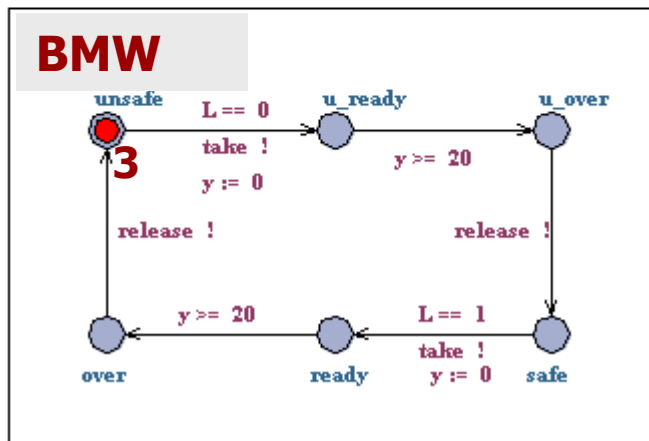
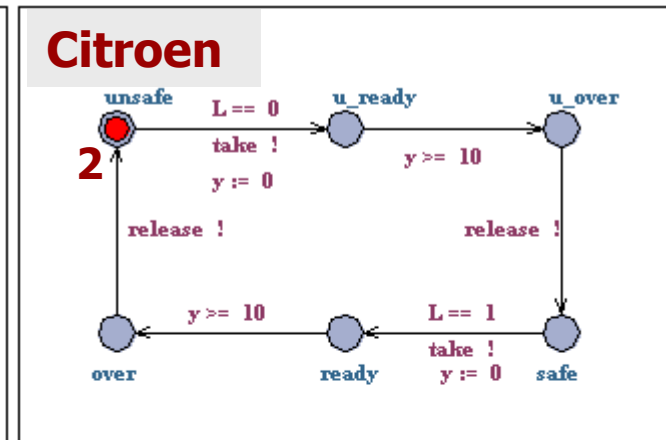
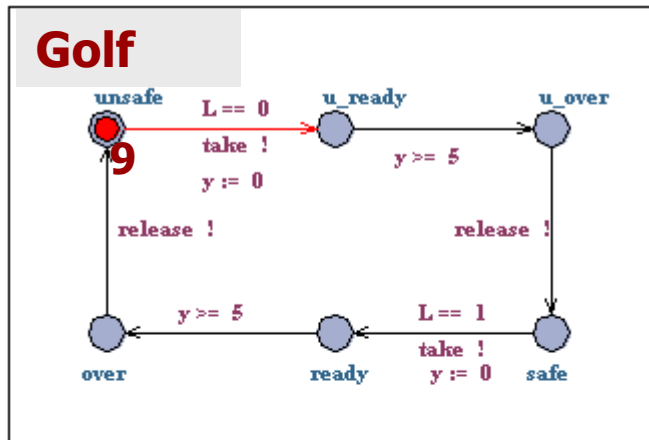
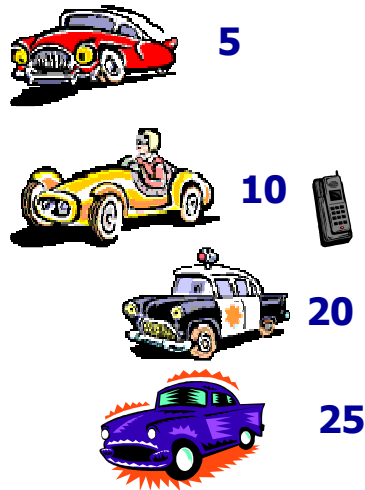
Abdeddaïm, Kerbaa, Maler

Optimality

Priced Timed Automata

with Paul Pettersson, Thomas Hune, Judi Romijn,
Ansgar Fehnker, Ed Brinksma, Frits Vaandrager,
Patricia Bouyer, Franck Cassez, Henning Dierks
Emmanuel Fleury, Jacob Rasmussen, ..

EXAMPLE: Optimal rescue plan for cars with different subscription rates for city driving !



OPTIMAL PLAN HAS ACCUMULATED COST=195 and TOTAL TIME=65!

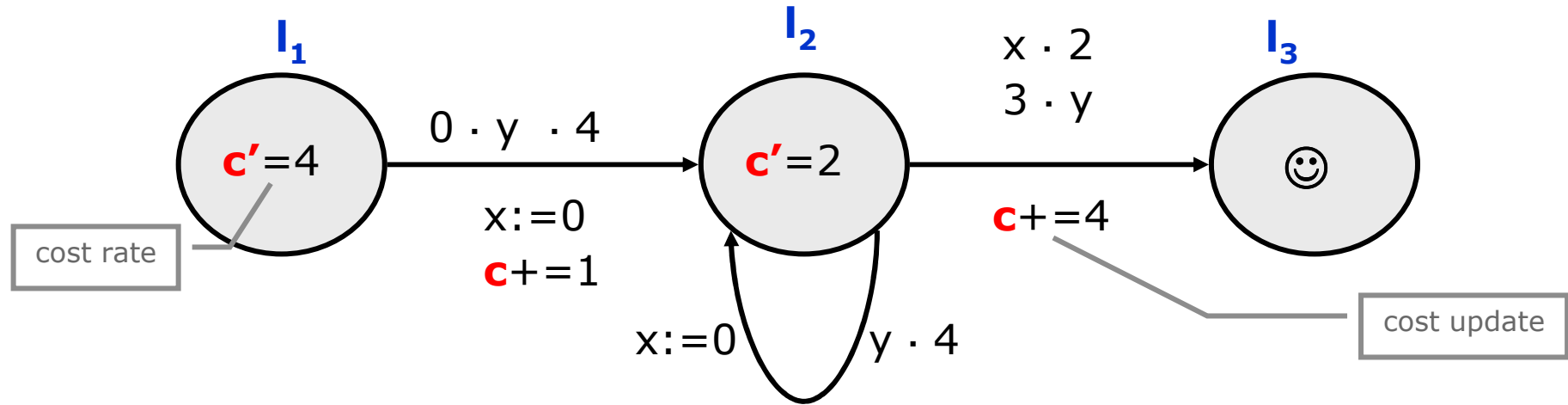
Experiments

COST-rates				SCHEDULE	COST	TIME	#Expl	#Pop'd
G	C	B	D					
Min Time				CG> G< BD> C< CG>		60	1762 1538	2638
1	1	1	1	CG> G< BG> G< GD>	55	65	252	378
9	2	3	10	GD> G< CG> G< BG>	195	65	149	233
1	2	3	4	CG> G< BD> C< CG>	140	60	232	350
1	2	3	10	CD> C< CB> C< CG>	170	65	263	408
1	20	30	40	BD> B< CB> C< CG>	975 1085	85 time<85	-	-
0	0	0	0	-	0	-	406	447

Priced Timed Automata

Timed Automata + **COST** variable

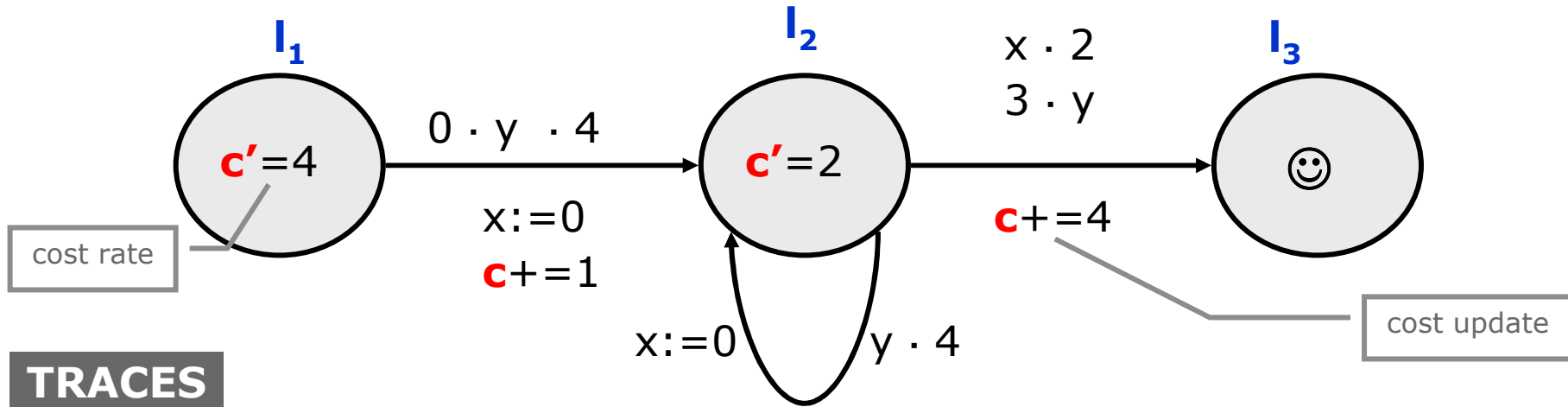
Behrmann, Fehnker, et al (HSCC'01)
 Alur, Torre, Pappas (HSCC'01)



Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et al (HSCC'01)
Alur, Torre, Pappas (HSCC'01)



TRACES

$$(l_1, x=y=0) \xrightarrow[12]{\varepsilon(3)} (l_1, x=y=3) \xrightarrow[1]{} (l_2, x=0, y=3) \xrightarrow[4]{} (l_3, \dots)$$

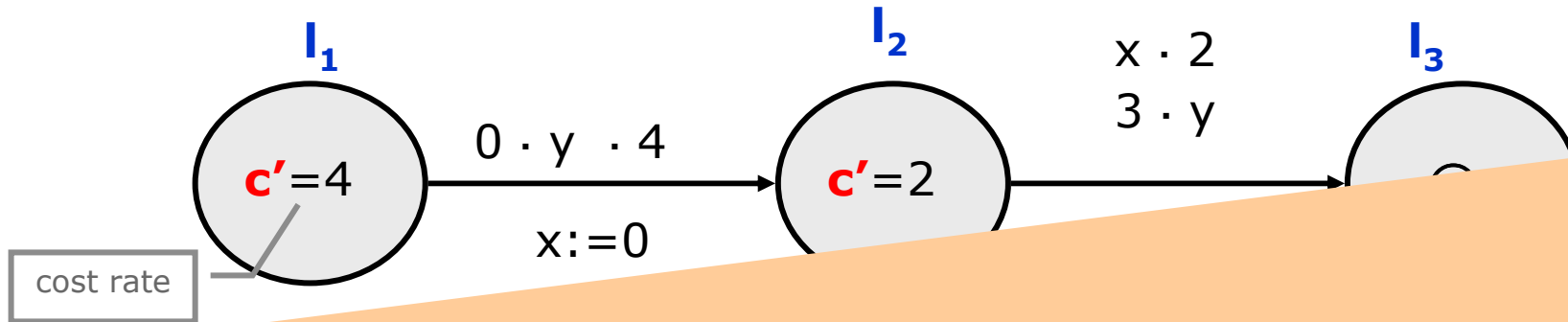
$\Sigma c=17$

Priced Timed Automata

Timed Automata + **COST** variable

Behrmann, Fehnker, et al (HSCC'01)

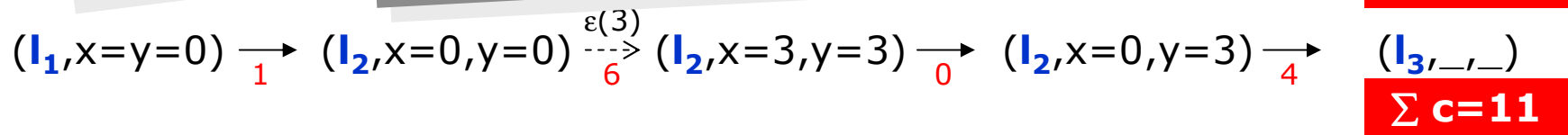
Alur, Torre, Pappas (HSCC'01)



TRA

Problem :
Find the **minimum** cost of reaching location l_3

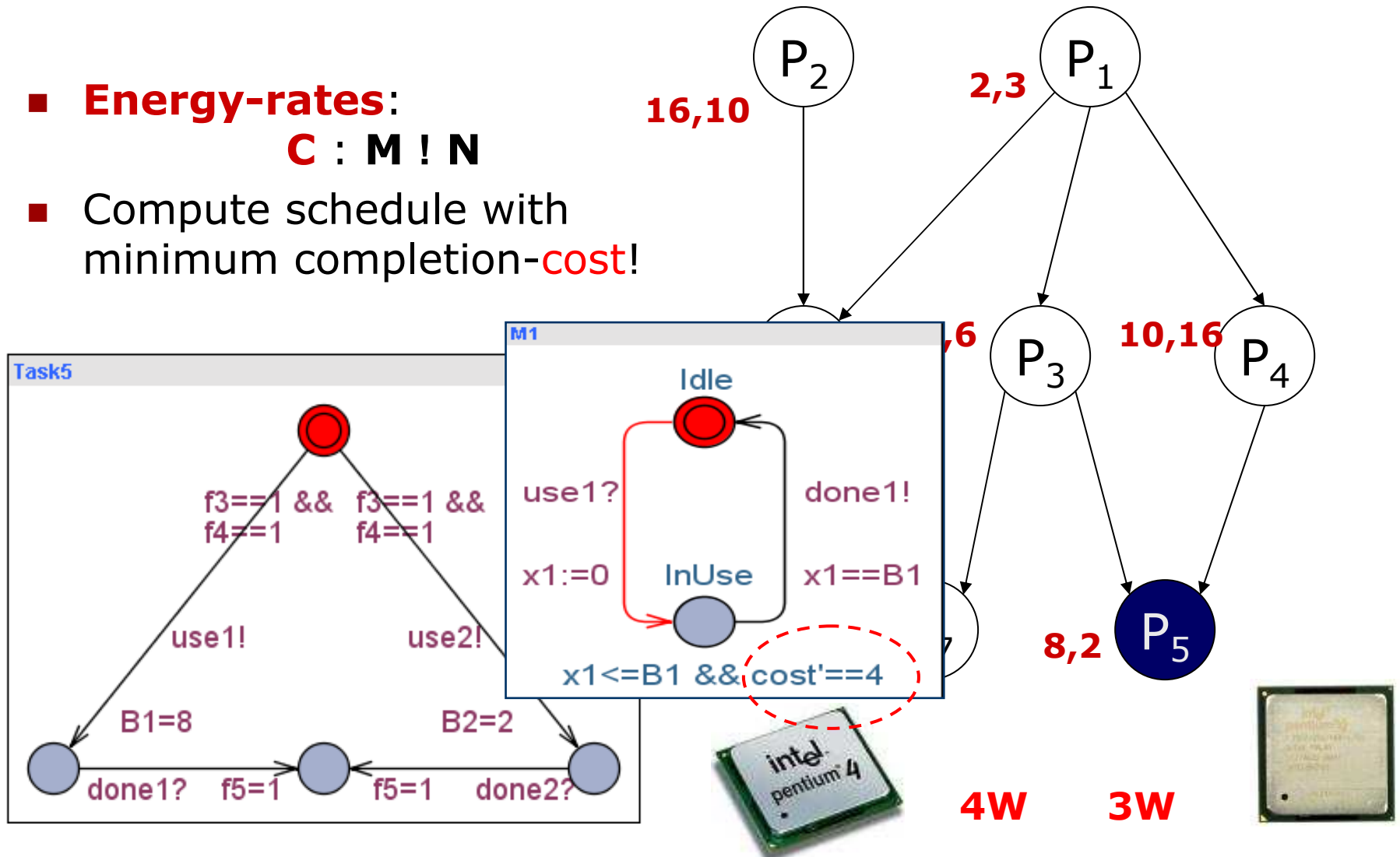
Efficient Implementation:
CAV'01 and TACAS'04



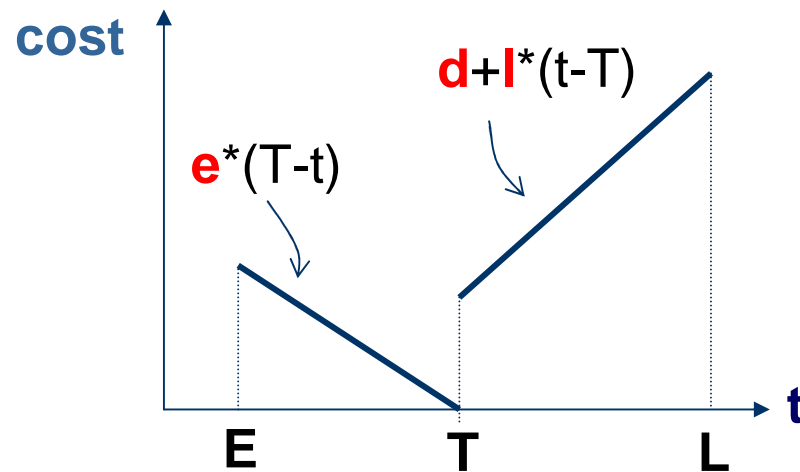
Optimal Task Graph Scheduling

Power-Optimality

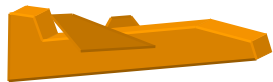
- Energy-rates:
 $C : M ! N$
- Compute schedule with minimum completion-cost!



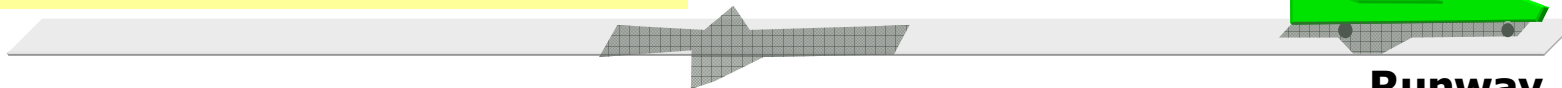
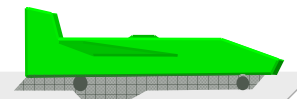
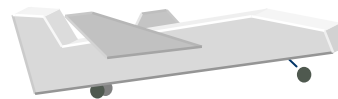
Aircraft Landing Problem



- E** earliest landing time
- T** target time
- L** latest time
- e** cost rate for being early
- l** cost rate for being late
- d** fixed cost for being late



Planes have to keep separation distance to avoid turbulences caused by preceding planes

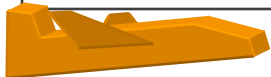
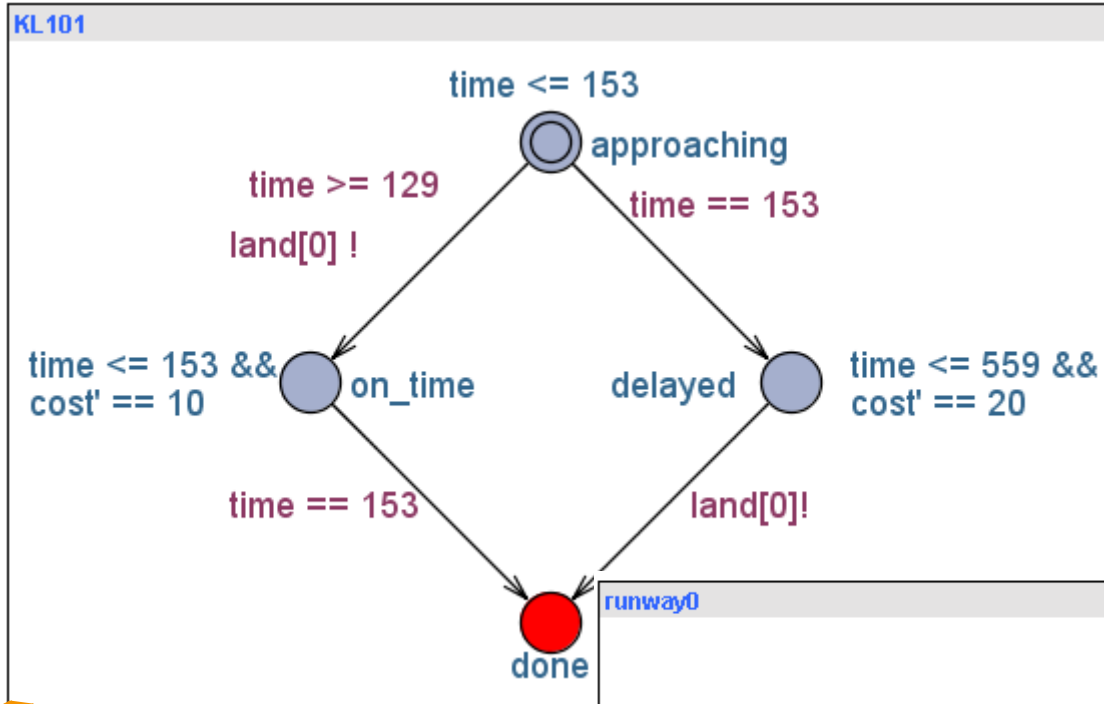


Runway

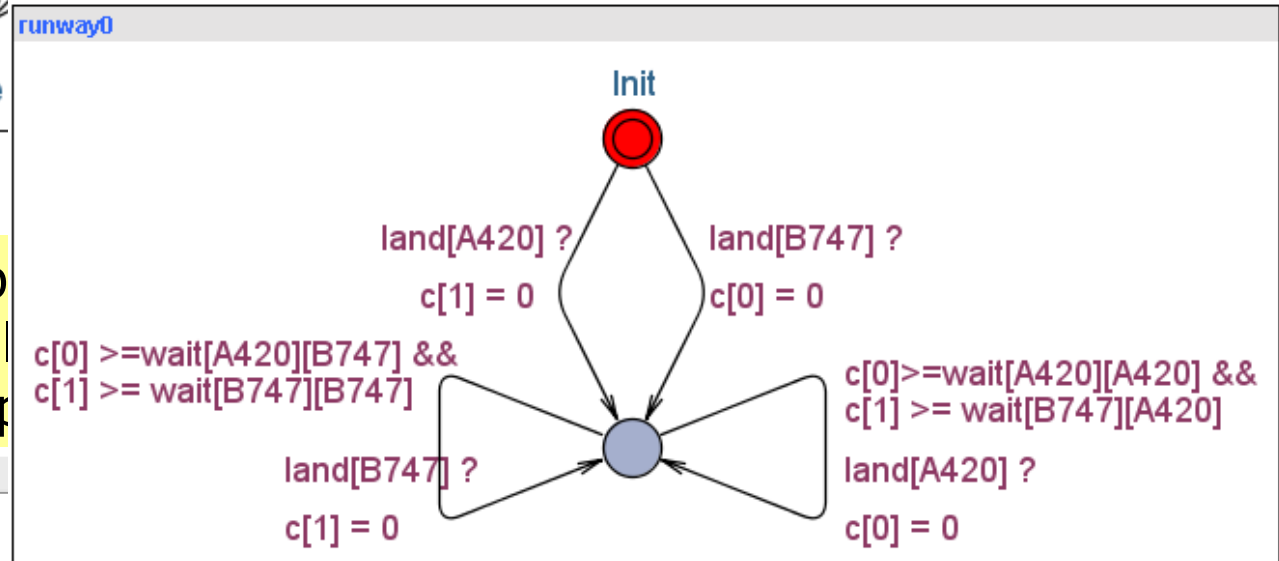
Modeling ALP with PTA

- 129: Earliest landing time
- 153: Target landing time
- 559: Latest landing time
- 10: Cost rate for early
- 20: Cost rate for late

Runway handles 2 types of planes



Planes have to keep sep distance to avoid turbul caused by preceding p



Aircraft Landing

CAV'01

Source of examples:
 Baesley et al'2000

	problem instance	1	2	3	4	5	6	7
	number of planes	10	15	20	20	20	30	44
	number of types	2	2	2	2	2	4	2
1	optimal value	700	1480	820	2520	3100	24442	1550
	explored states	481	2149	920	5693	15069	122	662
	cputime (secs)	4.19	25.30	11.05	87.67	220.22	0.60	4.27
2	optimal value	90	210	60	640	650	554	0
	explored states	1218	1797	669	28821	47993	9035	92
	cputime (secs)	17.87	39.92	11.02	755.84	1085.08	123.72	1.06
3	optimal value	0	0	0	130	170	0	
	explored states	24	46	84	207715	189602	62	N/A
	cputime (secs)	0.36	0.70	1.71	14786.19	12461.47	0.68	
4	optimal value				0	0		
	explored states	N/A	N/A	N/A	65	64	N/A	N/A
	cputime (secs)				1.97	1.53		

Symbolic "A*"

Zones

Definition

A *zone* Z over a set of clocks C is a finite conjunction of simple constraints of the forms:

$$x \geq l \quad x \leq u \quad x - y \geq l' \quad x - y \leq u'$$

where $x, y \in C$, $l, u \in \mathbb{N}$ and $l', u' \in \mathbb{Z}$.

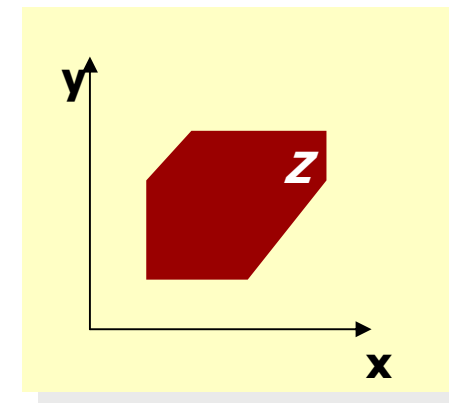
For $u \in \mathbb{R}^C$ and Z a zone we write $u \models Z$ if u satisfies all constraints of Z .

Operations

Reset: $\{x\}Z = \{u[0/x] \mid u \models Z\}$

Delay: $Z^\uparrow = \{u + d \mid u \models Z\}$

Offset: $\Delta_Z \models Z$ such that $\forall u \models Z. \forall x \in C. \Delta_Z(x) \leq u(x)$.



Priced Zone

Definition

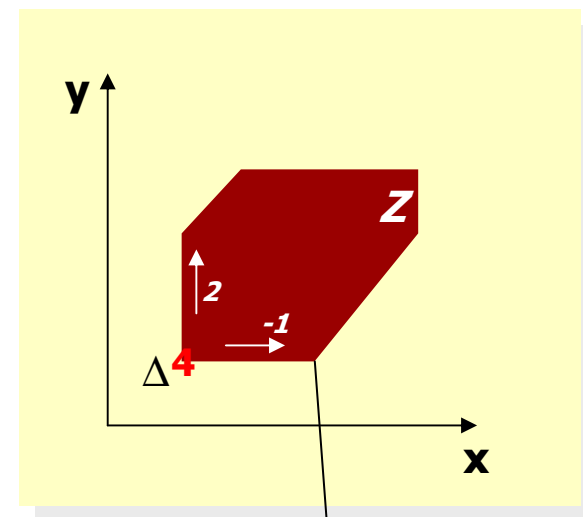
A priced zone P is a tuple (Z, c, r) , where:

- Z is a zone
- $c \in \mathbb{N}$ describes the cost of Δ_Z
- $r : C \rightarrow \mathbb{Z}$ gives a *rate* for any clock $x \in C$.

We write $u \models P$ whenever $u \models Z$. For $u \models P$ we define $\text{Cost}(u, P)$ as follows:

$$\text{Cost}(u, P) = c + \sum_{x \in C} r(x) \cdot (u(x) - \Delta_Z(x))$$

$$\text{Cost}(x, y) = 2y - x + 2$$



Branch & Bound Algorithm

```

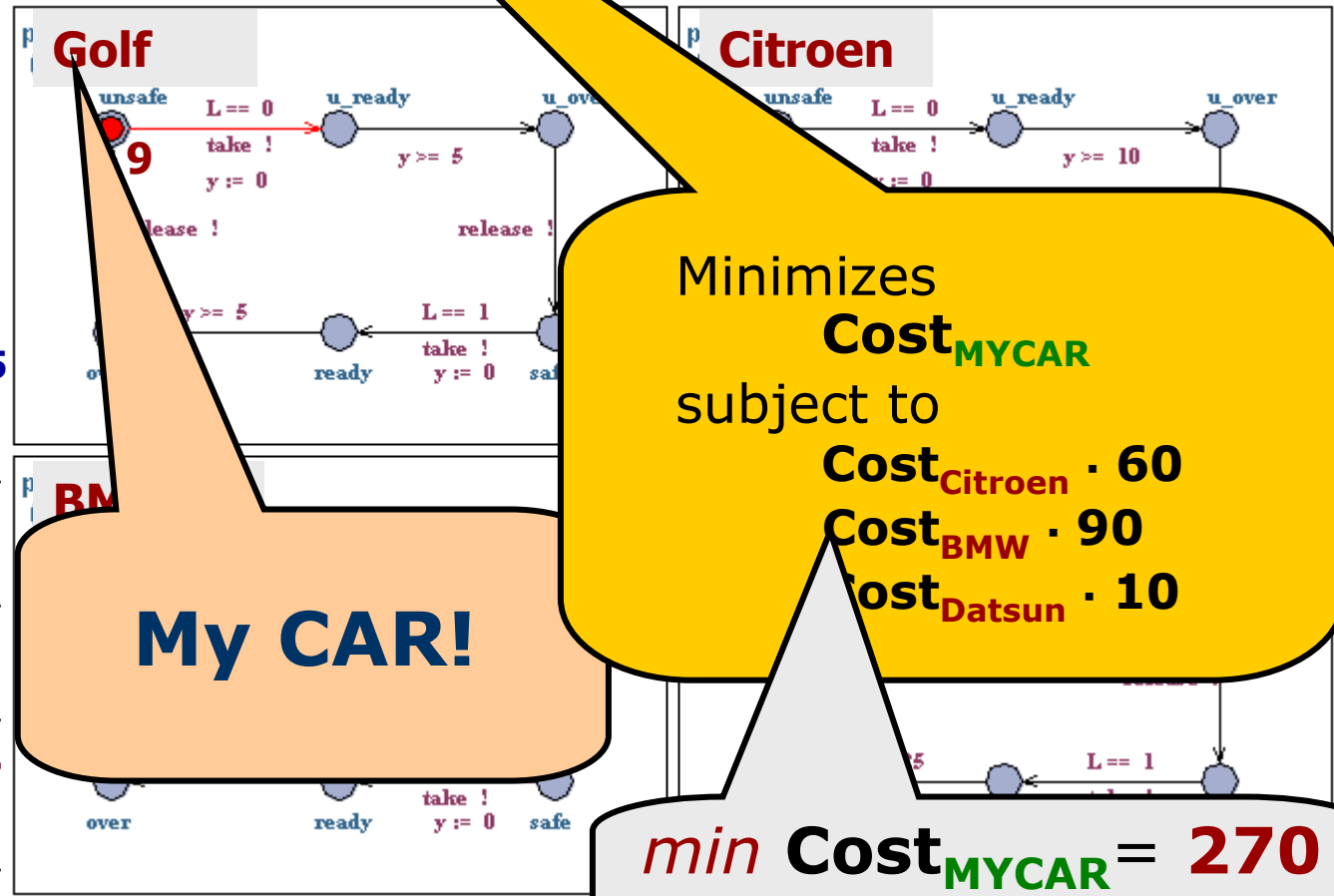
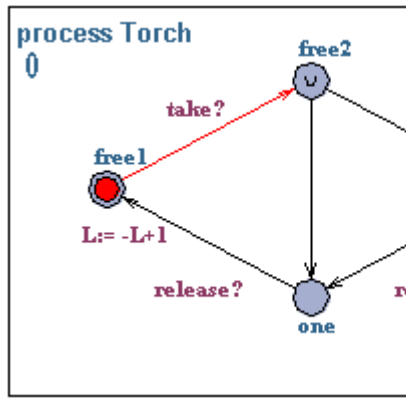
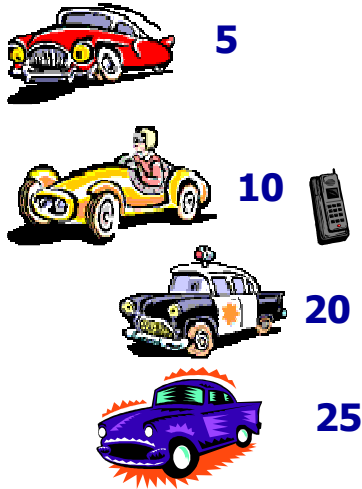
Cost := ∞
Passed := ∅
Waiting := {(l0, Z0)}
while Waiting ≠ ∅ do
    select (l, Z) from Waiting
    if l = lg and minCost(Z) < Cost then
        Cost := minCost(Z)
    if minCost(Z) + Rem(l,Z) ≥ Cost then break
    if for all (l, Z') in Passed: Z' ≠ Z then
        add (l, Z) to Passed
        add all (l', Z') with (l, Z) → (l', Z') to Waiting
return Cost
  
```

Optimization with Multi Objectives

with Jacob I. Rasmussen

EXAMPLE: **CONDITIONAL** Optimal rescue plan for cars with different subscription rates for city driving !

UNSAFE

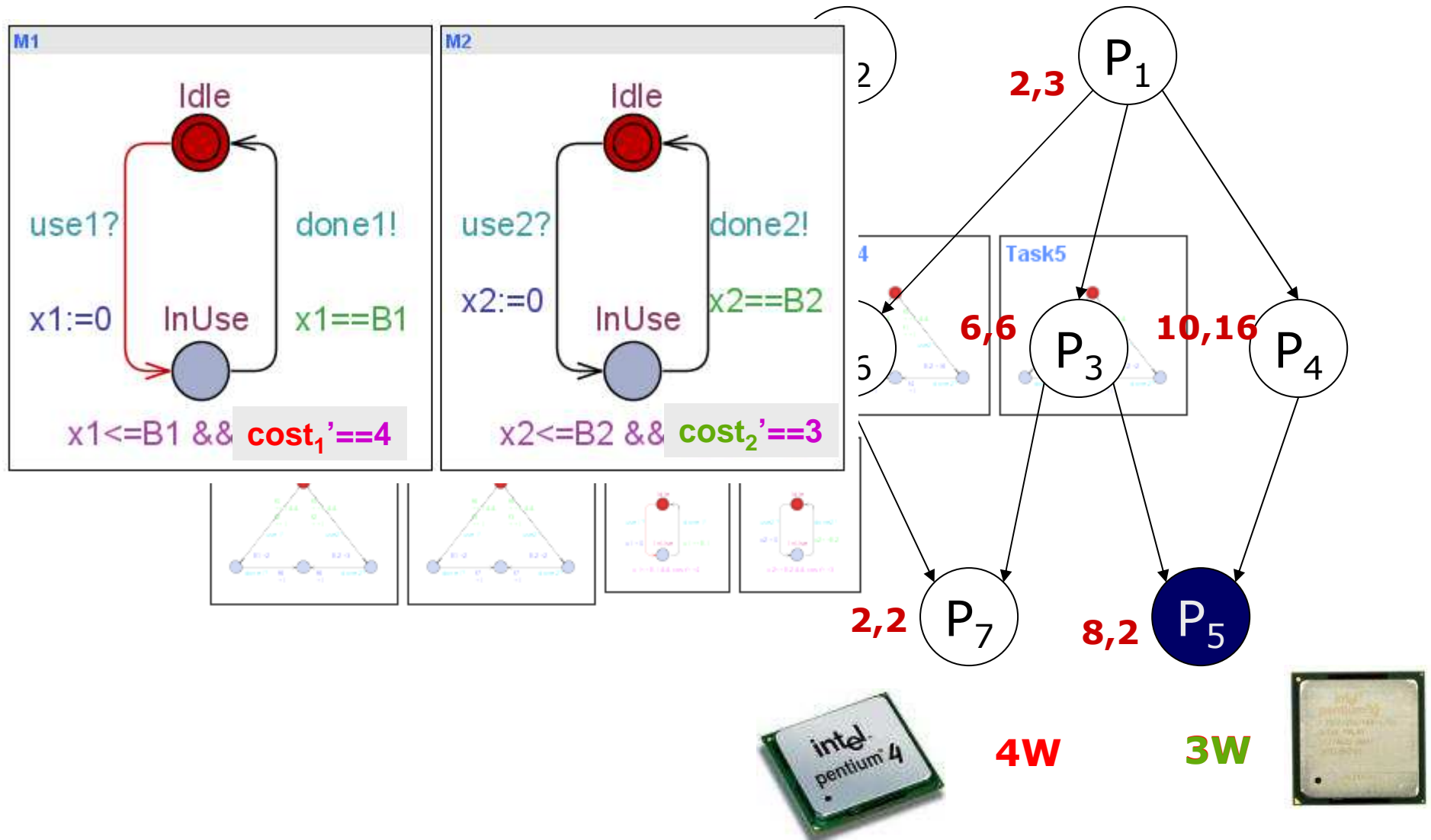


My CAR!

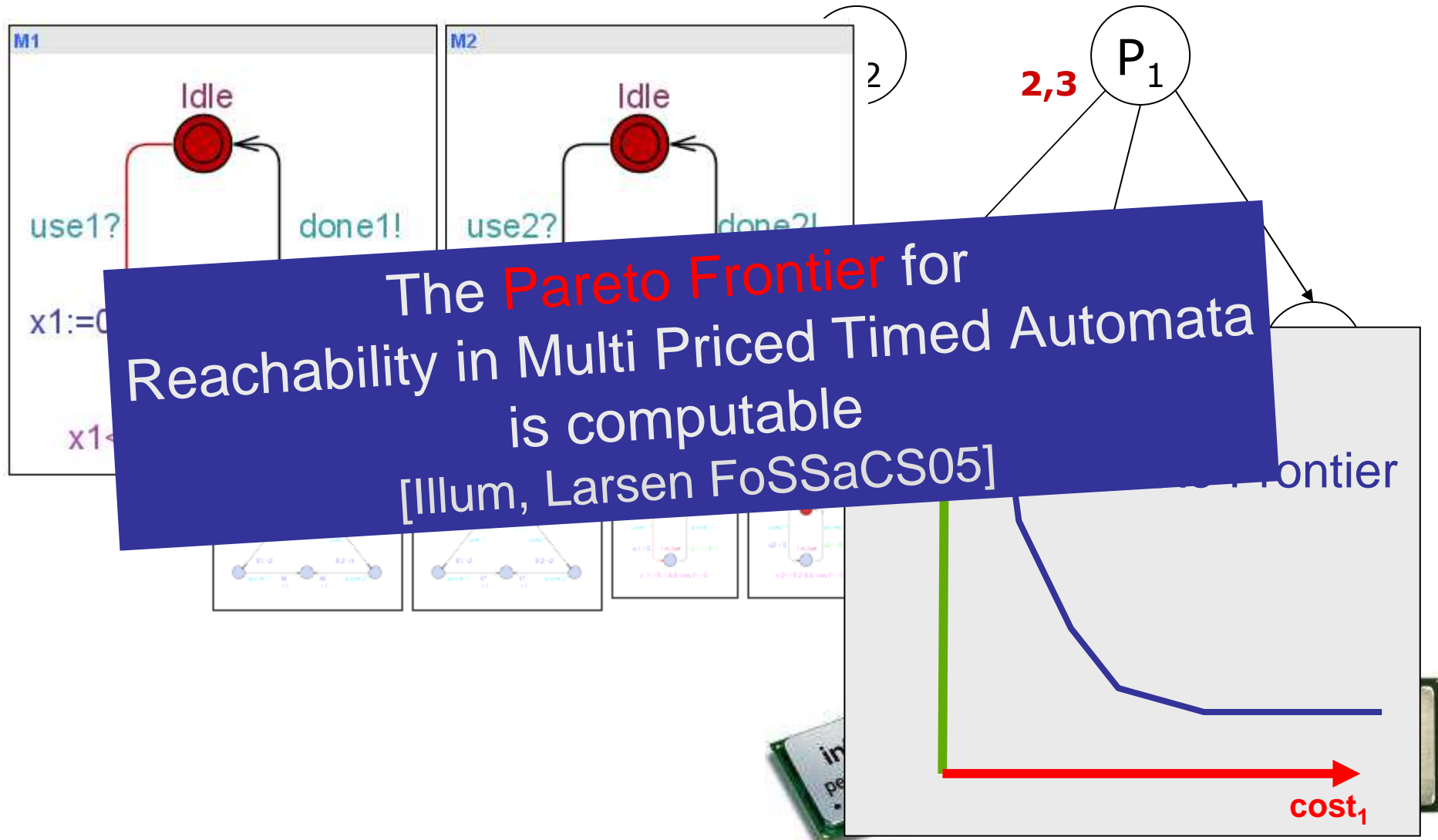
Minimizes **Cost_{MYCAR}**
 subject to
 Cost_{Citroen} · 60
 Cost_{BMW} · 90
 Cost_{Datsun} · 10

min Cost_{MYCAR} = **270**
 time = 70

Multiple Objective Scheduling



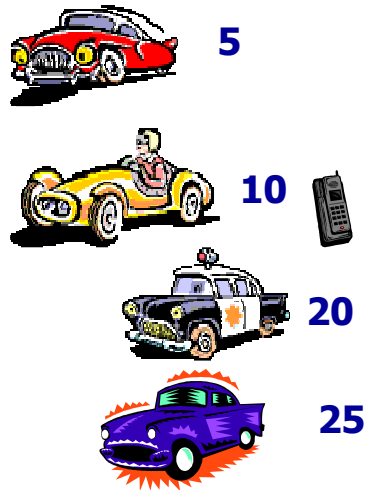
Multiple Objective Scheduling



Optimal Infinite Scheduling

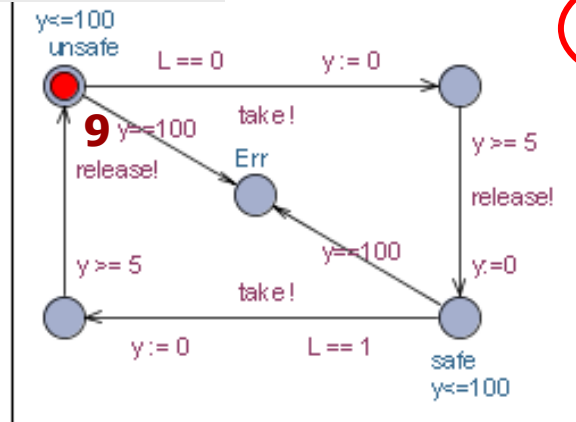
**with Ed Brinksma
Patricia Bouyer
Arne Skou
Ulrich Fahrenberg**

EXAMPLE: Optimal **WORK** plan for cars with different subscription rates for city driving !

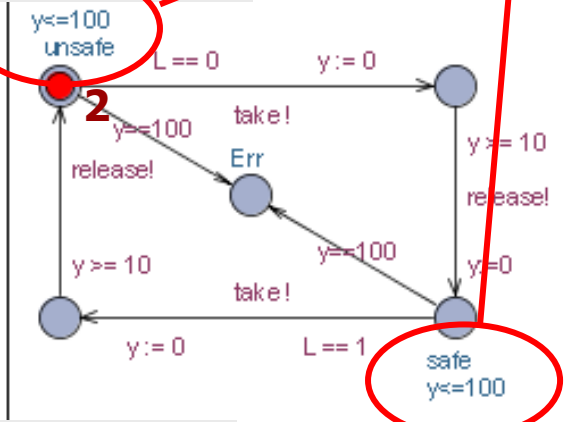


maximal 100 min. at each location

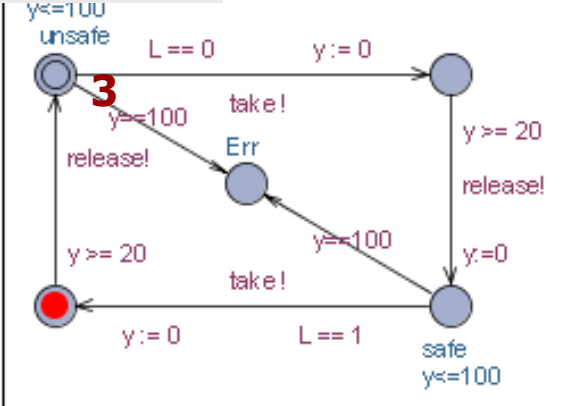
Golf



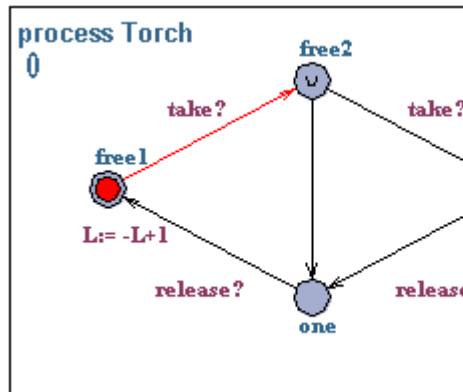
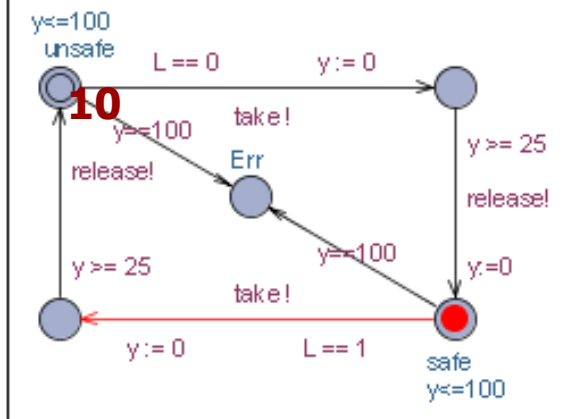
Citroen



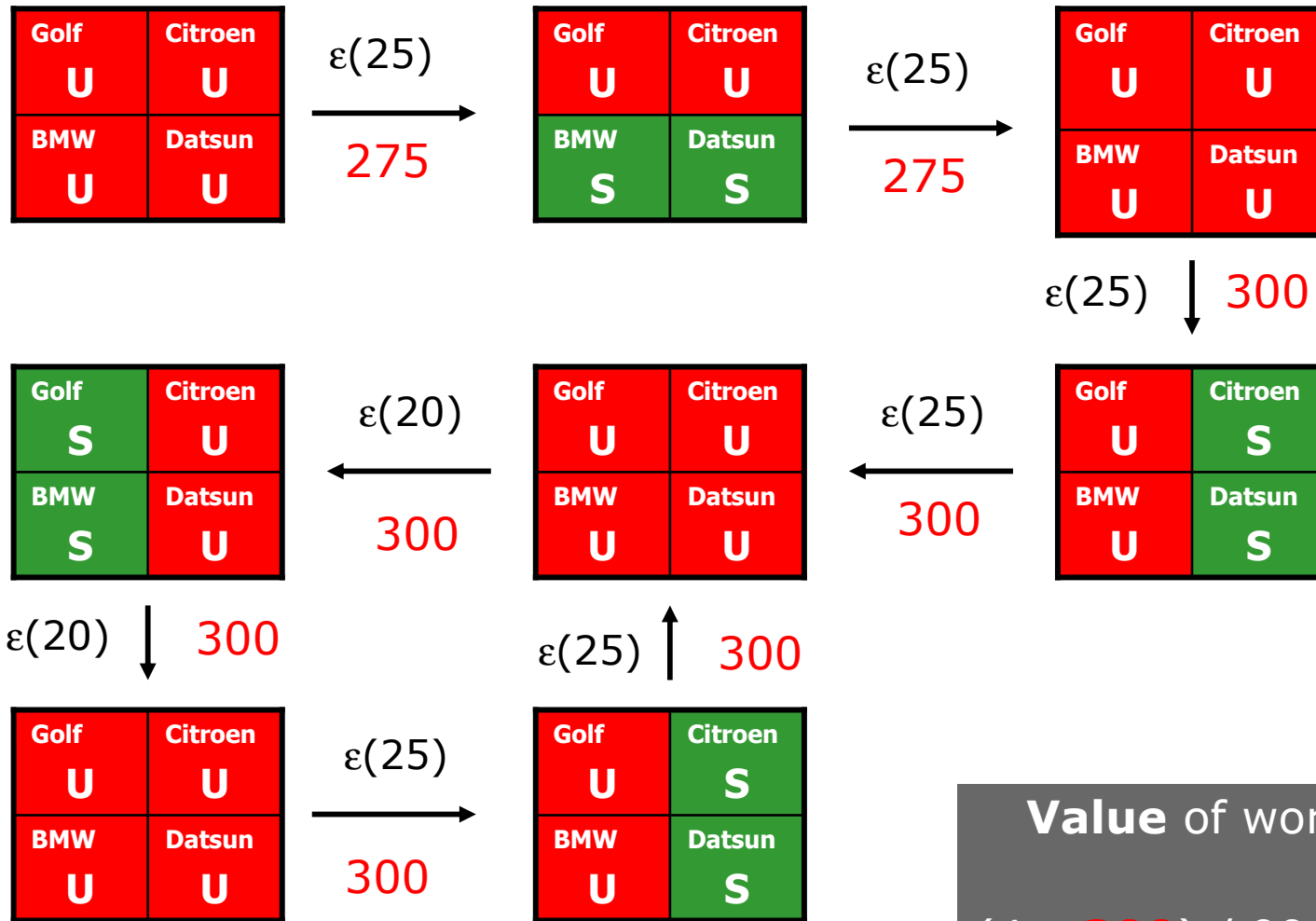
BMW



Datsun



Workplan I



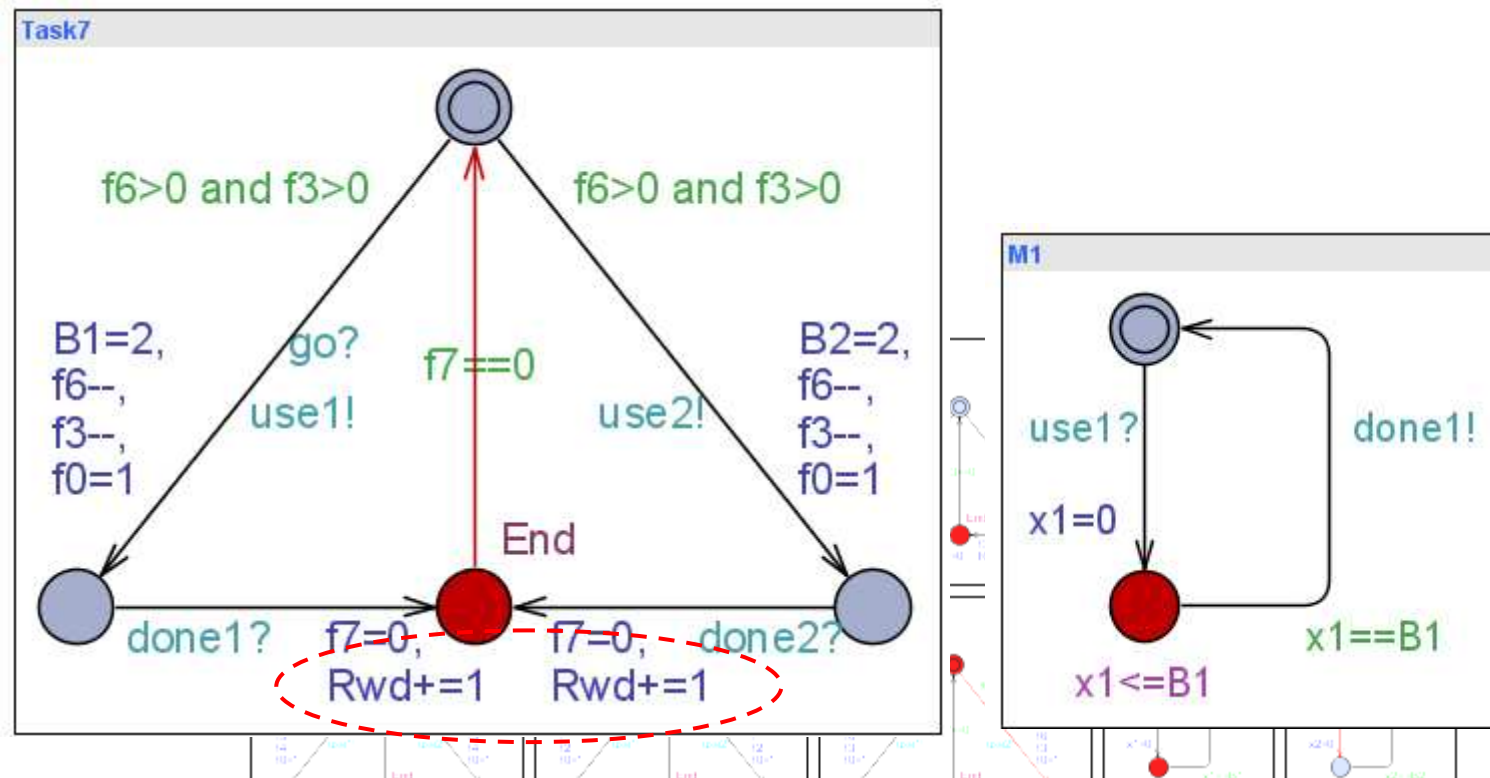
Value of workplan:
 $(4 \times 300) / 90 = 13.33$

Workplan II



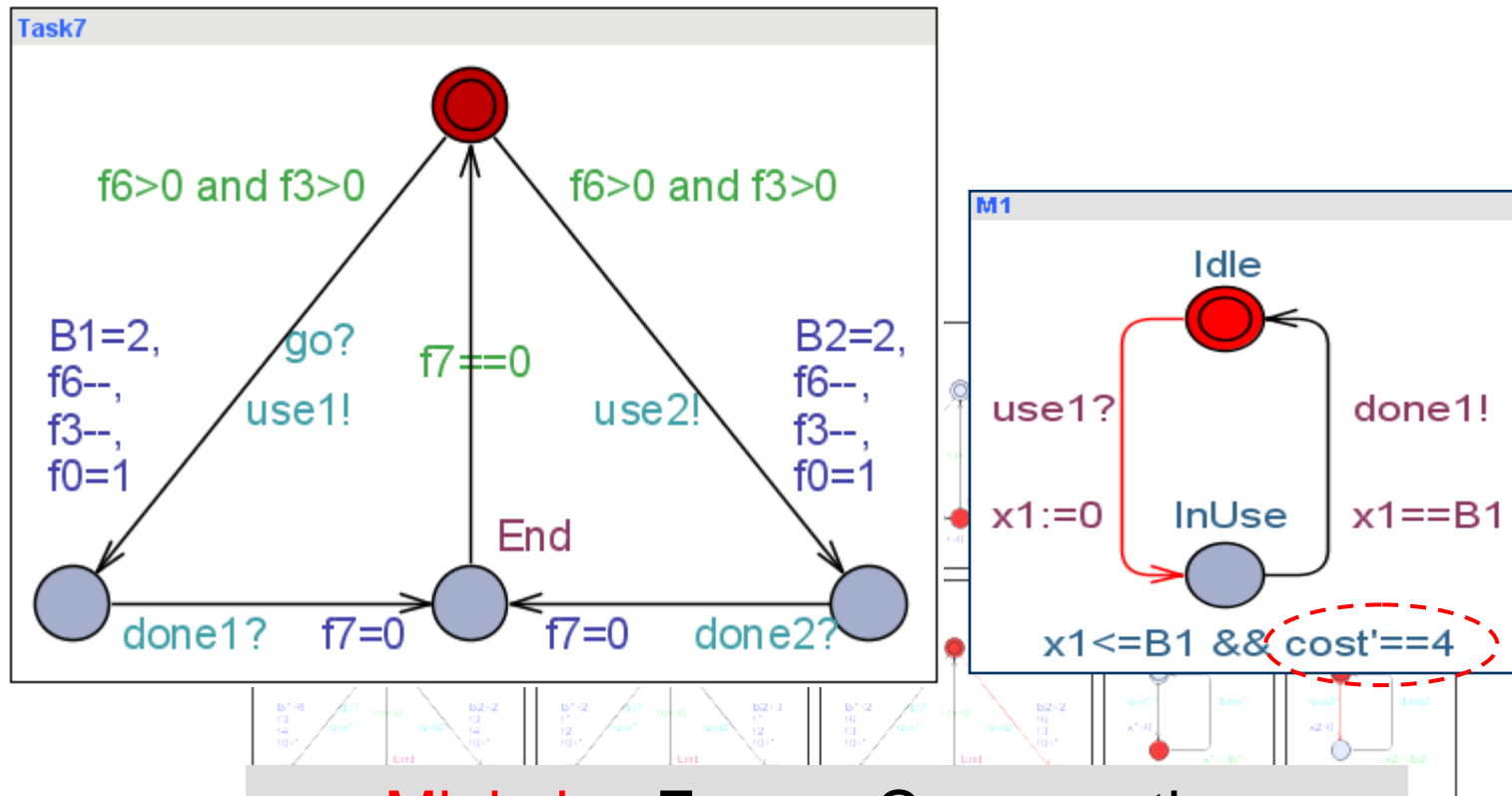
Value of workplan:
 $560 / 100 = 5.6$

Optimal Infinite Scheduling



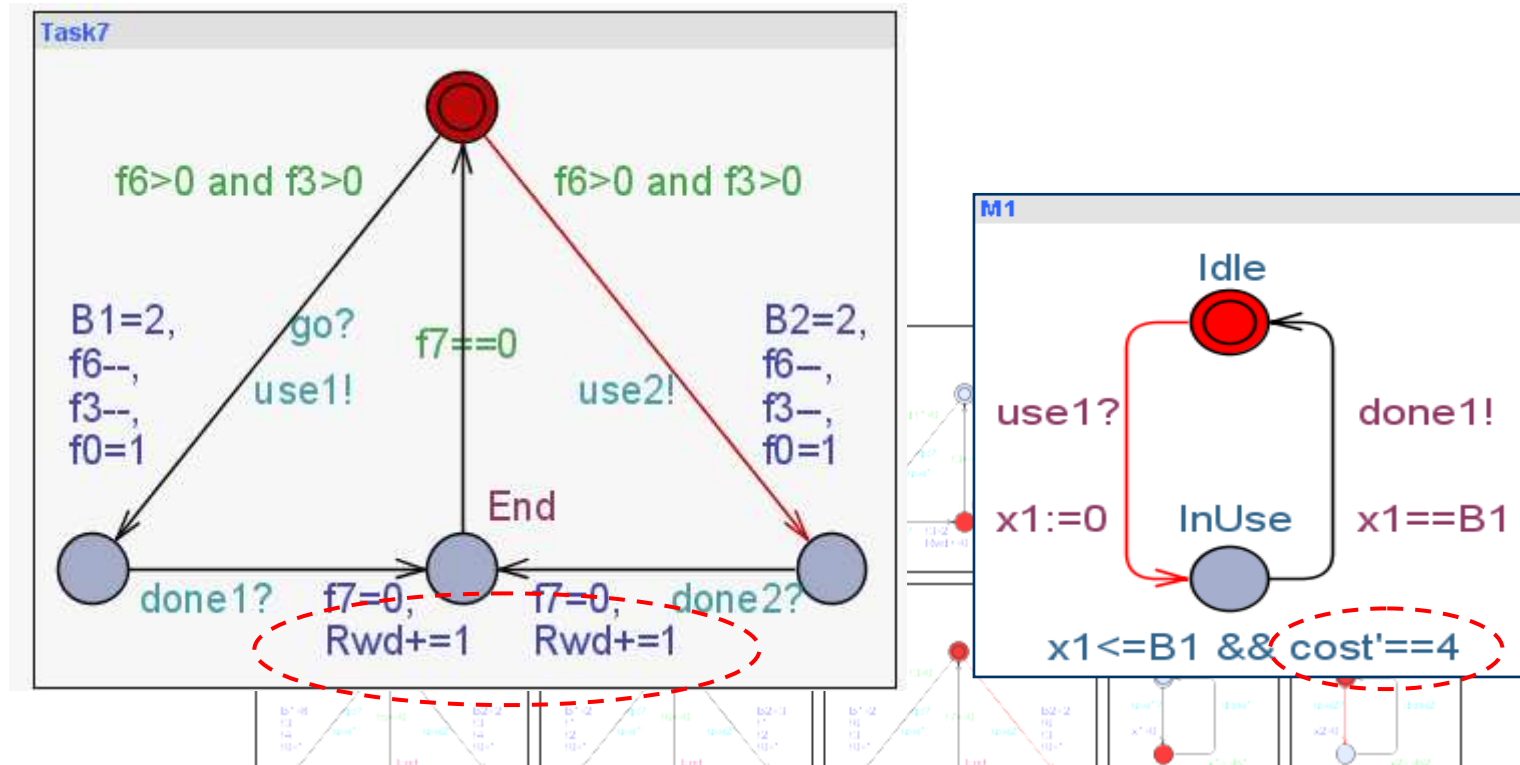
Maximize throughput:
 i.e. maximize **Reward** / Time in the long run!

Optimal Infinite Scheduling



Minimize Energy Consumption:
i.e. minimize **Cost** / Time in the long run

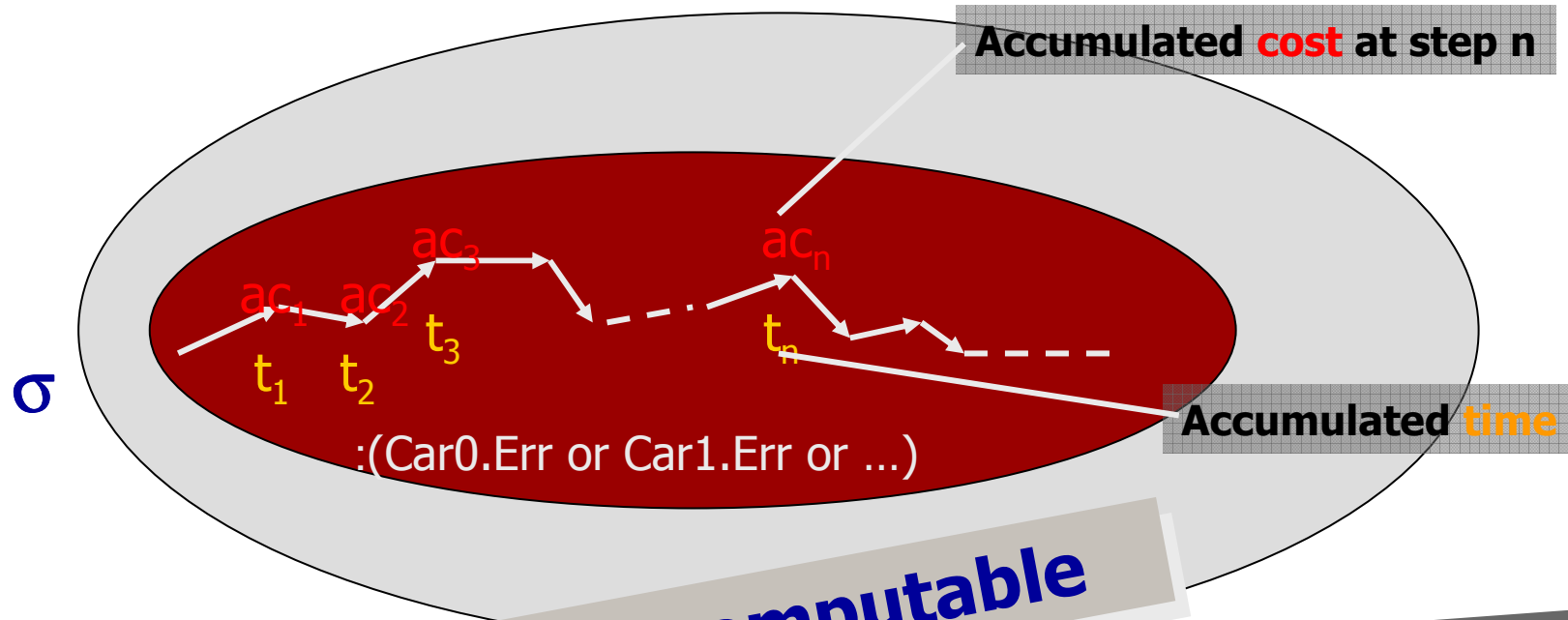
Optimal Infinite Scheduling



Maximize throughput:
 i.e. maximize **Reward** / **Cost** in the long run

Optimal Infinite Scheduling I

Limit Ratio (Pay-off)



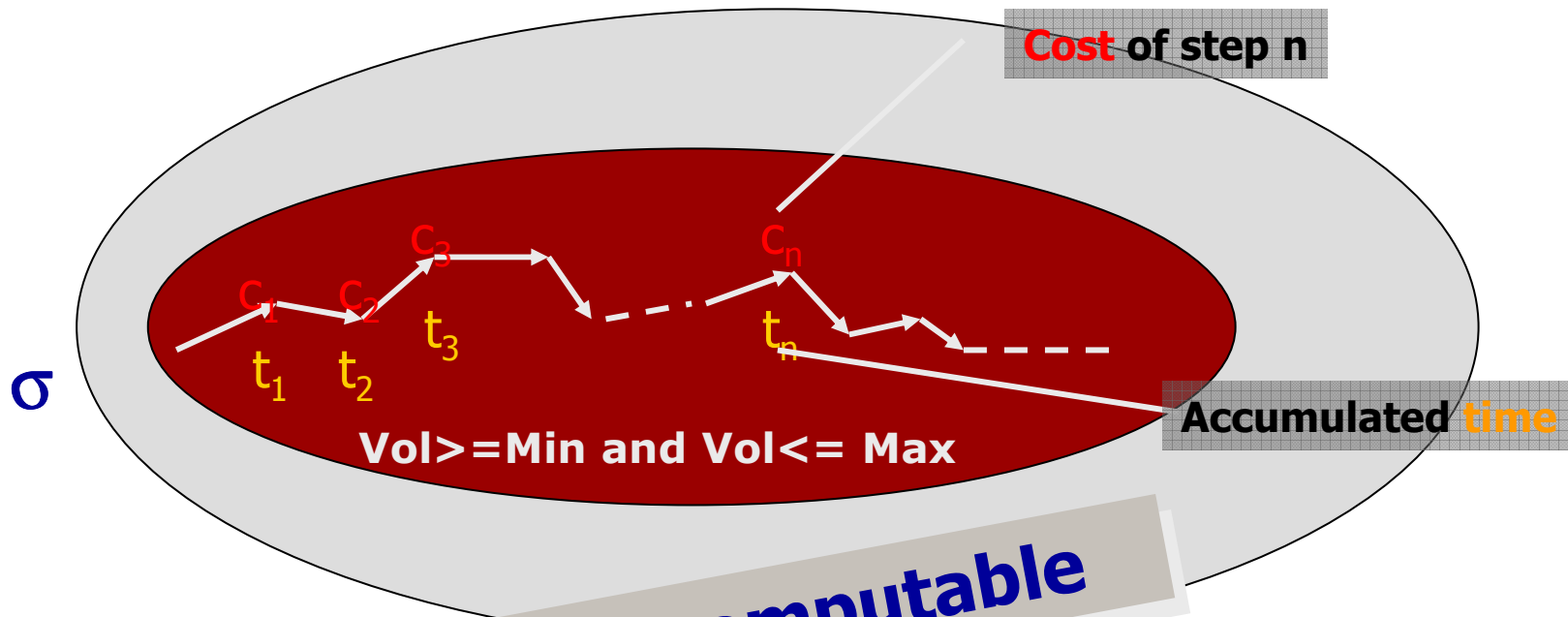
For path σ : $val(\sigma) = \lim_{n \rightarrow \infty} ac_n / t_n$

Optimal Schedule σ^* : $val(\sigma^*) = \inf_{\sigma} val(\sigma)$

Optimal Infinite Scheduling I

Discounting

$\lambda < 1$: discounting factor



THEOREM: σ^* is computable

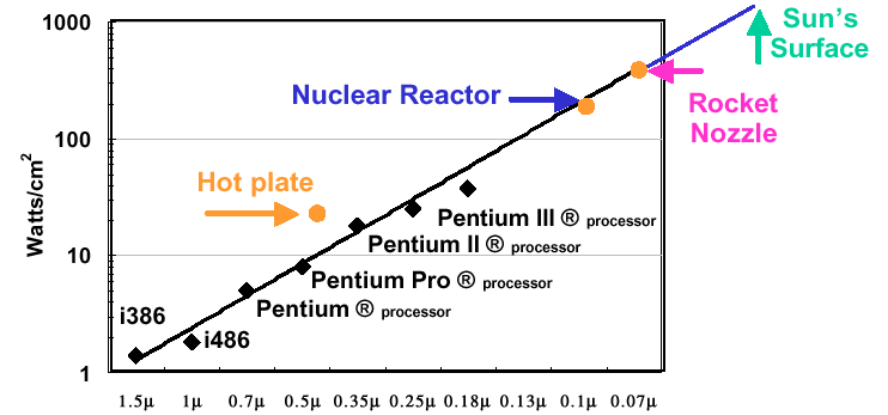
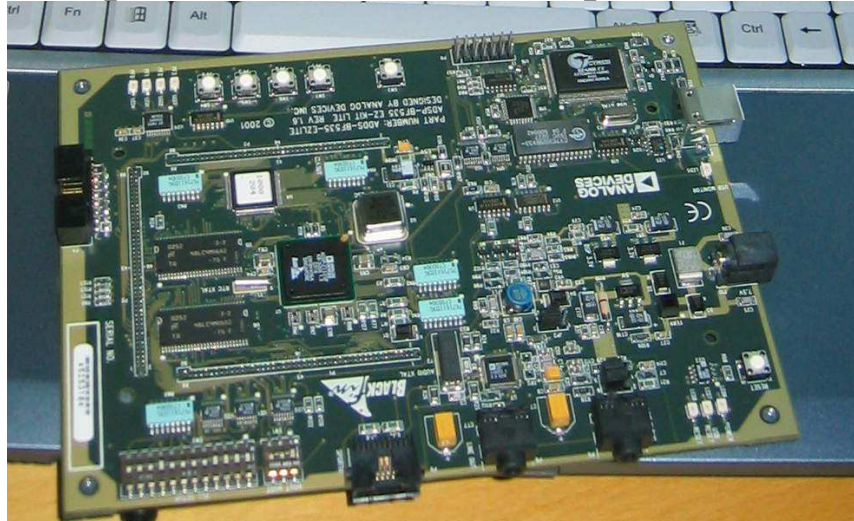
Fahrenberg, Larsen: INFINITY'08

$$\text{val}(\sigma) = \sum_{i=1}^{\infty} c_i \cdot \lambda^{t_i}$$

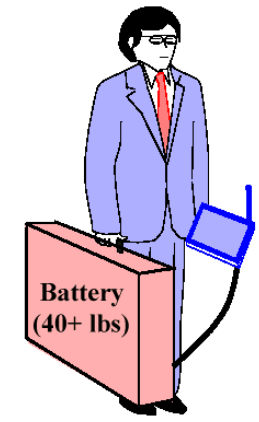
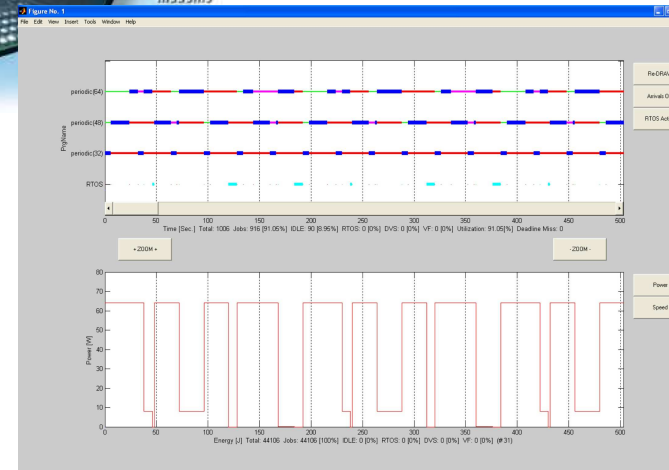
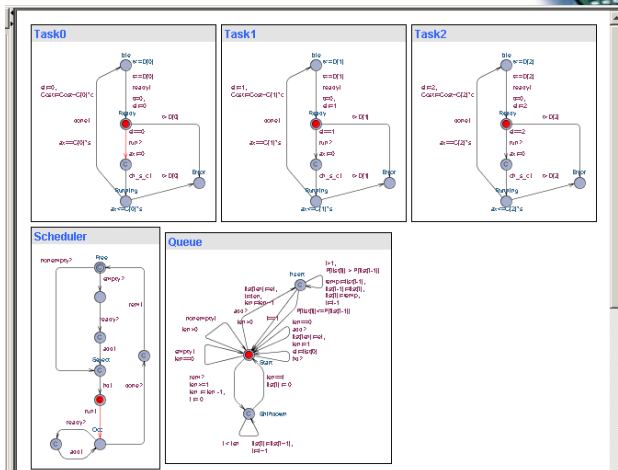
Optimal Schedule σ^* : $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$

Application

Dynamic Voltage Scaling



Automotive Electronics
 Biometrics
 Security and Surveillance
 Information Appliances
 Embedded Modems



Future Work & Challenges

☹️ Optimal Constrained Infinite Strategies for Multi-
 Priced TA

☹️ / 😊 Model Checking wrt **P**TCTL

[Raskin et al, FORMATS'04]
 [Bouyer, Larsen, Markey FoSSACS07]

☹️ Efficient implementations of Optimal Infinite
 Scheduling

☹️ Negative Cost Rates → Cost Bounded Infinite
 Runs

[Bouyer, Fahrenberg, Larsen, Markey, Srba
 FORMATS08]

Further Information

UPPAAL
 [Start | About | Documentation | Download | Examples | Internal]

Welcome to the UPPAAL home page. UPPAAL is an integrated tool environment for modeling, validation and verification of real-time systems modeled as networks of timed automata, extended with data types (bounded integers, arrays, etc.).

The tool is developed in collaboration between the [Design and Analysis of Real-Time Systems](#) group at Uppsala University, Sweden and [Basic Research in Computer Science](#) at Aalborg University, Denmark.

Download: UPPAAL2k 3.2.11 (released September 6, 2002) is available from the [download](#) page. More information about the 3.2 version of can be found in the [press release](#). In addition to the 3.2 version, a snapshot of version 3.3.23 is also available from the [download](#) page.

License: The UPPAAL tool is **free** for non-profit applications but we have a [license agreement](#) that all users must fill in before downloading and using the tool. To find out more about UPPAAL, read this short [introduction](#). Further information may be found at this web site in the pages [About](#), [Documentation](#), [Download](#), and [Examples](#).

Mailing lists: UPPAAL has an open mailinglist intended for users of the tool. To join the list, email uppaal-subscribe@yahoogroups.com, to post use uppaal@yahoogroups.com. For more information see the [group page](#) at Yahoo! Groups. To email the development team directly, please use bug-uppaal@docs.uu.se for bug reports and uppaal@docs.uu.se otherwise.

www.uppaal.com

Page created by Paul Pettersson. Last modified: Sep 6, 2002 (by Paul Pettersson).

Figure 1: UPPAAL2k on screen.



TIGA

Real Time Controller Synthesis

with

Gerd Behrmann, Patricia Bouyer,
Franck Cassez, *Agnes Counard*, *Alexandre David*
Emmanuel Fleury, *Didier Lime*, Nicolas Markey,
Jean-Francois Raskin



BRICS

Basic Research
in Computer Science



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

UPPAAL TIGA

UPPAAL for Timed Games

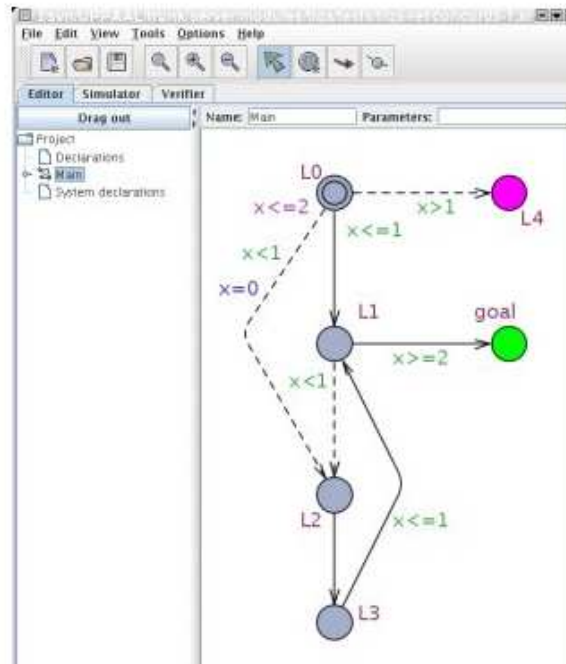
www.uppaal.com

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Welcome!

UPPAAL TIGA (Fig. 1) is an extension of UPPAAL [BDL04] and it implements the first efficient on-the-fly algorithm for solving games based on timed game automata with respect to reachability and safety properties. Though timed games for long have been known to be decidable there has until now been a lack of efficient and truly on-the-fly algorithms for their analysis.

The algorithm we propose [CDFLL05] is a symbolic extension of the on-the-fly algorithm suggested by Liu & Smolka [LS98] for linear-time model-checking of finite-state systems. Being on-the-fly, the symbolic algorithm may terminate long before having explored the entire state-space. Also the individual steps of the algorithm are carried out efficiently by the use of so-called zones as the underlying data structure. Our tool implements various optimizations of the basic symbolic algorithm, as well as methods for obtaining time-optimal winning strategies.



Latest News

Versions 0.10 and 0.11 released.

7 July 2007

Versions 0.10 and 0.11 are released today. Version 0.11 contains a new concrete simulator that allows the user to play strategies from the GUI. Both versions fix the following bugs: maximal constants in the formula are now taken into account, the command line simulator is new and works better, delay when no clock was used, better user feedback, end-of-game detection fixed, other bugs involving delays in the strategy, precision problems in the simulator, and leak in the DBM library. These new versions have also the following new features: options to control the type of strategy output, better control on the search ordering (forward and backward), cooperative strategies, and time optimal strategies. The manual has been updated to

CONCUR 2005, TACAS07, CAV07



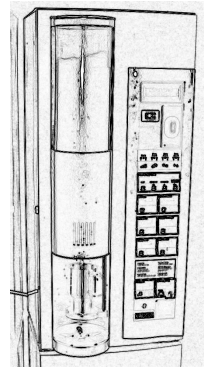
UPPSALA
UNIVERSITET



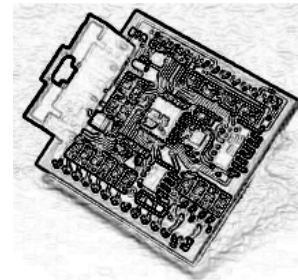
AALBORG UNIVERSITY

Model Checking

Plant
Continuous



Controller Program
Discrete

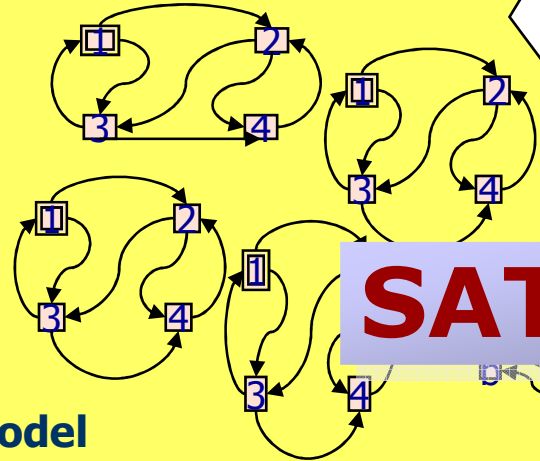
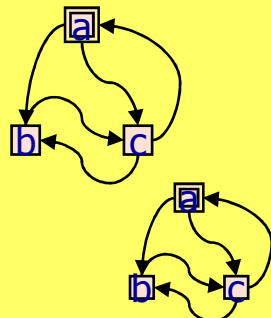


sensors

actuators

Model of tasks (automatic?)

Model of environment (user-supplied / non-determinism)

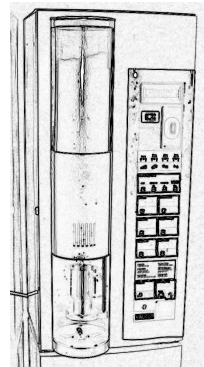


SAT \emptyset ??

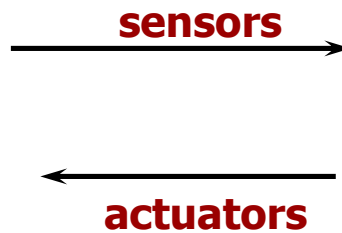
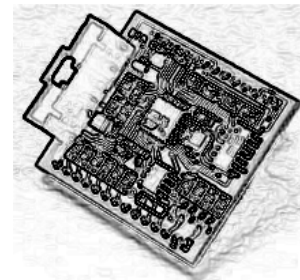
UPPAAL Model

Synthesis

Plant
Continuous

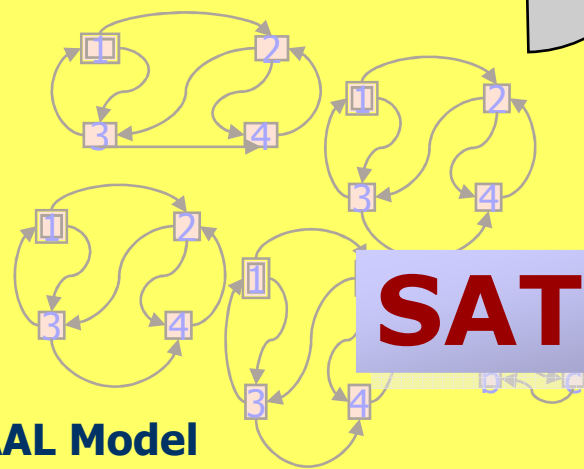
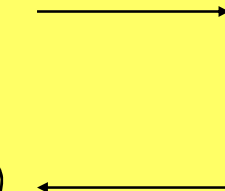
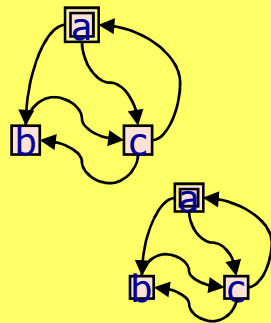


Controller Program
Discrete



Synthesis
of
tasks/scheduler
(automatic)

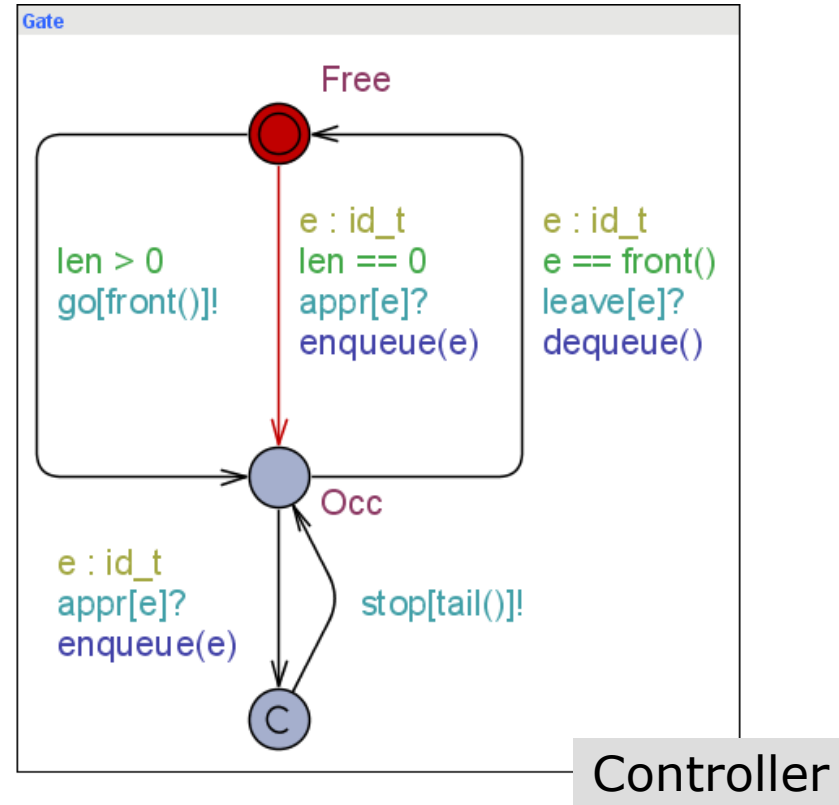
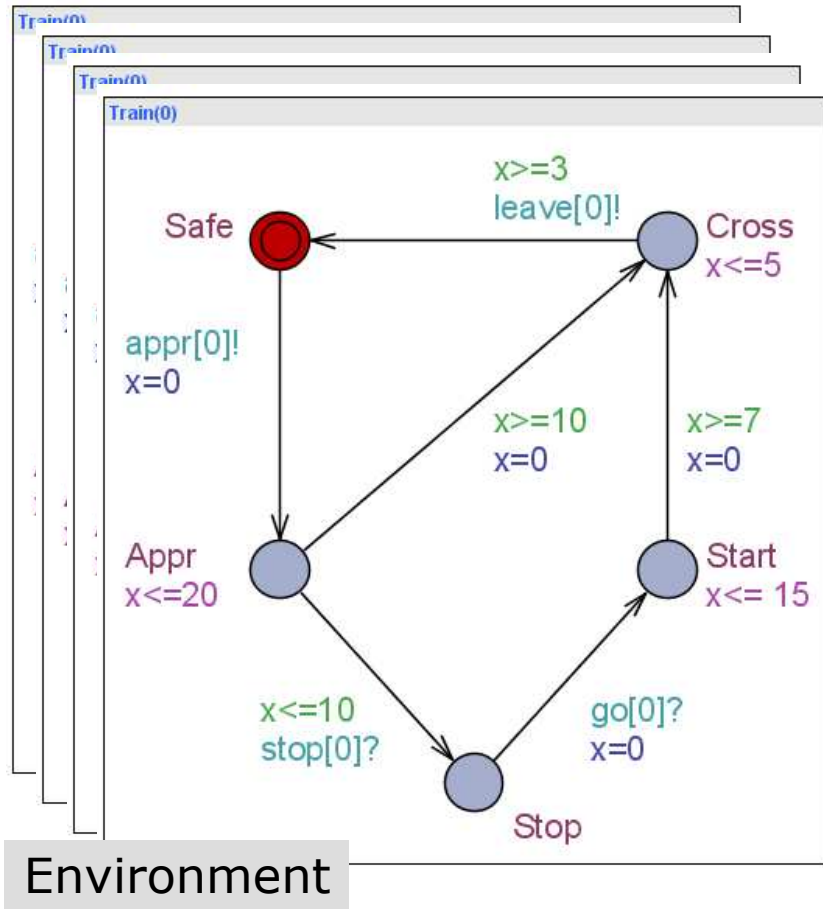
Model
of
environment
(user-supplied)



SAT \emptyset !!

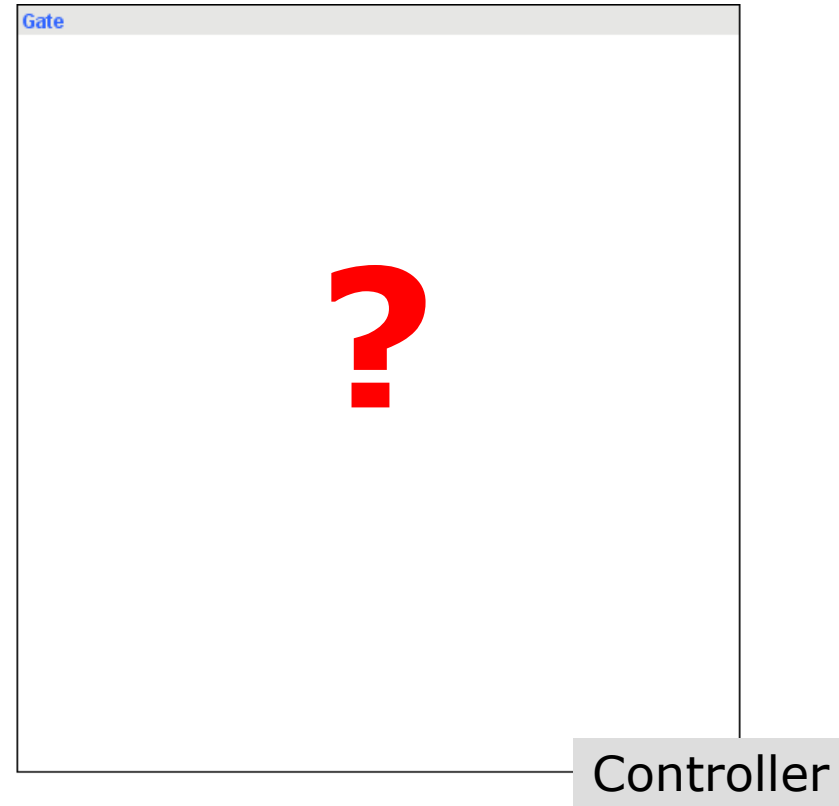
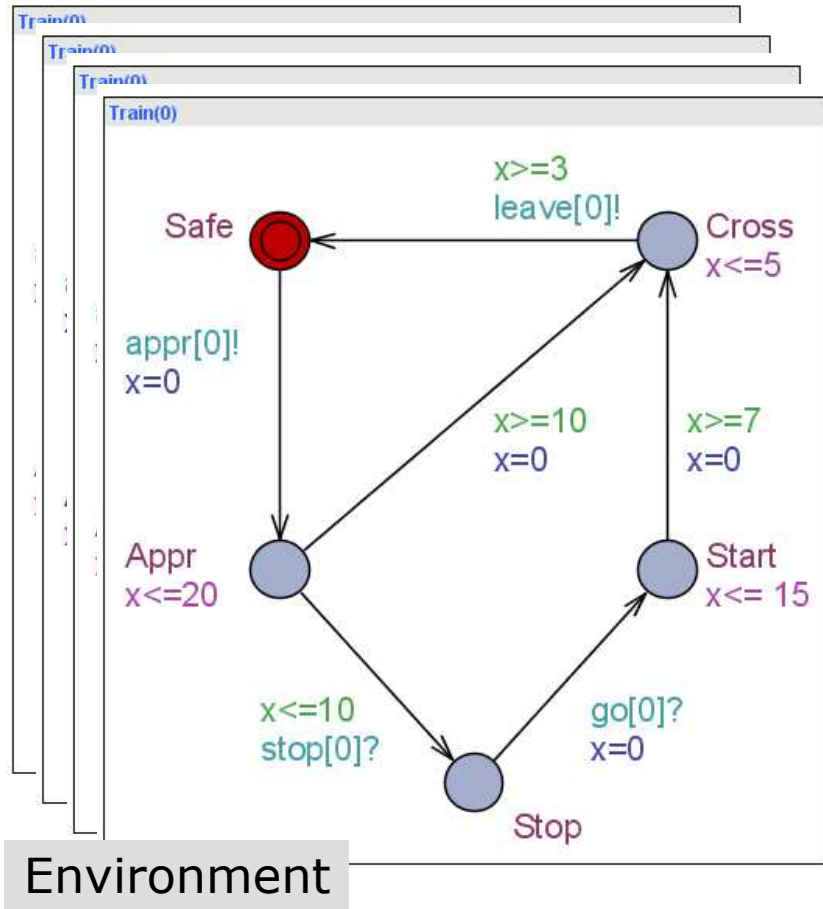
Partial UPPAAL Model

Model Checking



ϕ : Never two trains at the crossing at the same time

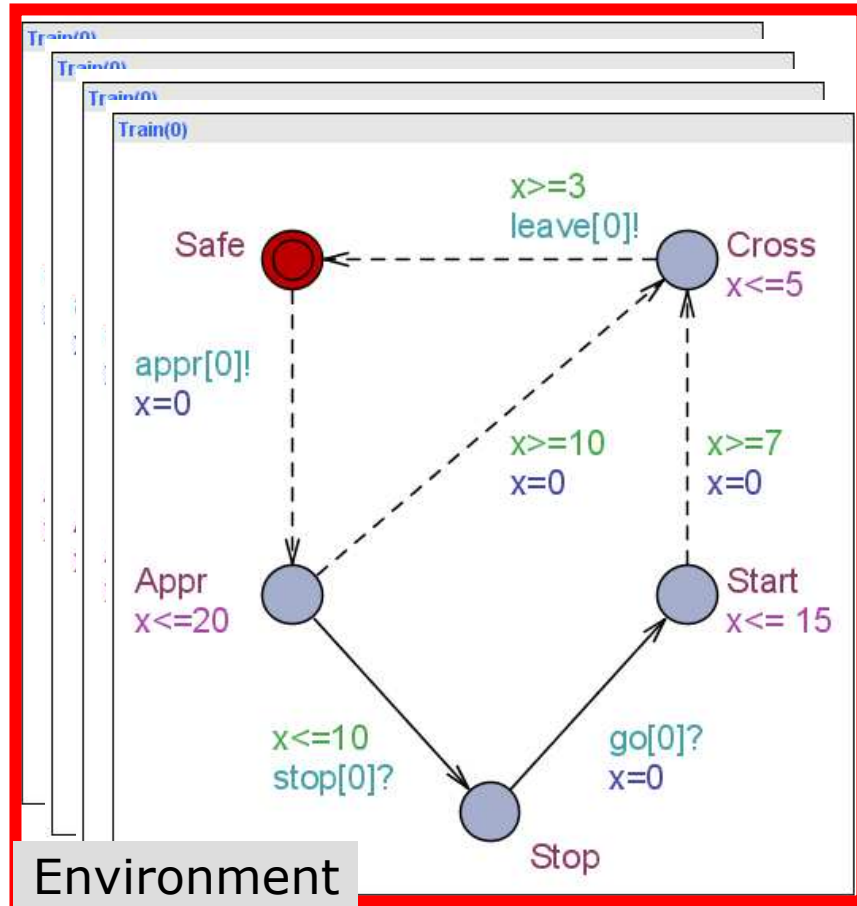
Synthesis



ϕ : Never two trains at the crossing at the same time

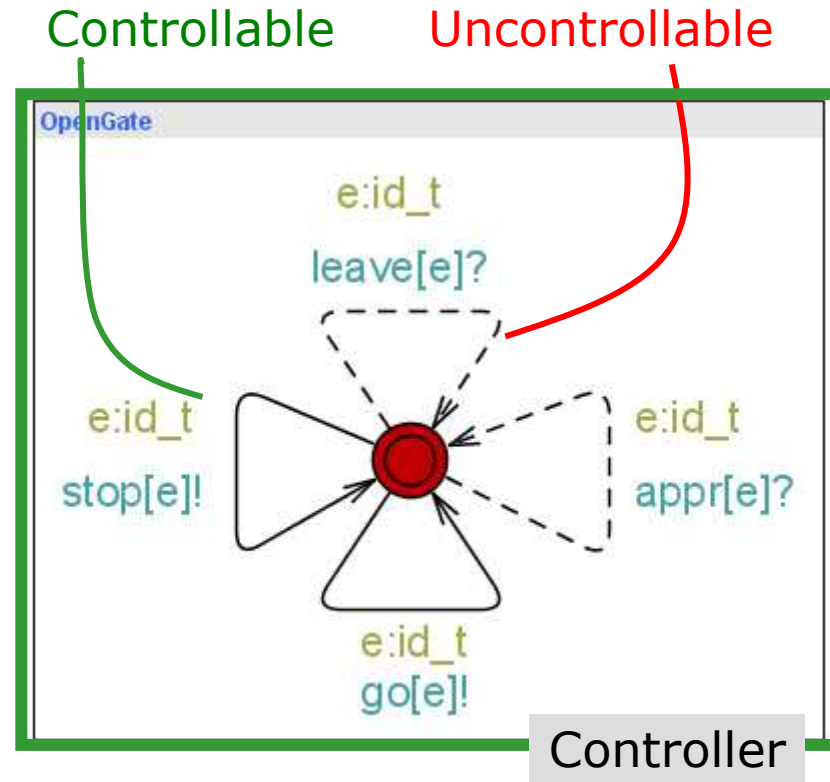
Synthesis

Two Player Game



Environment

Find strategy for controllable actions st behaviour satisfies ϕ

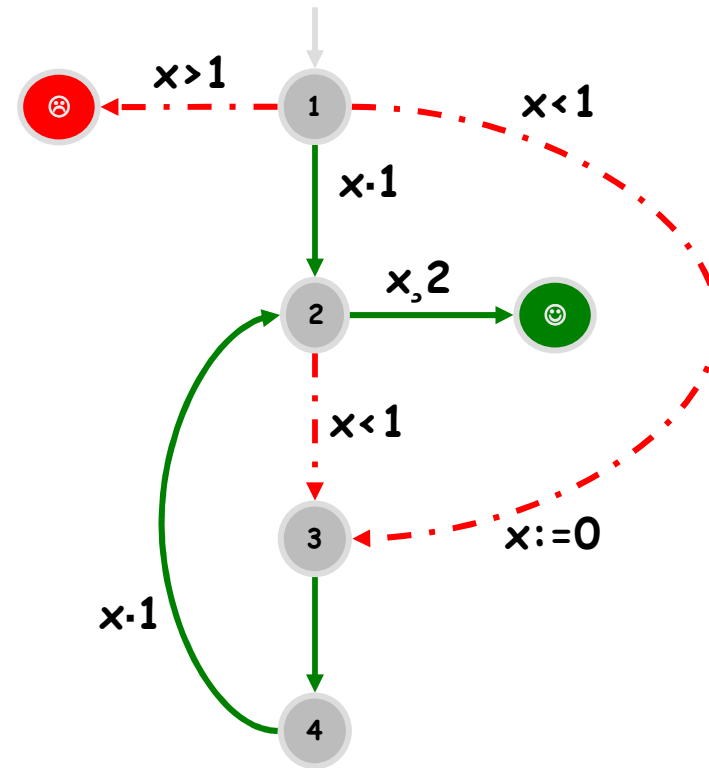
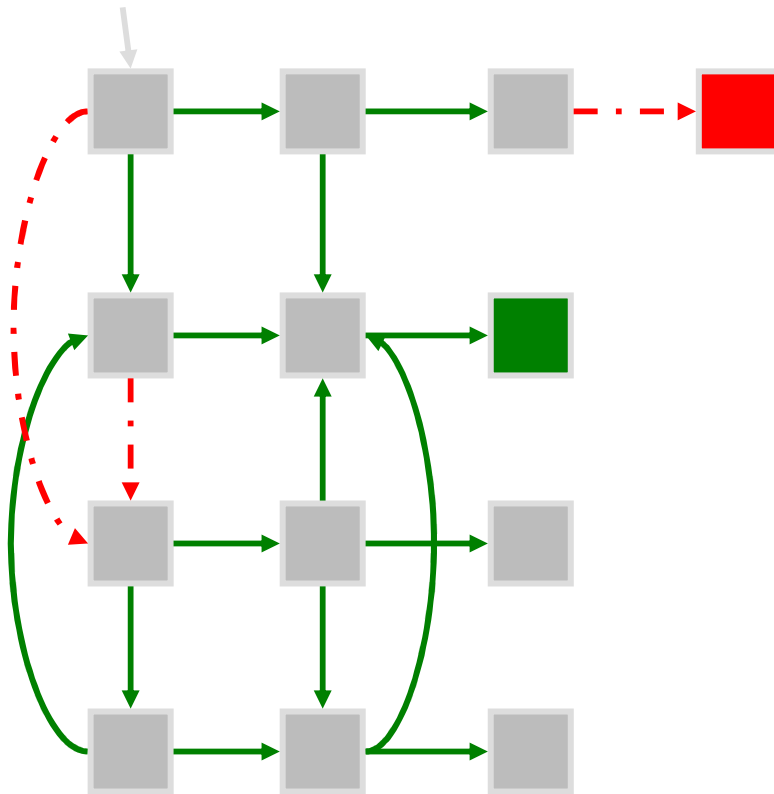


Controller

ϕ : Never two trains at the crossing at the same time

Untimed and Timed Games

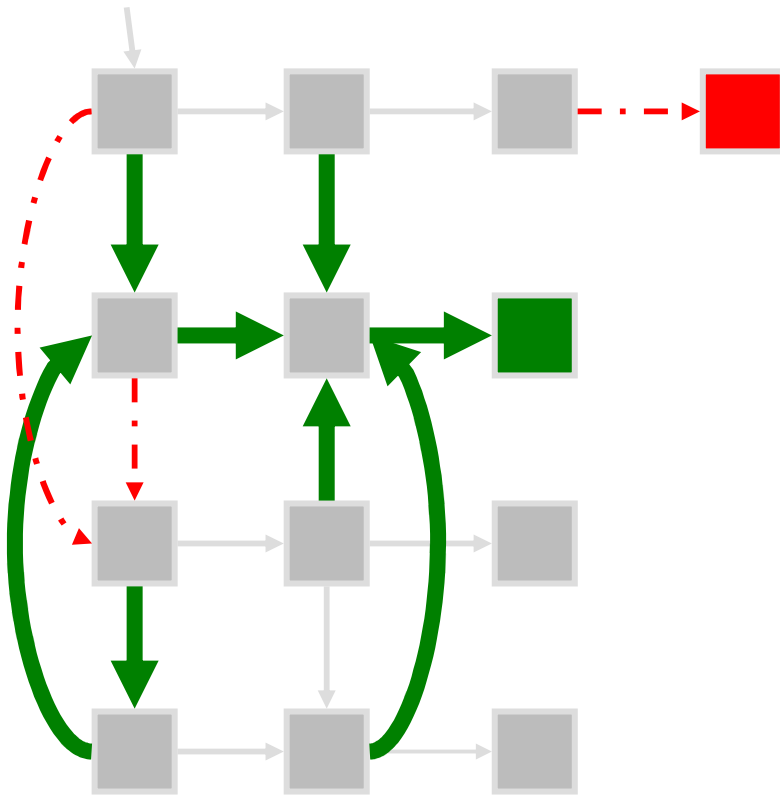
Reachability / Safety Games



- - > Uncontrollable
- - > Controllable

Untimed Games

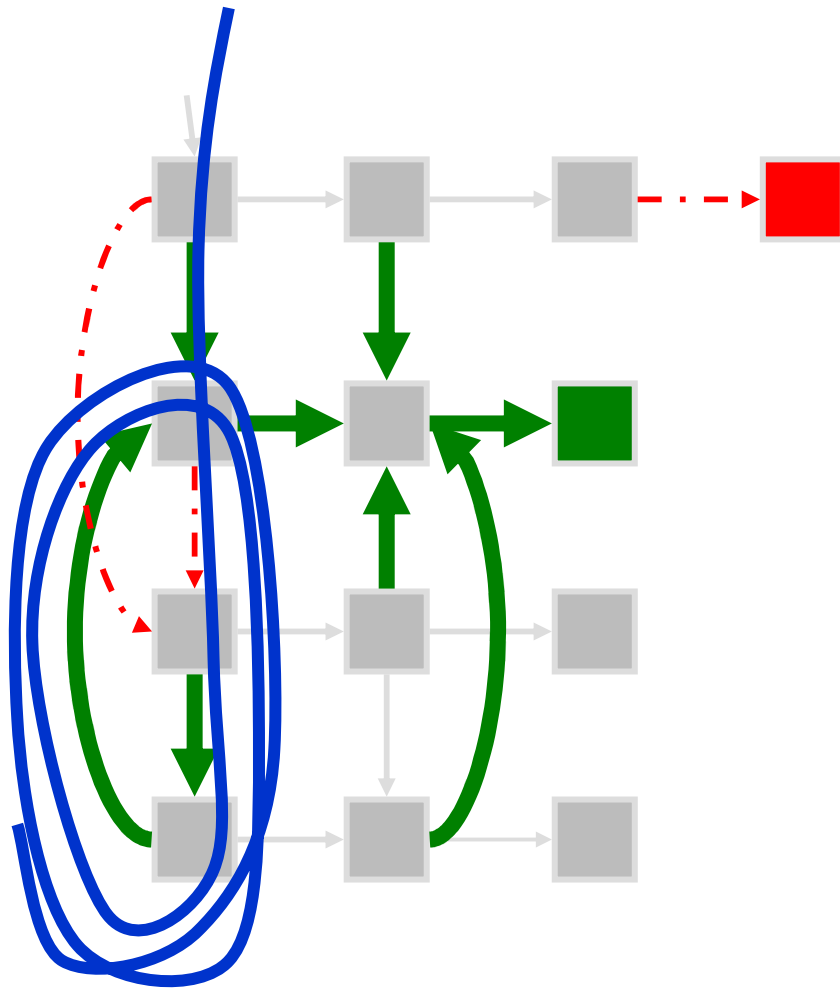
Memoryless Strategy:
 $F : Q \rightarrow E_c$



- -> Uncontrollable
- -> Controllable

UCb

Untimed Games



UCb

Memoryless Strategy:

$$F : Q \rightarrow E_c$$

Winning Run ρ :

$$\text{States}(\rho) \overset{\Delta}{\cap} G \neq \emptyset$$

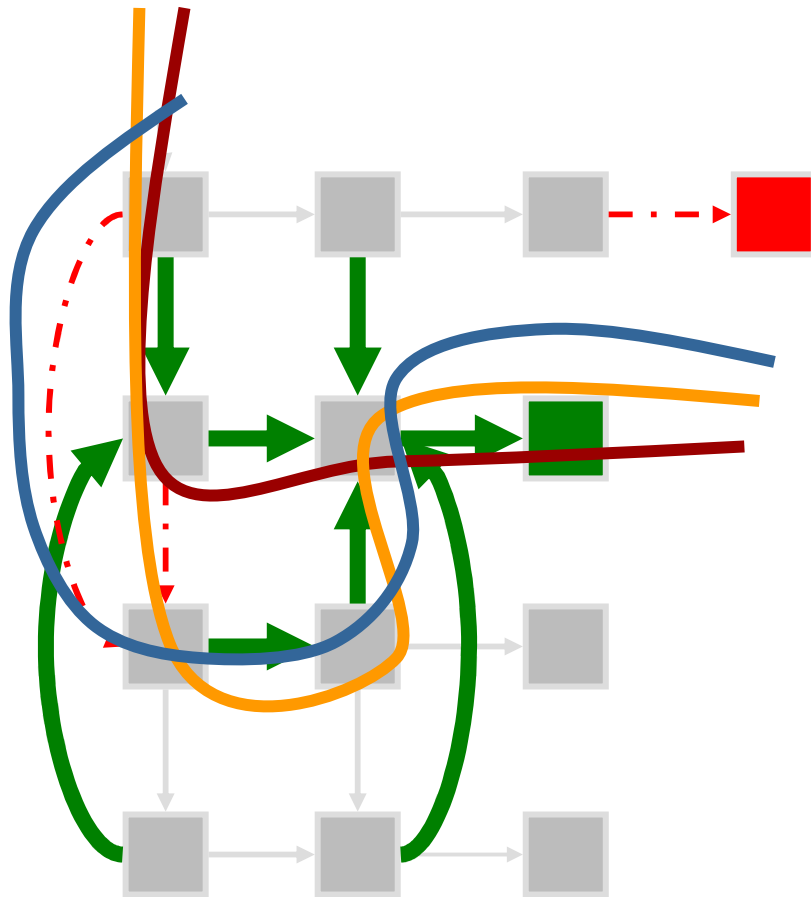
Winning Strategy:

$$\text{Runs}(F) \mu \text{ WinRuns}$$

- - -> Uncontrollable

- - -> Controllable

Untimed Games



Memoryless Strategy:

$$F : Q \rightarrow E_c$$

Winning Run ρ :

$$\text{States}(\rho) \overset{\Delta}{\cap} G \neq \emptyset$$

Winning Strategy:

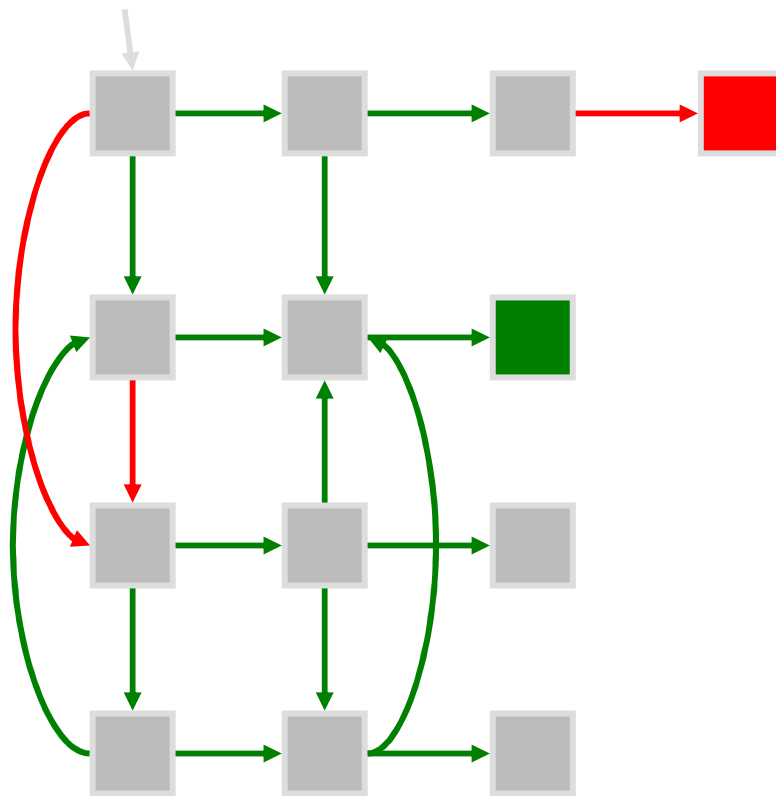
$$\text{Runs}(F) \mu \text{ WinRuns}$$

- - -> Uncontrollable

- - -> Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

$$\pi(X) = cPred(X) \setminus uPred(X^c)$$

Theorem:

The set of winning states is obtained as the least fixpoint of the function:

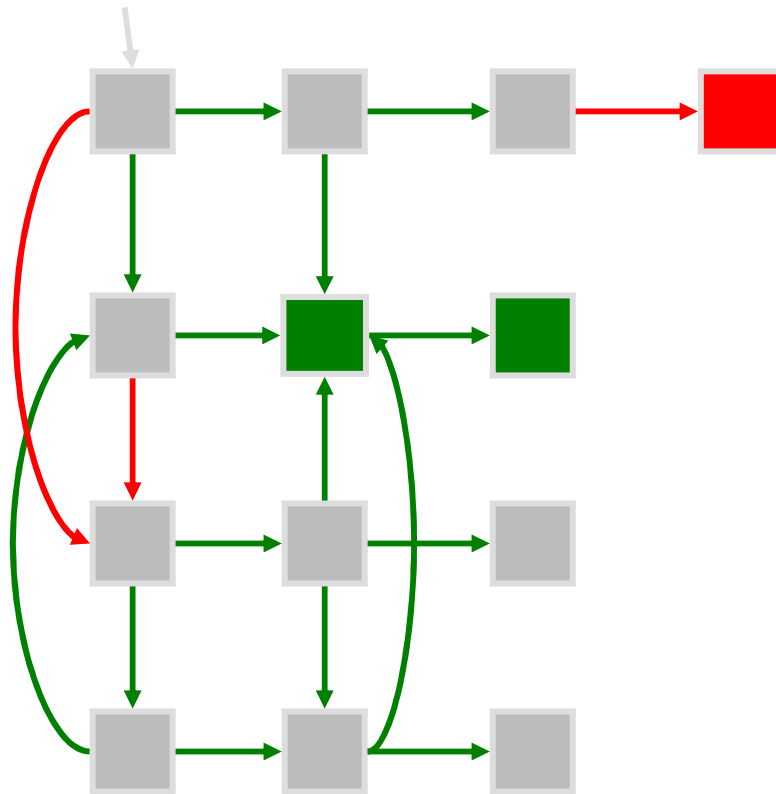
$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

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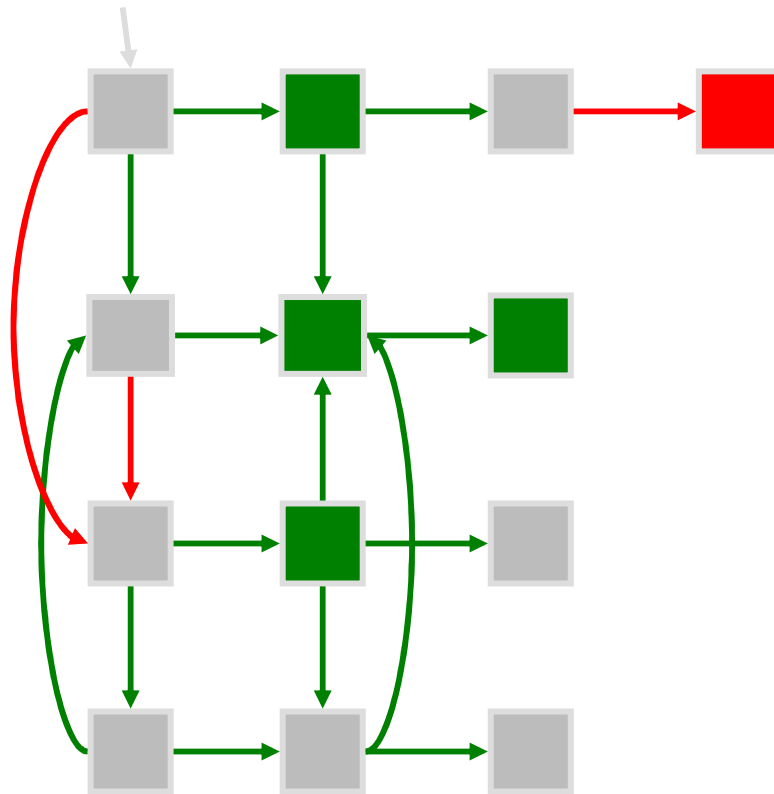
$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

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Theorem:

The set of winning states is obtained as the least fixpoint of the function:

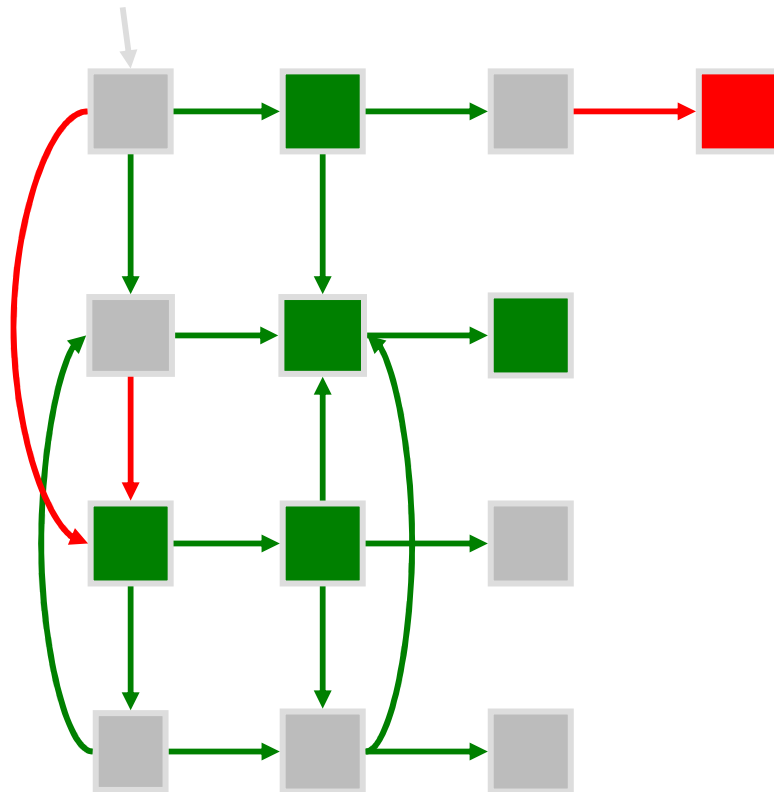
$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

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The set of winning states is obtained as the least fixpoint of the function:

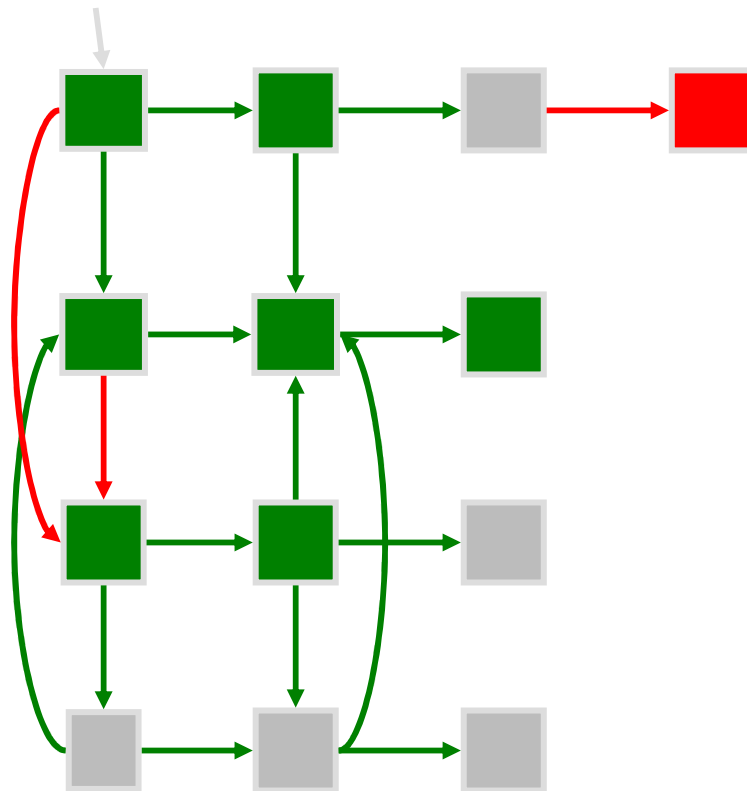
$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

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Theorem:

The set of winning states is obtained as the least fixpoint of the function:

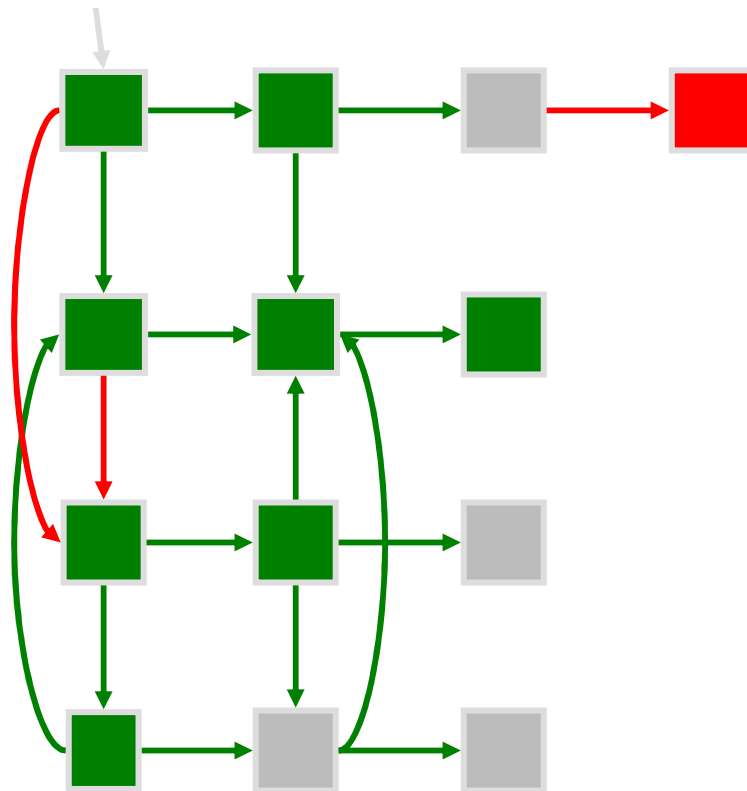
$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Untimed Games

Backwards Fixed-Point Computation



$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

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The set of winning states is obtained as the least fixpoint of the function:

$$X \mapsto \pi(X) \text{ [Goal]}$$

→ Uncontrollable

→ Controllable

Timed Games

Memoryless Strategy:

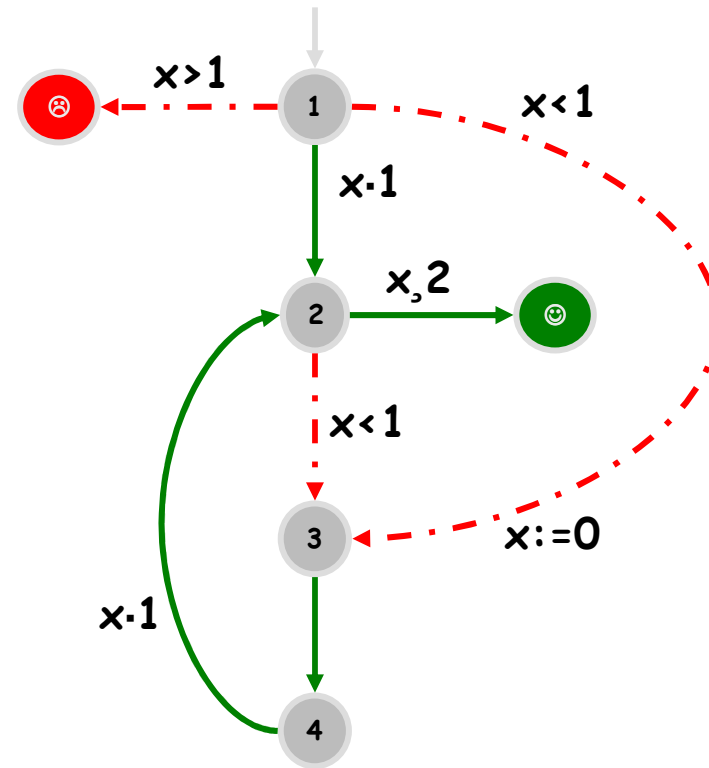
$$F : Q \rightarrow E_c[\lambda]$$

Winning Run:

$$\text{States}(\rho) \stackrel{\circ}{\Delta} G \neq \emptyset$$

Winning Strategy:

$$\text{Runs}(F) \mu \text{ WinRuns}$$



- - -> Uncontrollable

- - -> Controllable

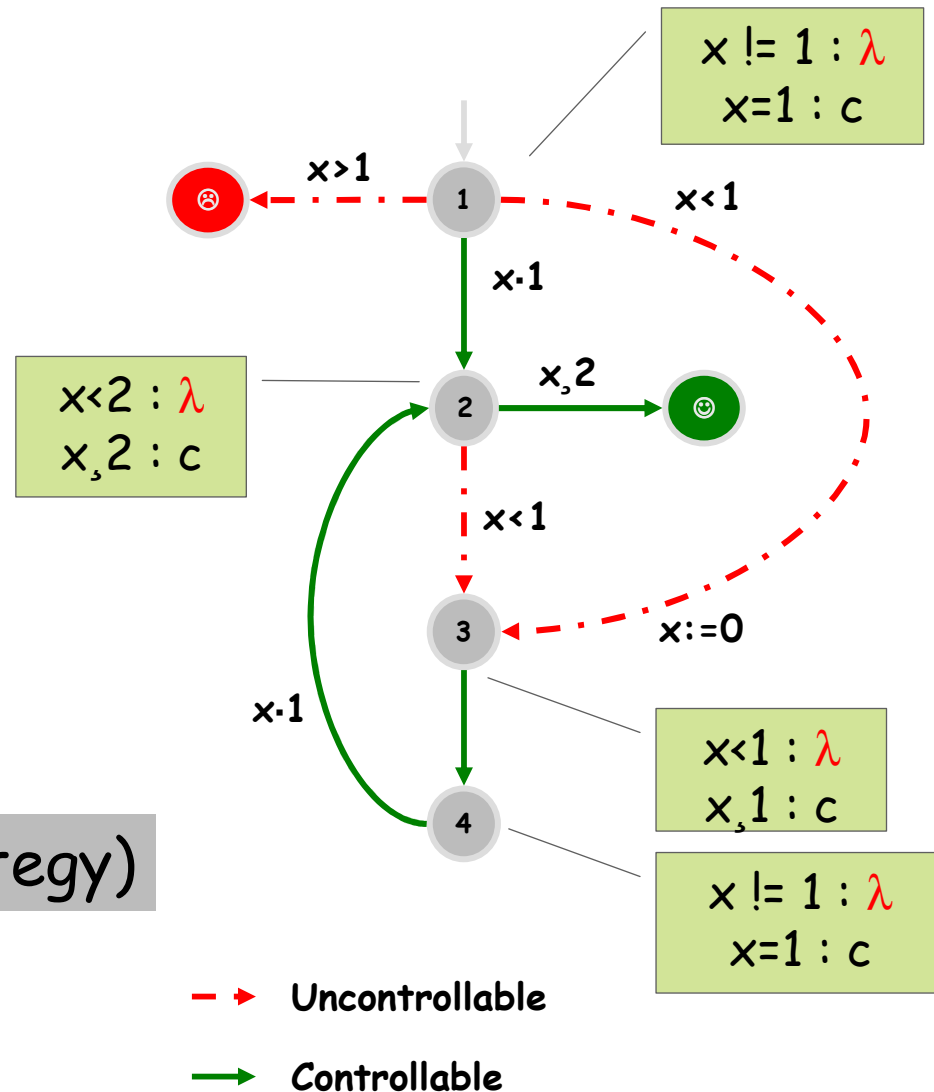
Timed Games

Memoryless Strategy:
 $F : Q \rightarrow E_c[\lambda]$

Winning Run:
 $\text{States}(\rho) \dot{\wedge} G \neq \emptyset$

Winning Strategy:
 $\text{Runs}(F) \mu \text{WinRuns}$

Winning (memoryless) strategy)



Timed Games – State-of-the-Art

Backwards Fixed-Point Computation

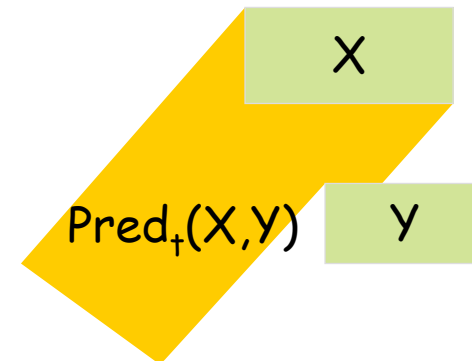
Definitions

$$cPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{c} q' \}$$

$$uPred(X) = \{ q \in Q \mid \exists q' \in X. q \xrightarrow{u} q' \}$$

$$Pred_+(X, Y) = \{ q \in Q \mid \exists t. q \in X \text{ and } \exists s. t. q \in Y^c \}$$

$$\pi(X) = Pred_+[X [cPred(X) , uPred(X^c)]$$



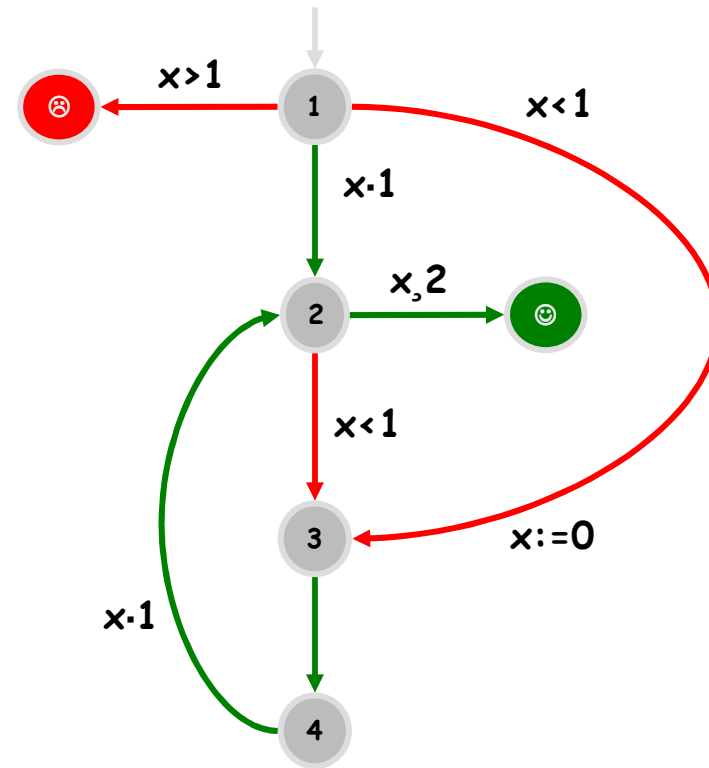
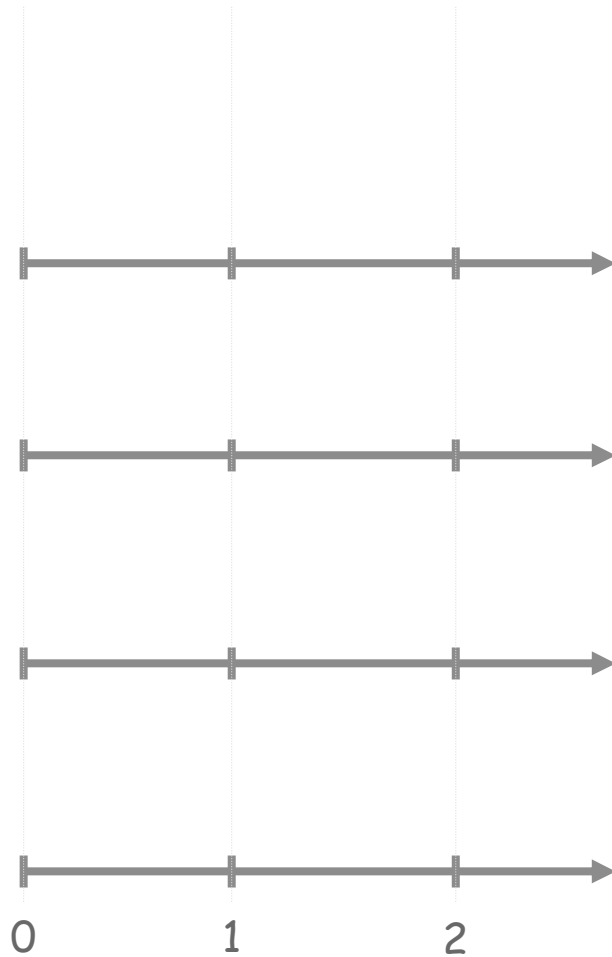
Theorem:

The set of winning states is obtained as the least fixpoint of the function:

$$X \mapsto \pi(X) \text{ [Goal]}$$

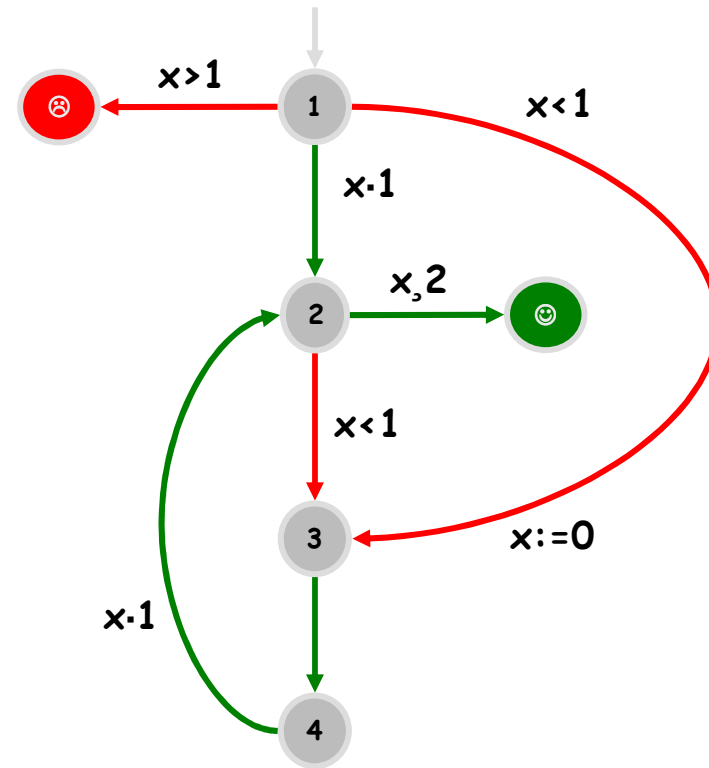
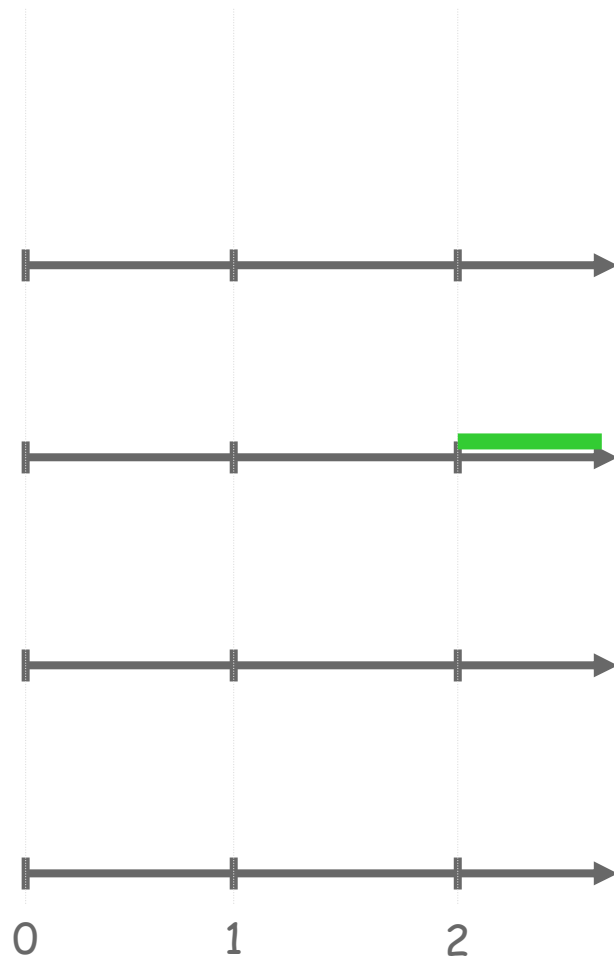
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



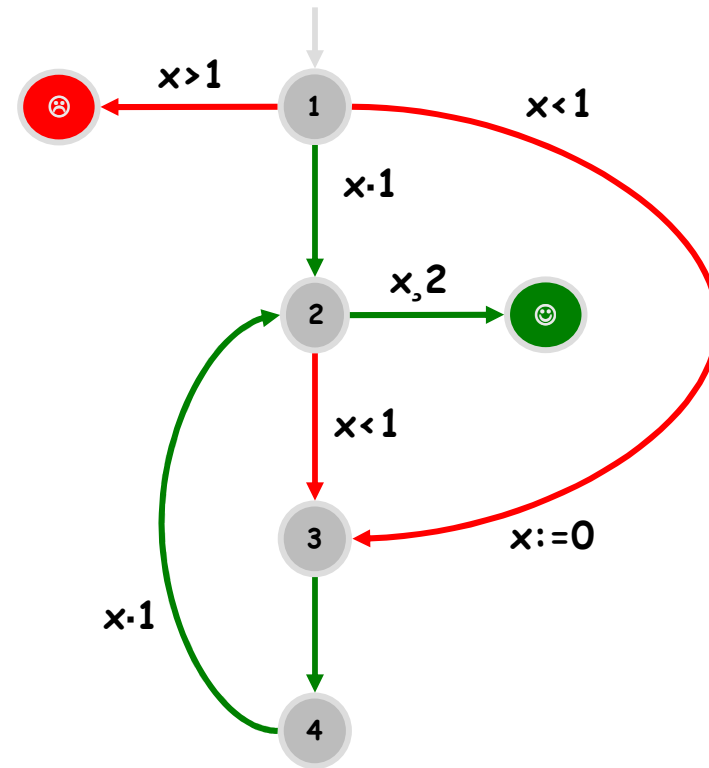
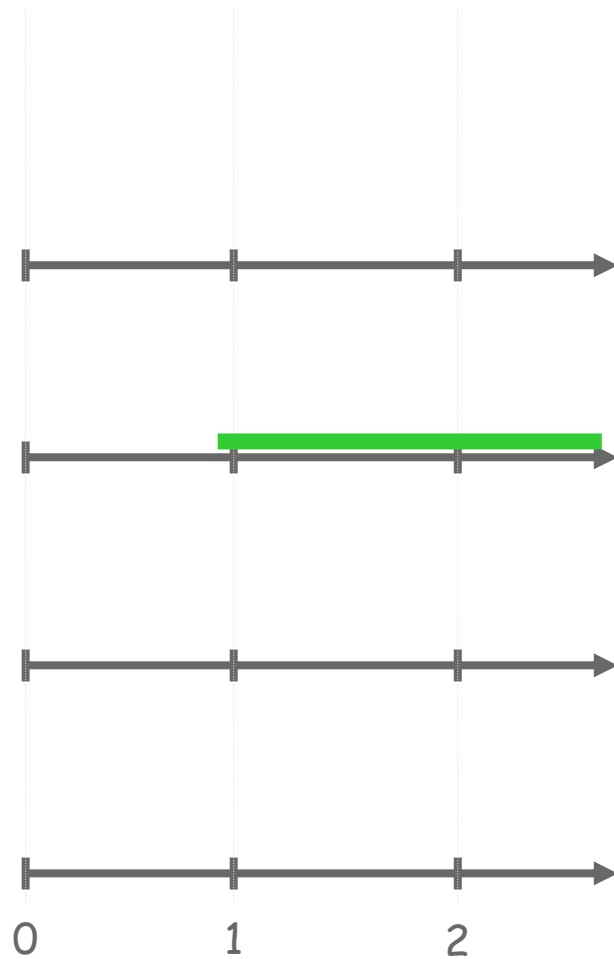
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



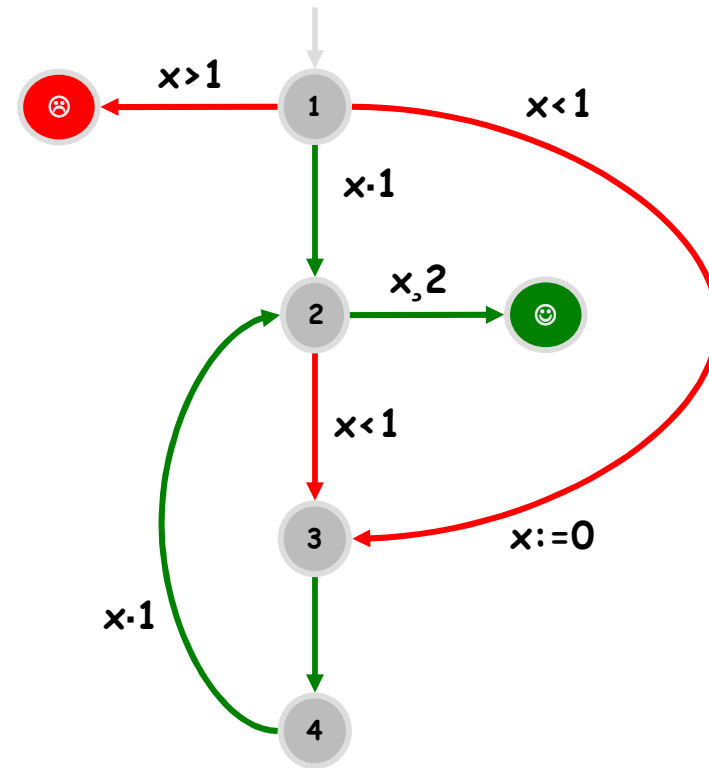
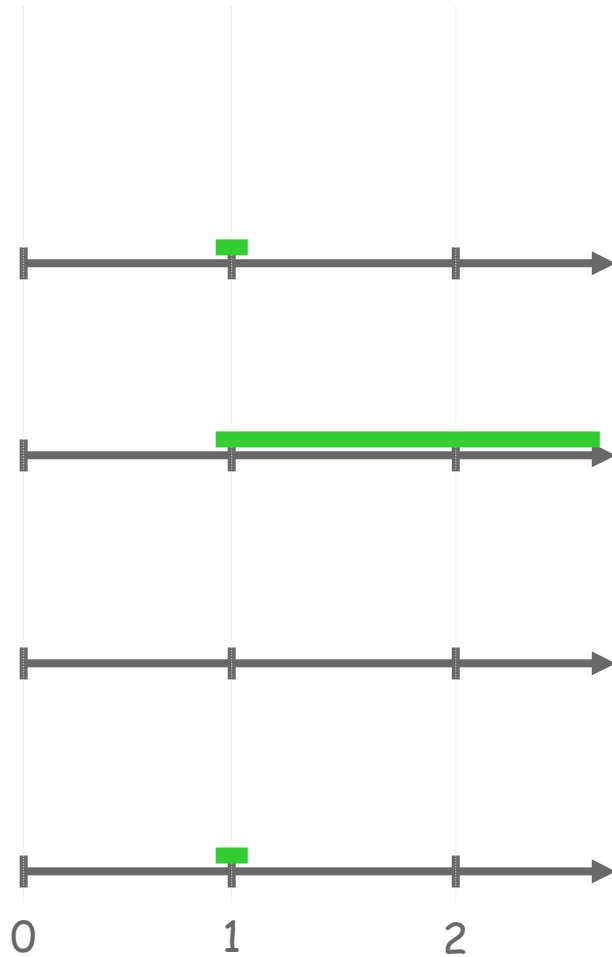
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



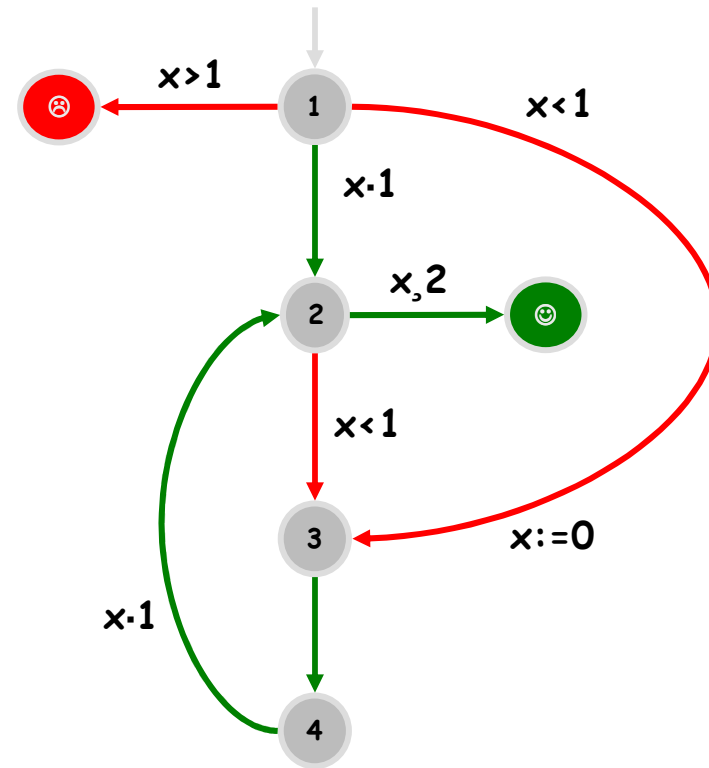
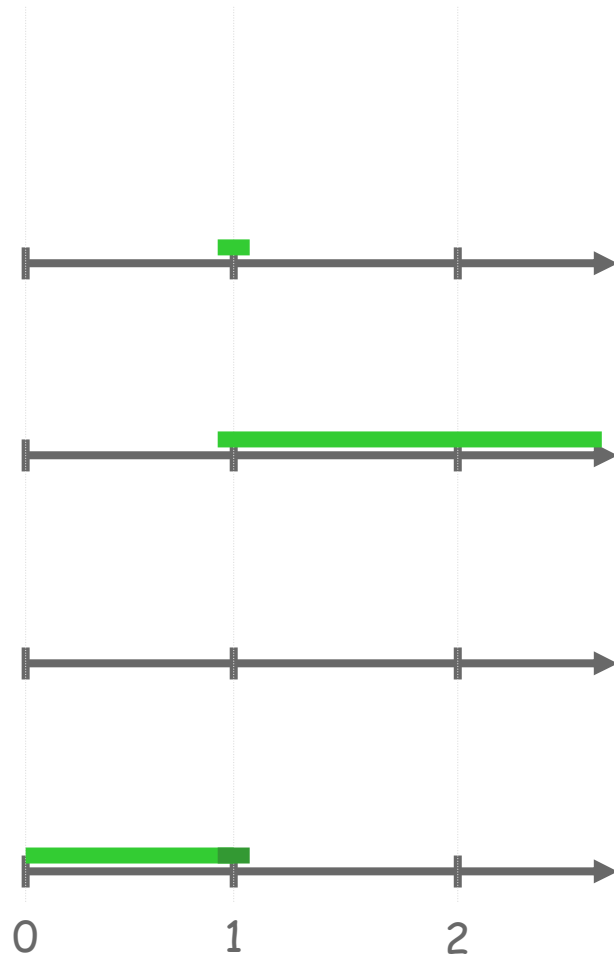
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



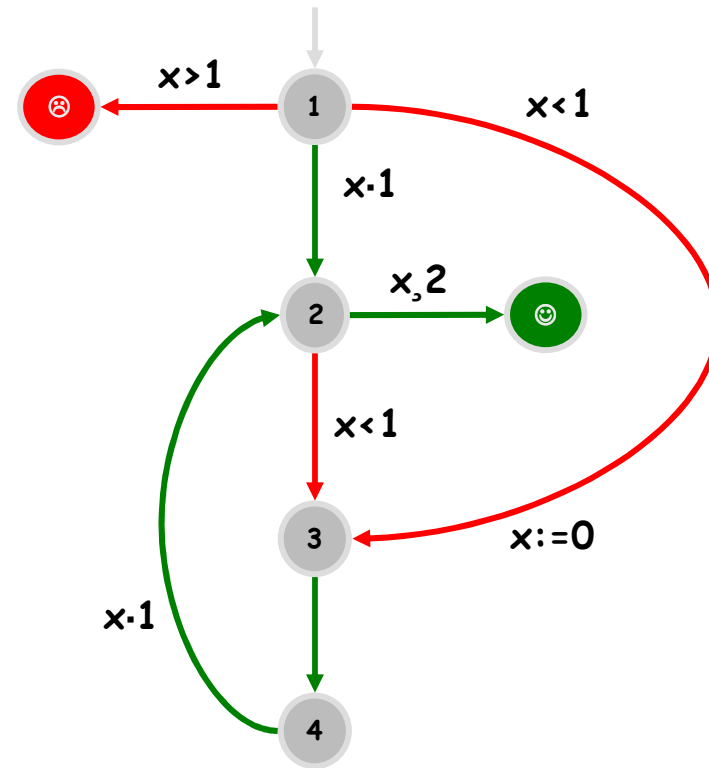
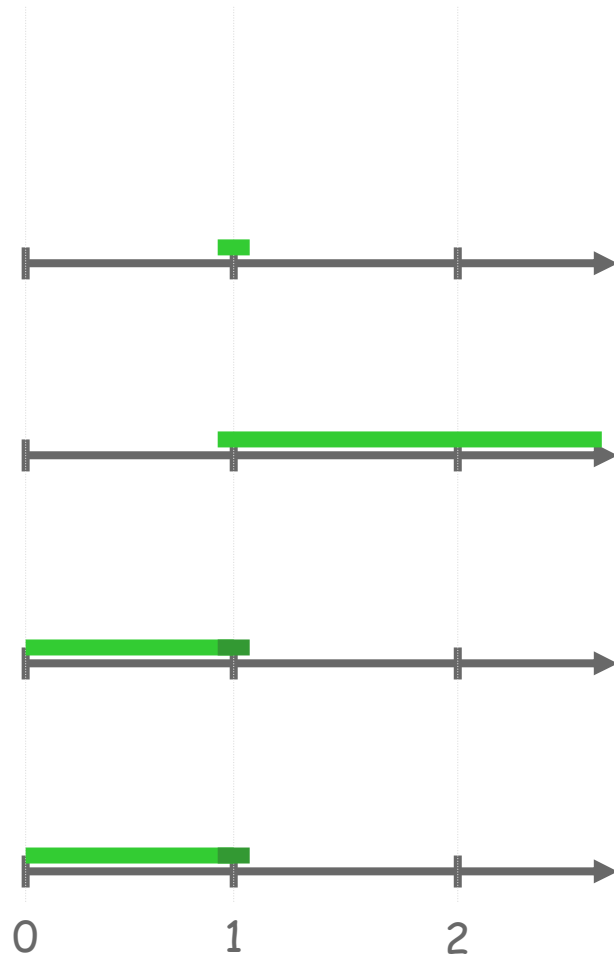
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



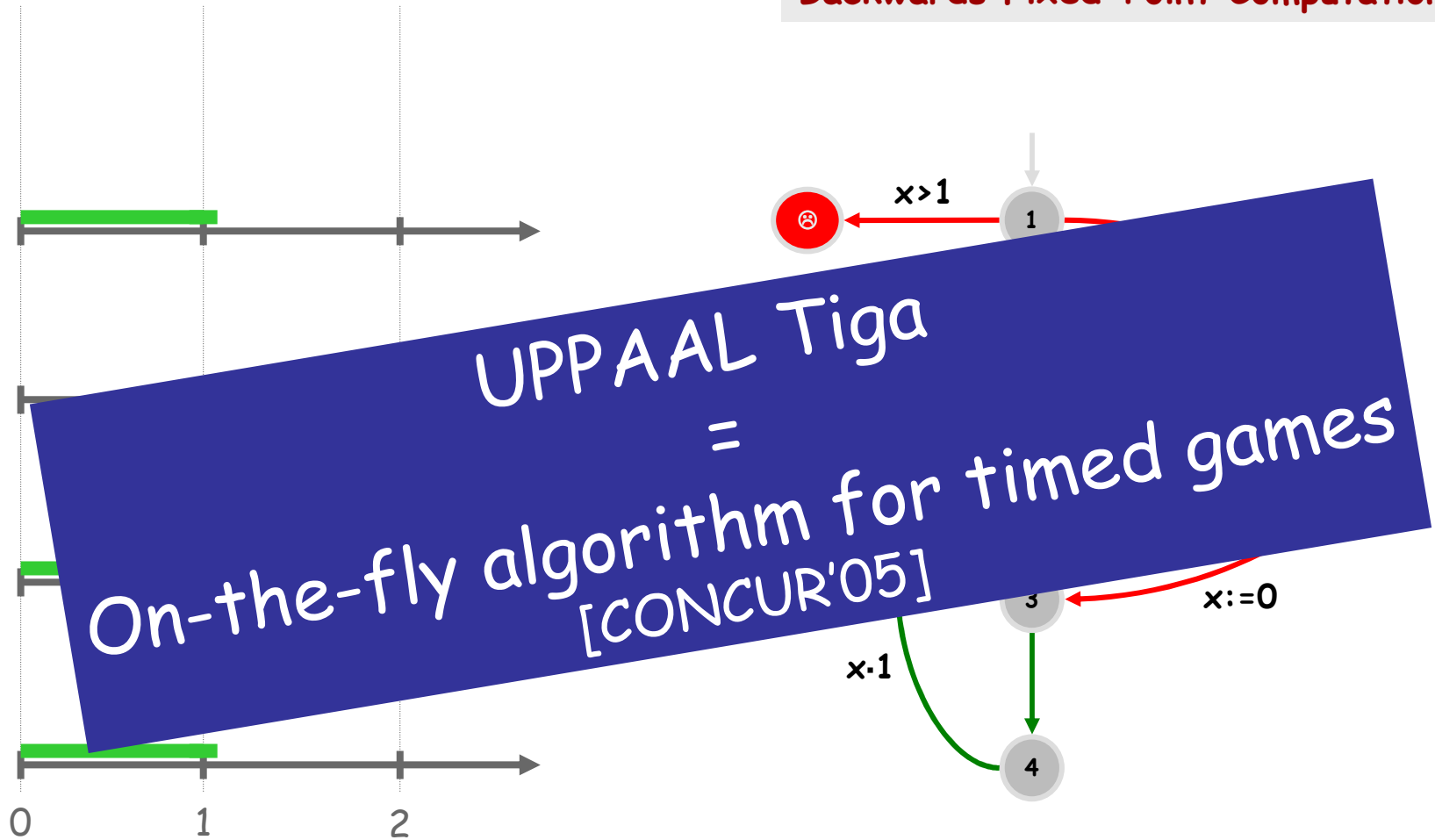
Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



Timed Games – State-of-the-Art

Backwards Fixed-Point Computation



UPPAAL Tiga

Editor
Simulator
Verifier

Drag out

Enabled Transitions

0.0 ,3.33 ,6.66

Main

Main

Drag out

t(0) = 0
Main.x = 0.630000

Main

CAV 2007

Current time: A

Delay:

UPPAAL Tiga

CTL Control Objectives

■ Reachability properties:

- control: $A[p U q]$ *until*
- control: $A\langle \rangle q \Leftrightarrow \text{control: } A[\text{true} U q]$

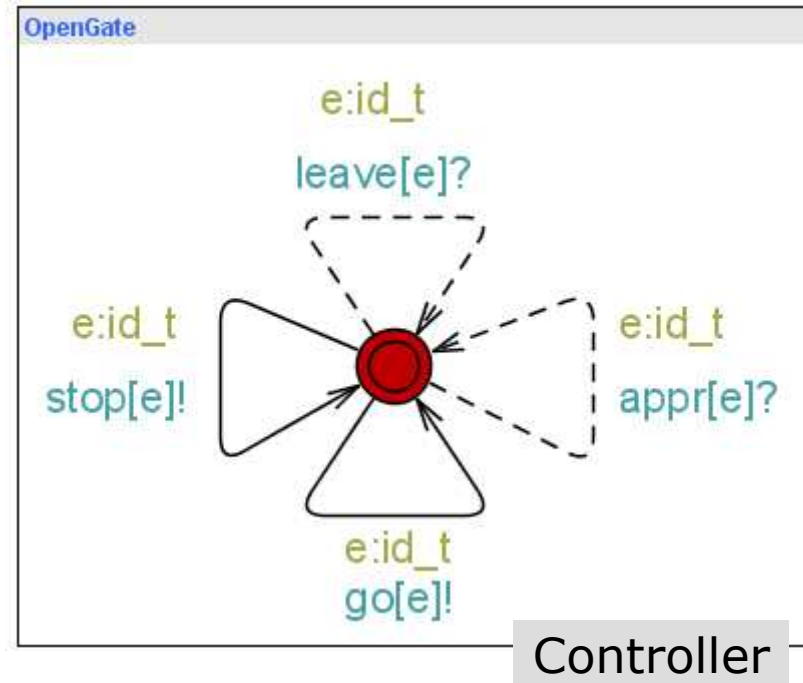
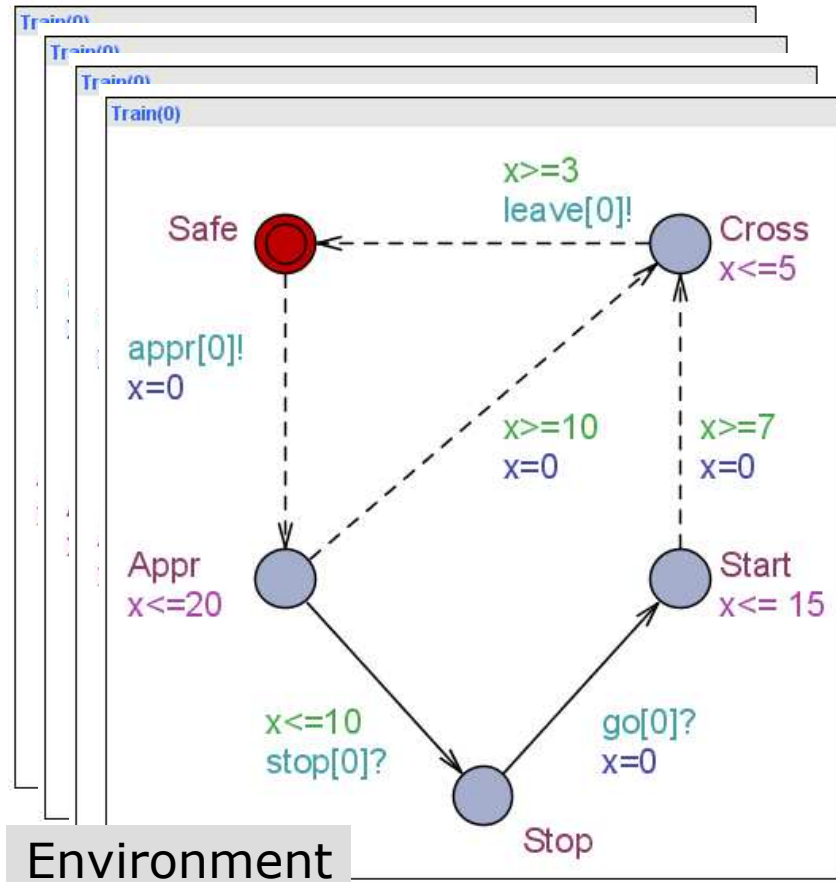
■ Safety properties:

- control: $A[p W q]$ *weak until*
- control: $A[] p \Leftrightarrow \text{control: } A[p W \text{false}]$

■ Time-optimality :

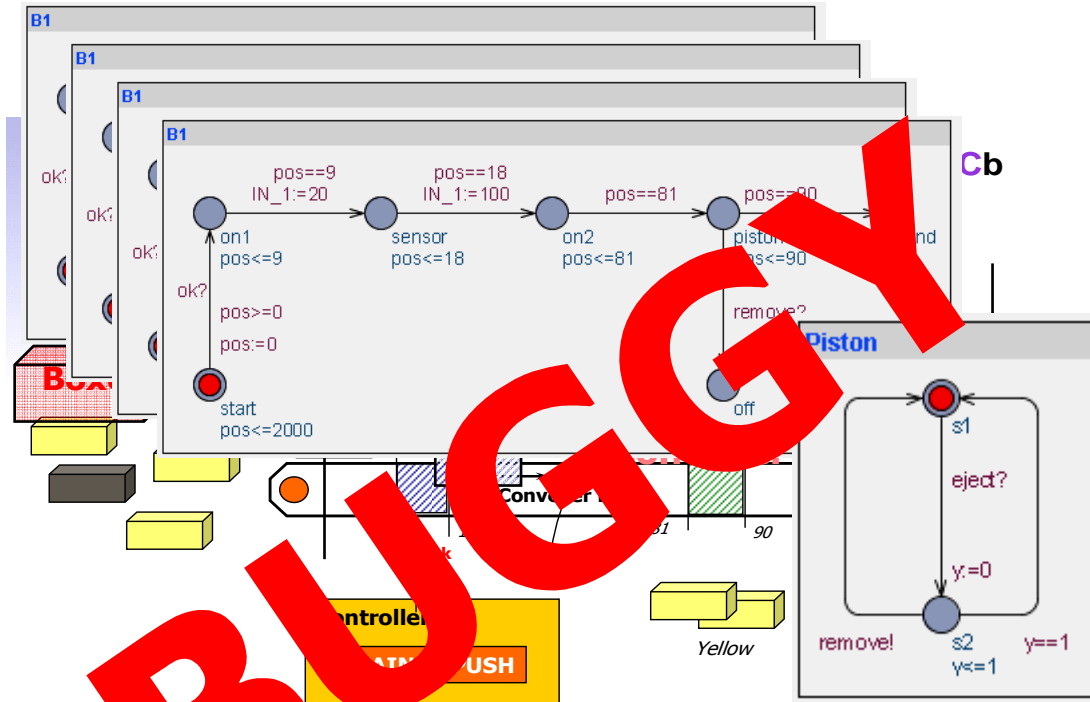
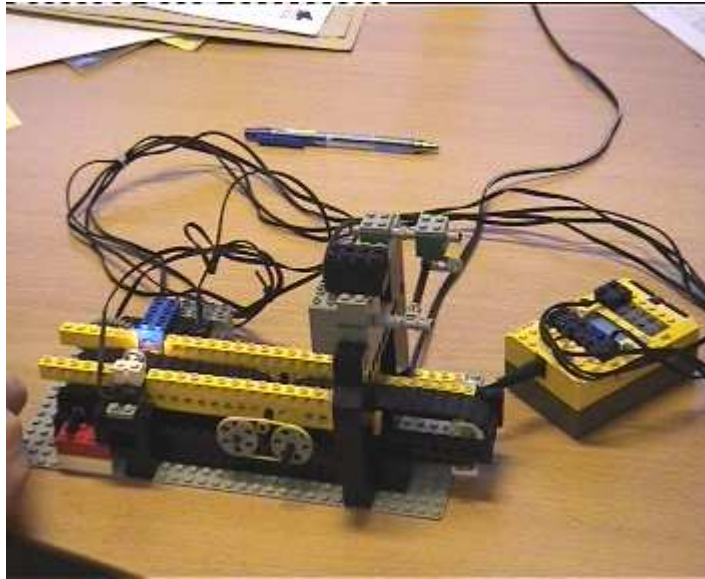
- control_{t*}(u,g): $A[p U q]$
 - u is an upper-bound to prune the search
 - g is the time to the goal from the current state

Train Crossing

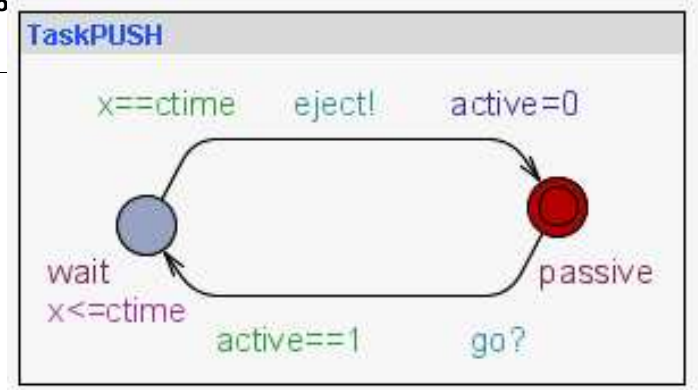
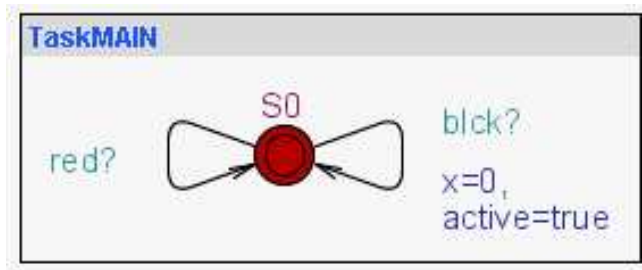


ϕ : Never two trains at the crossing at the same time

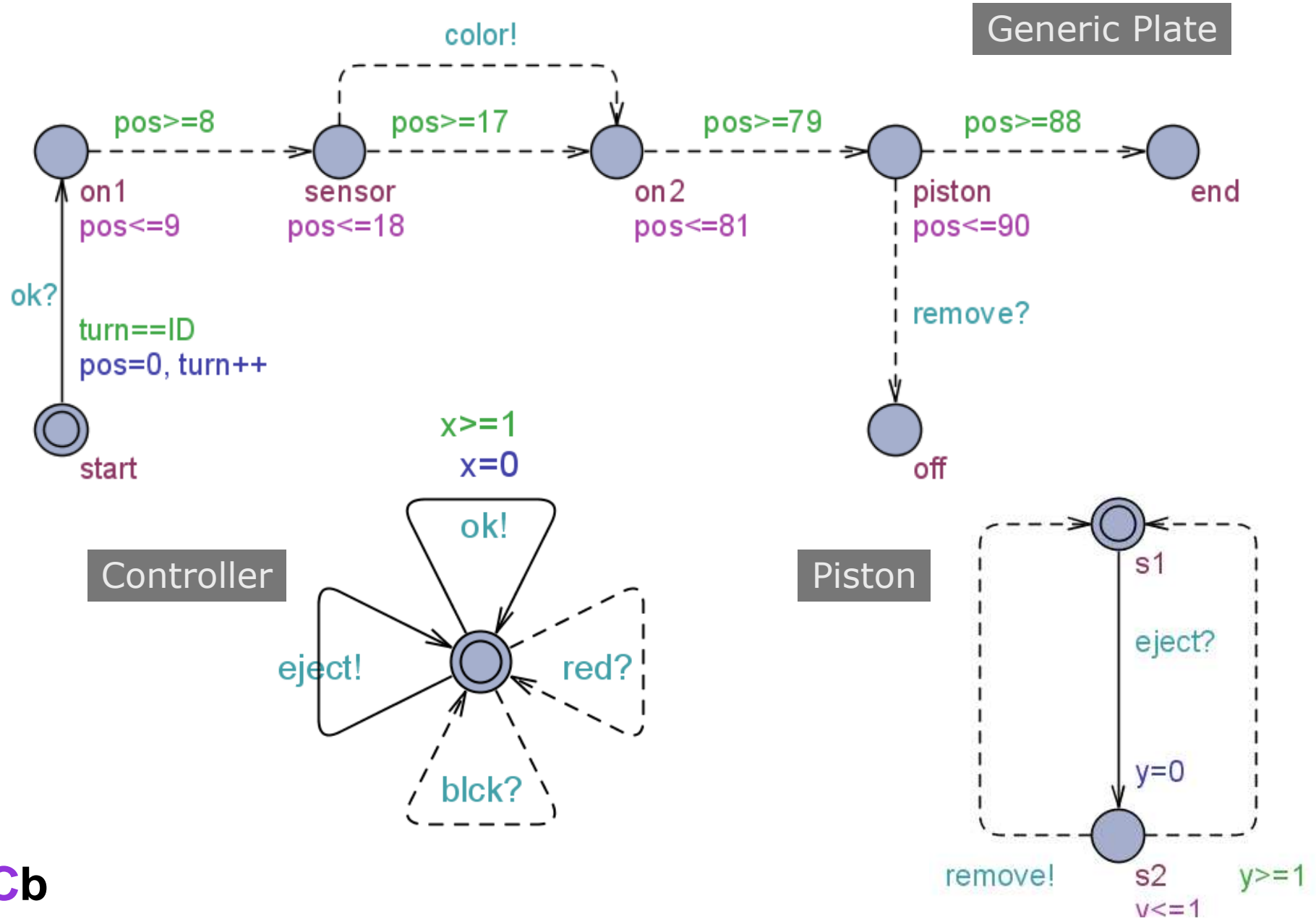
A Buggy Brick Sorting Program



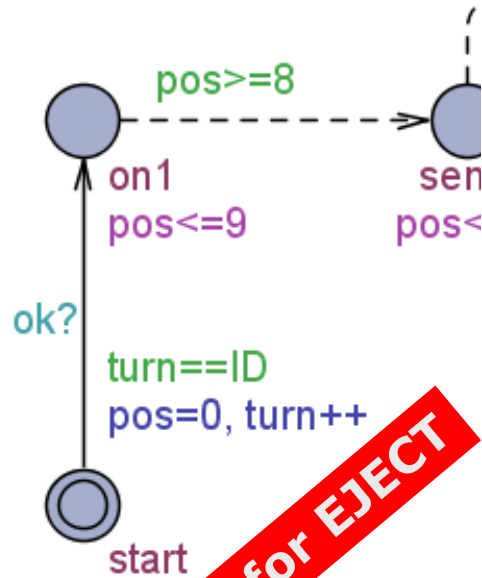
MCD 2001, Tw



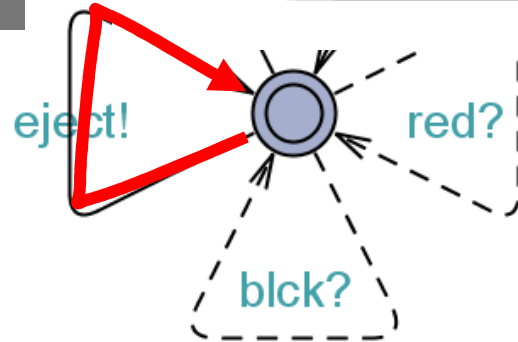
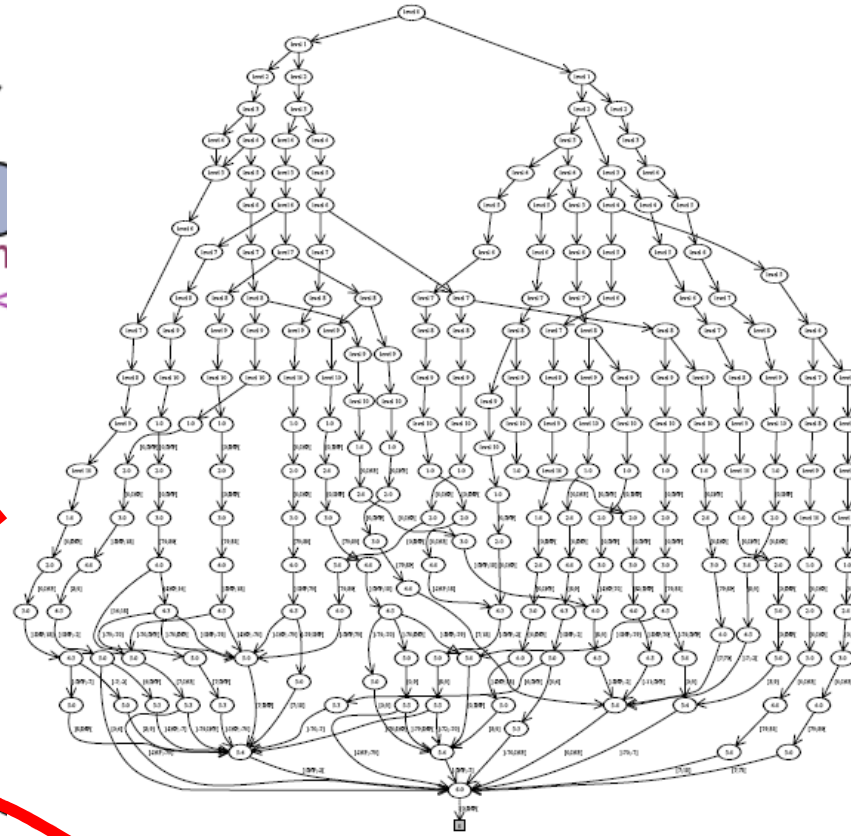
Brick Sorting



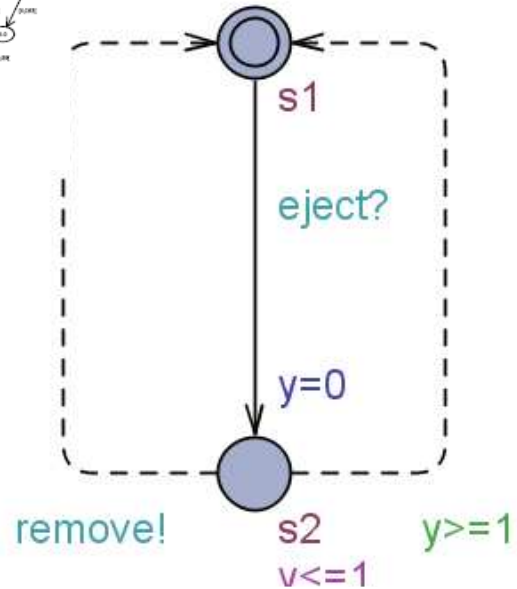
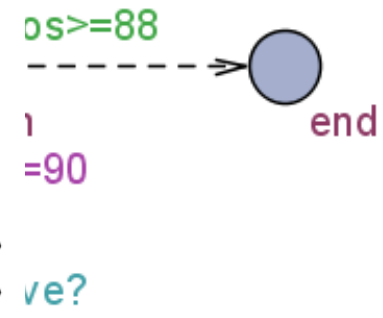
Brick Sorting



Strategy for EJECT



Generic Plate



Production Cell

Plates		Basic		Basic +inc		Basic +inc +pruning		Basic+lose +inc +pruning		Basic+lose +inc +topt	
		time	mem	time	mem	time	mem	time	mem	time	mem
2	win	0.0s	1M	0.0s	1M	0.0s	1M	0.0s	1M	0.04s	1M
	lose	0.0s	1M	0.0s	1M	0.0s	1M	0.0s	1M	n/a	n/a
3	win	0.5s	19M	0.0s	1M	0.0s	1M	0.1s	1M	0.27s	4M
	lose	1.1s	45M	0.1s	1M	0.0s	1M	0.2s	3M	n/a	n/a
4	win	33.9s	1395M	0.2s	8M	0.1s	6M	0.4s	5M	1.88s	13M
	lose	-	-	0.5s	11M	0.4s	10M	0.9s	9M	n/a	n/a
5	win	-	-	3.0s	31M	1.5s	22M	2.0s	16M	13.35s	59M
	lose	-	-	11.1s	61M	5.9s	46M	7.0s	41M	n/a	n/a
6	win	-	-	89.1s	179M	38.9s	121M	12.0s	63M	220.3s	369M
	lose	-	-	699s	480M	317s	346M	135.1s	273M	n/a	n/a
7	win	-	-	3256s	1183M	1181s	786M	124s	319M	6188s	2457M
	lose	-	-	-	-	16791s	2981M	4075s	2090M	n/a	n/a

Model		3		6		12		50		100	
Old	c	0.1s	1M	12s	63M	-	-	-	-	-	-
	u	0.2s	3M	235s	273M	-	-	-	-	-	-
New	c	0.05s	3.5M	0.05s	3.5M	0.14s	55M	2.79s	104M	18.5s	426M
	u	0.02s	3.5M	0.04s	3.5M	0.12s	55M	2.32s	94M	15.6s	340M

Climate Control

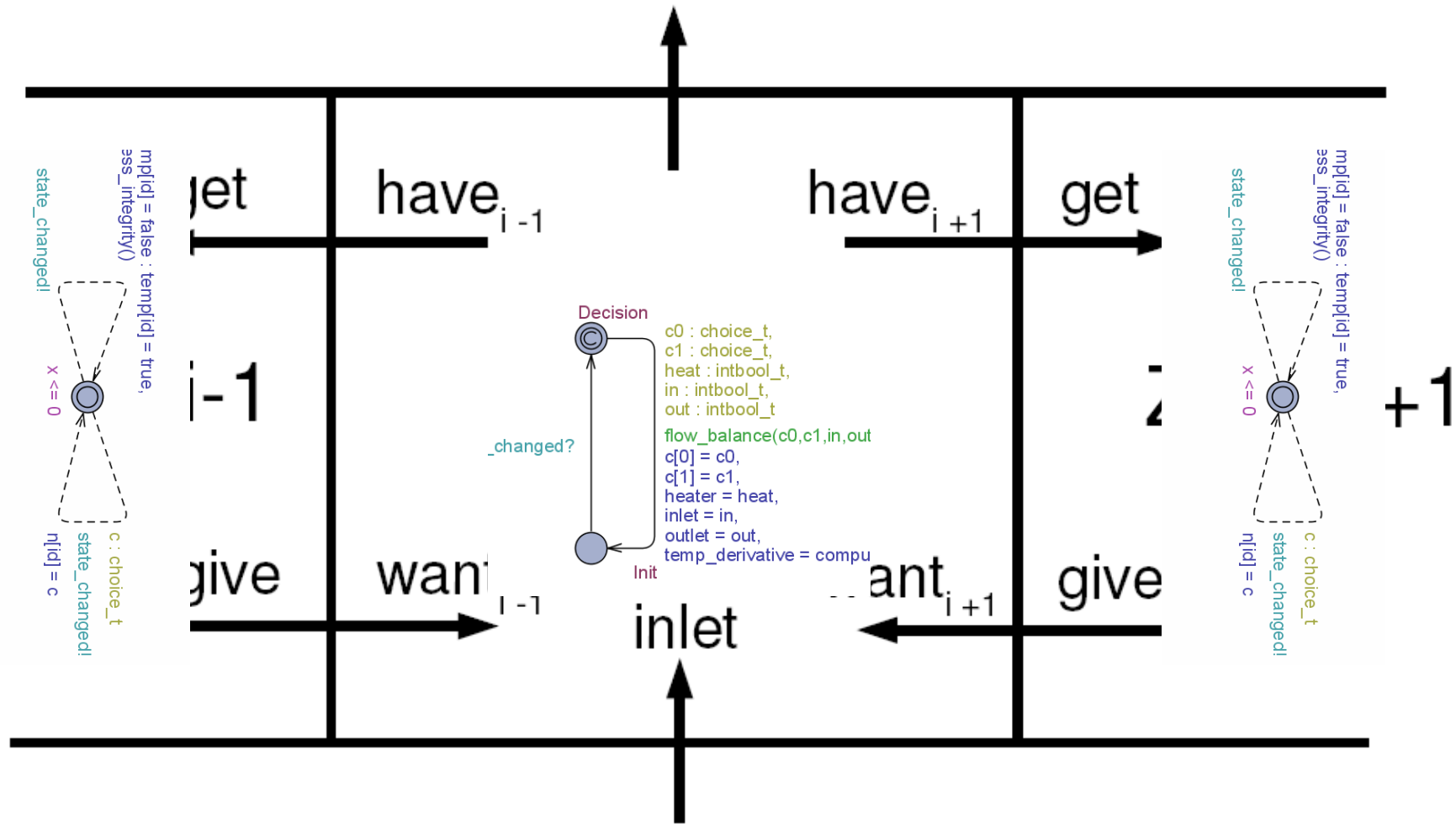


Syvsten,
Northern Jutland,
DK

With Jan J. Jessen
Jacob I. Rasmussen



Climate Control



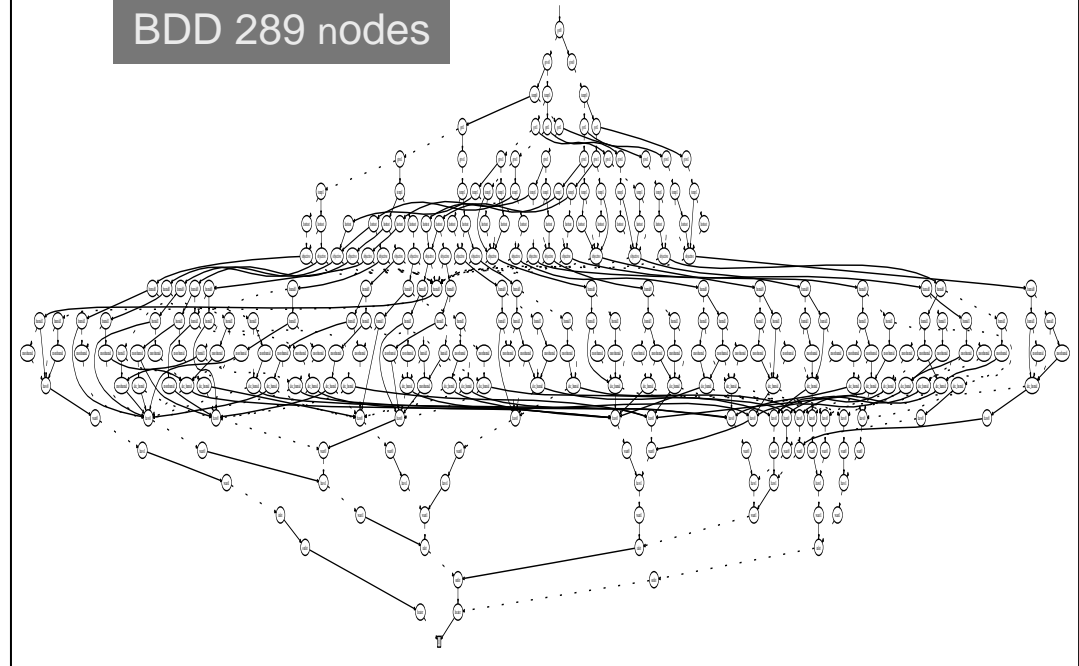
Obtaining executable code

```

-----
Strategy for state:
  Zone i-1: (Temp. lower/equal, wants flow)
  Zone i+1: (Temp. lower/equal, no interaction)
  Hottest neighbor: i-1
  Objective: heat
is:
  Wants flow from i-1
  Wants flow from i+1
  inlet closed
  outlet off
  heater on
-----
Strategy for state:
  Zone i-1: (Temp. greater, offers flow)
  Zone i+1: (Temp. greater, offers flow)
  Hottest neighbor: i+1
  Objective: cool
is:
  Has flow for i-1
  Has flow for i+1
  inlet open
  outlet on
  heater off
-----
Strategy for state:
  Zone i-1: (Temp. lower/equal, no interaction)
  Zone i+1: (Temp. greater, no interaction)
  Hottest neighbor: i+1
  Objective: cool
  
```

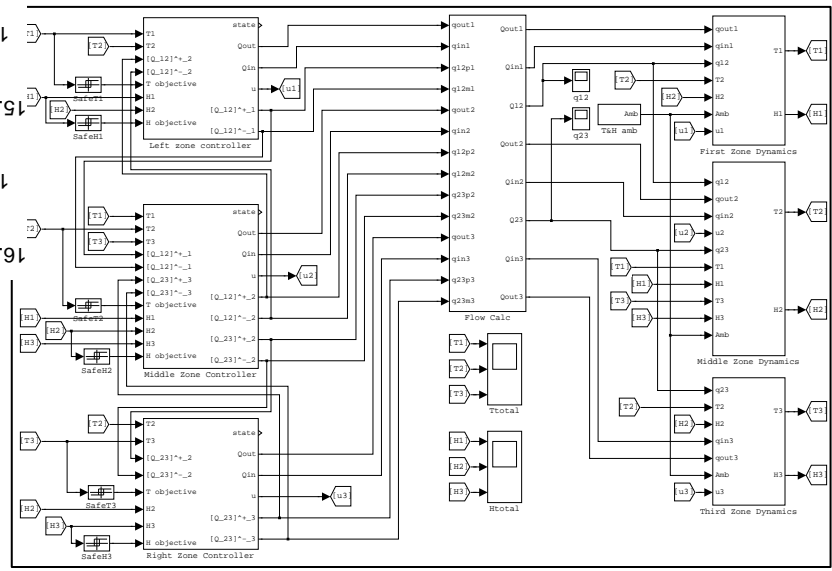
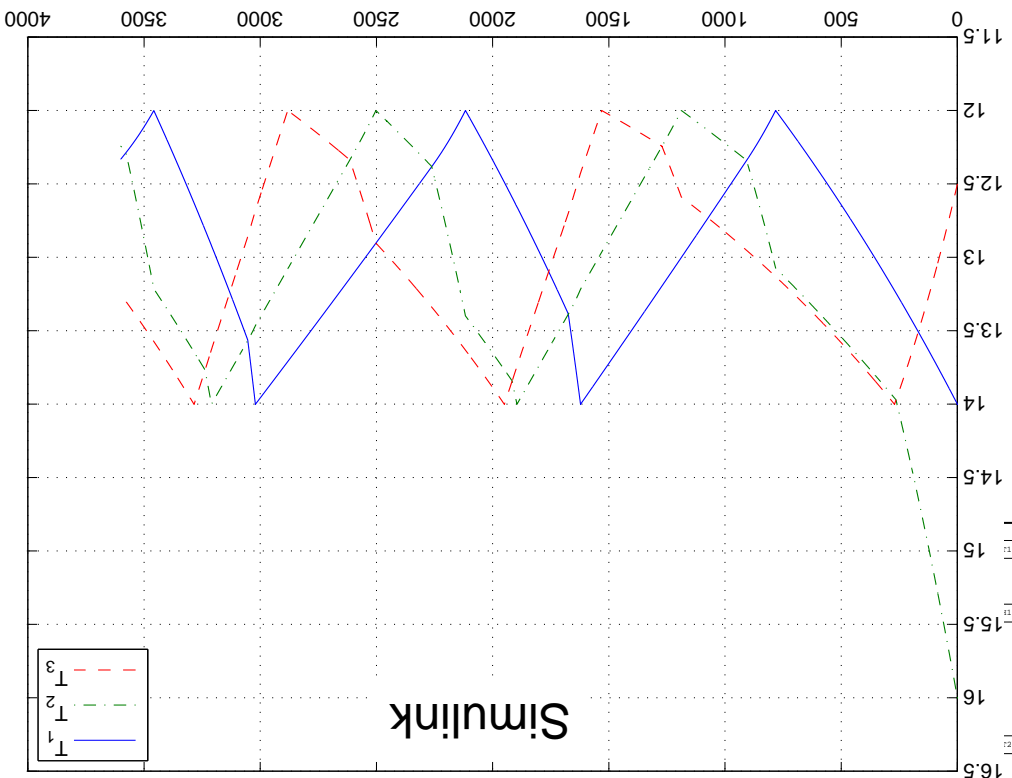
Strategy

1296 cases



```

control : A[]
((ZC.Init && objective) imply temp_derivative > 0) &&
((ZC.Init && !objective) imply temp_derivative < 0)
  
```



Obtaining executable code

Open Problems

Decidability

- Priced Timed Games
 - Reachability & Model Checking
 - Safety
 - Negative cost rates
- Probabilistic Priced Timed Automata



- Priced Timed Games

Thanks for your attention!
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www.cs.aau.dk

Efficiency

- Timed Automata
 - Fully Symbolic
 - Static Analysis & Slicing of C-code
- Timed Games
 - Alternating Logics
 - Partial Observability
 - CEGAR

- APPLICATIONS
- Live Sequence Charts
- Gantt Chart
- Code Generation
 - From TA models of controllers
 - From strategies

Usability