

# Model Checking Higher-Order Computation

C.-H. Luke Ong  
Oxford University, United Kingdom

Over the past decade, model checking and allied methods have made great strides in the verification of first-order imperative programs, producing impressive tools that can readily verify properties of hundreds of thousands of lines of code. A challenge facing software verification research is to extend these methods and techniques to the formal analysis of higher-order programs. This lecture course presents recent progress in the application of semantic methods to the verification of higher-order computation, covering mainly foundational work but also offering a glimpse of experimental tool construction.

Game semantics has emerged as a powerful method to construct highly accurate models of programming languages that lend themselves to algorithmic analysis. As a case study, we consider the fully abstract game semantics of the 3rd-order fragment of Idealized Algol (a higher-order procedural language), which is representable as visibly pushdown automata (VPA). Thanks to game semantics, we show that observational equivalence is decidable by reduction to the VPA equivalence problem. [4, 2]

Models of computation and their algorithmics. There has been a recent revival of interests in higher-order recursion schemes as generators of infinite structures such as infinite trees and graphs. As a definitional device, they are highly expressive and robust: they are equi-expressive with a new class of higher-order collapsible pushdown automata. The structures they generate enjoy strong model-checking properties. We present a new type-theoretic proof that the modal  $\mu$ -calculus model checking of these trees are n-EXPTIME complete. [5, 1, 3]

Flow analysis and reachability. We consider reachability in higher-order programs and its connections with flow analysis. We formulate reachability as a decision problem in PCF, and show that even in the recursion-free fragment generated from a finite base type, several versions of the reachability problem are undecidable from order 4 onwards, several other versions are reducible to each other; and identify a decidable case. We characterise a version of the reachability problem in terms of a new class of tree automata introduced by Stirling called alternating dependency tree automata and examine some consequences. [6]

## References

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