Abstraction for system verification

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Marktoberdorf, August 2010

Outline

1. Motivation
2. Property preserving abstractions: semantic level
   - Galois connexions between lattices
   - Abstractions for transition systems
3. Effectively computing abstractions
4. Verification of composed systems

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2. Property preserving abstractions: semantic level
   - Galois connexions between lattices
   - Abstractions for transition systems

3. Effectively computing abstractions

4. Verification of composed systems

What is verification?

We consider a specification / verification setting to be given by:

(1) a set of potential *design specifications*, called "models" $\mathcal{M}$, with $M \in \mathcal{M}$ (how)

(2) a set of potential *requirements* $\mathcal{L}$, with $\varphi \in \mathcal{L}$ (what)

(3*) a *satisfaction* or *conformance* relation $\models \subseteq 2^{\mathcal{M} \times \mathcal{L}}$ relating models and properties. We write $M \models \varphi$ and $M \not\models \varphi$.

(4) an *algorithm* to check $M \models \varphi$ (*model-checking*)

* Sometimes, we also consider the cases

- $M \models M'$ (refinement)
- $\varphi \models \varphi'$ (requirements engineering)

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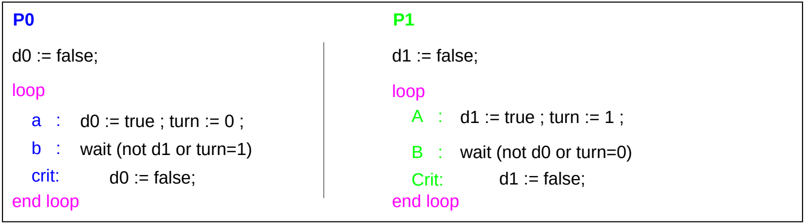
Example for M and φ: Peterson mutex algorithm



P0

```
d0 := false;
loop
   a   :   d0 := true ; turn := 0 ;
   b   :   wait (not d1 or turn=1)
   crit:        d0 := false;
end loop
```

P1

```
d1 := false;
loop
   A   :   d1 := true ; turn := 1 ;
   B   :   wait (not d0 or turn=0)
   Crit:        d1 := false;
end loop
```

Does the *design* M guarantee the following *requirements*?

- *mutual exclusion*: at most one process is in critical section *crit*
- *deadlock freedom*: system is never definitively blocked
- *non-blocking*: always, any of the critical sections is *reachable* in a bounded number of steps
- *fairness*: when a process is engaged (in *b* resp. *B*) its critical section *must be reached* in a bounded number of steps.

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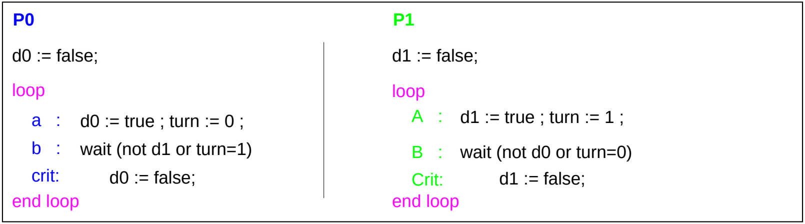

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d0 := false;                                d1 := false;

loop                                        loop
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(1) Design specifications

... express:

- a (set of) potential solution(s) (use a shared variable “turn” ....)
- i.e. specific algorithms/ components/ ...
- can (in principle) be “implemented”

Typical formalisms:

- programs, abstract programs
- (extended) automata, transition systems (TS), Kripke structures, Petri Nets, ...
- Composition: parallel composition ($P_0 \| P_1$)

Design specifications are often operational.

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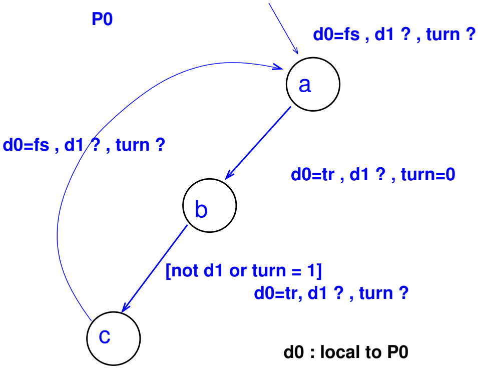
Example: Peterson as a composition of symb. TS

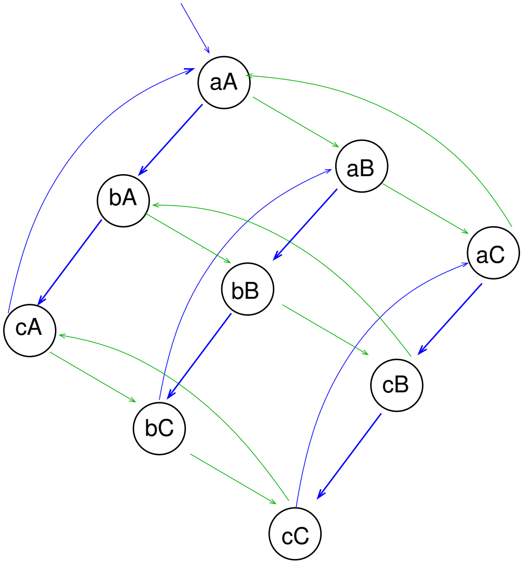


d0 : local to P0

d1 : local to P1

turn : shared

Example: Peterson as a global symbolic TS



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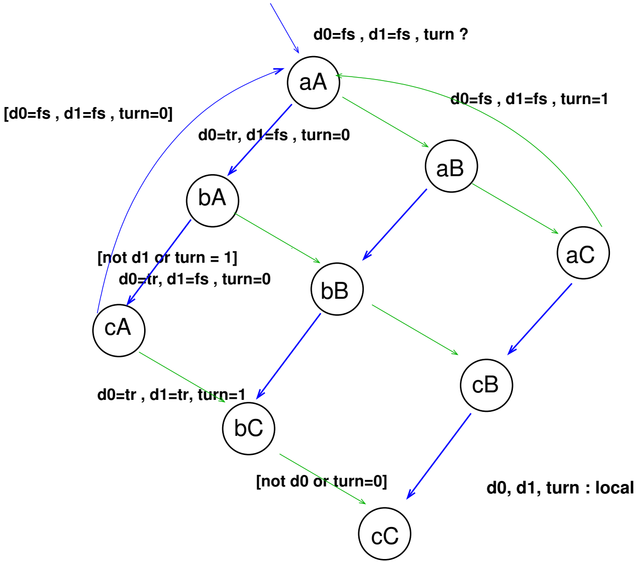

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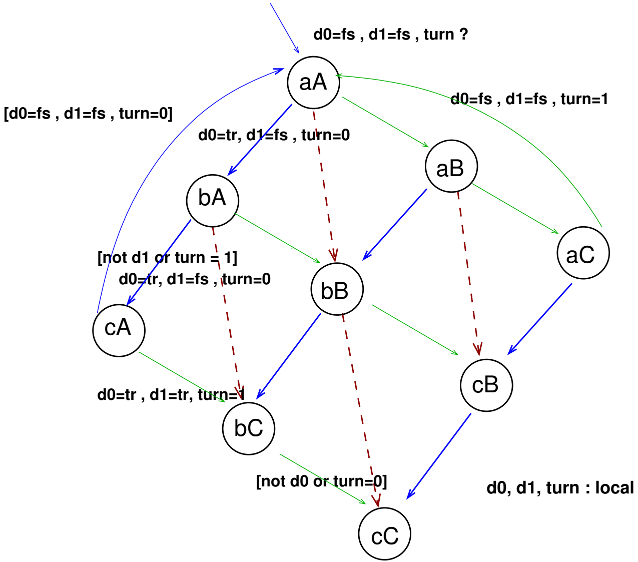


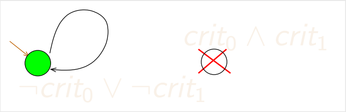
(2) Requirements

… express

- generic or specific properties that a component, system, algorithm, … should have (deadlock freedom, mutual exclusion, …)
- specifies "what" it does, its qualities, not how
- cannot generally not be meaningfully "implemented" (by a compiler)

Typical formalisms:

- (temporal) logic $\forall\Box(\neg crit_0 \vee \neg crit_1)$
- (extended) TS , …



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may be "executable" or not.

The difference is more in the intent than the form.

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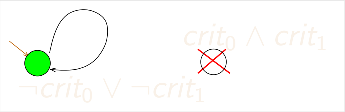

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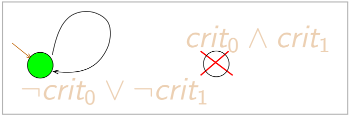

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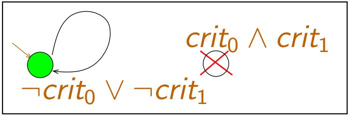

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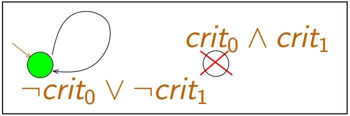

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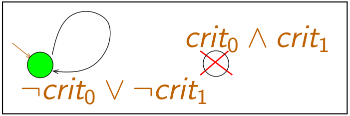

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(3) Satisfaction Relations

... defines what is means that M has property φ.

Typically defined on some *semantic domain*:

Semantic Property Domain	Relationship $\models$	Property class
Function relating input/output	equality	(correctness)
Reachable states	inclusion	(invariance)
Sets of executions/prefixes/streams	inclusion	(linear, LTL)
Refusal sets	inclusion	(reactivity)
TS	simulation	(structural)

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... an algorithm for checking the relation $\models$

$$\models \; : \; \mathcal{M} \times \mathcal{L} \mapsto \{tr, fs, fail\}$$

fail may be due to

- theoretical *undecidability* of $\models$
- excessive complexity (*state explosion*) of the algorithm used
- *incompleteness* of the algorithm

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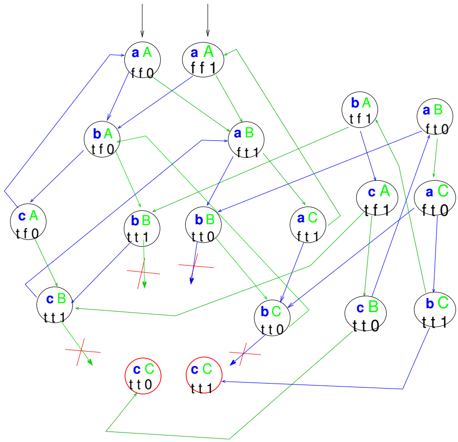
Example: Semantic model of the Peterson algorithm



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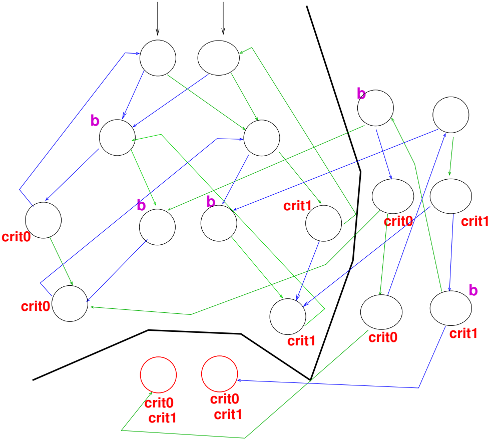

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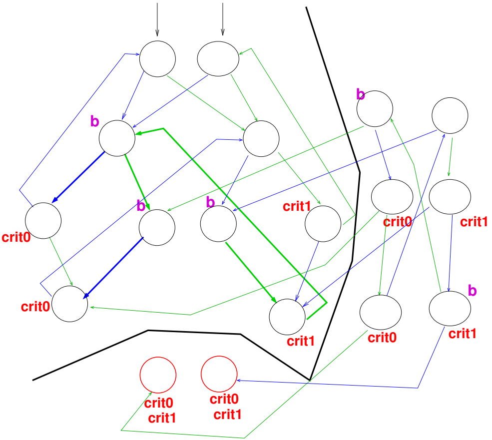

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$$b \subseteq |\Diamond crit_0|$$

The main difficulty of verification: complexity

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M, \varphi \arrow[d, "mapping"] & M \models \varphi \quad \text{(yes/no/?)} \\
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$\mathcal{M}$: compact syntax, program, timed automata, transition system (TS) with variables, composition, ...
$|\mathcal{M}|$: less structure, e.g. labeled TS $\{+$ constraints$\}$

$\Longrightarrow$ Model-checking algorithms $\models$ typically work in 2 steps:

- $\models^{\textit{step } 1}$: transformation into semantic models $|M|$, $|\varphi|$
- $\models^{\textit{step } 2}$: evaluate satisfaction based on $|M|$, $|\varphi|$

Main complexity: *step 1* (for M) $\longrightarrow$ **state explosion**

Note: performant procedures $\models$ mix steps 1 and 2: avoid computing $|M|$ exhaustively. But the problem of complexity explosion remains.

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An **abstraction** is a *property preserving transformation*

Given

- a verification setting: $(\mathcal{M}, \mathcal{L}, \models)$
- a transformation α : $\mathcal{M} \mapsto \mathcal{M}^A$ with $\mathcal{M}^A \subseteq \mathcal{M}$

Then

- α is an *abstraction* for $(\mathcal{M}, \mathcal{L}, \models)$ if

$$\forall M \in \mathcal{M}\ \forall \varphi \in \mathcal{L} \,.\, \alpha(M) \models \varphi \text{ implies } M \models \varphi$$

- α is called *strongly property preserving* if in addition

$$\forall M \in \mathcal{M}\ \forall \varphi \in \mathcal{L} \,.\, \alpha(M) \not\models \varphi \text{ implies } M \not\models \varphi$$

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What is abstraction?

An **abstraction** is a *property preserving transformation*

Given

- a verification setting: $(\ \mathcal{M}, \mathcal{L}, \models)$
- a transformation $\alpha\ :\ \mathcal{M} \mapsto \mathcal{M}^A$ with $\mathcal{M}^A \subseteq \mathcal{M}$

Then

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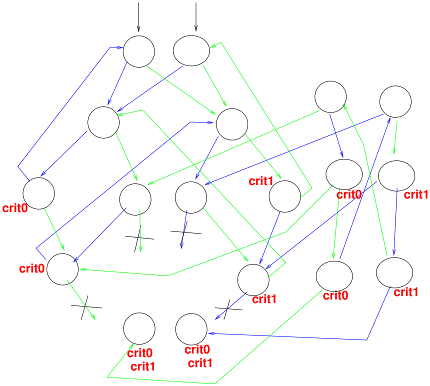
α: (1) group *states* to 5 abstract ones (black, green, blue, red, yellow), (2) draw a (green / blue) *transition* between abstract states if there is one between a corresponding pair of concrete ones



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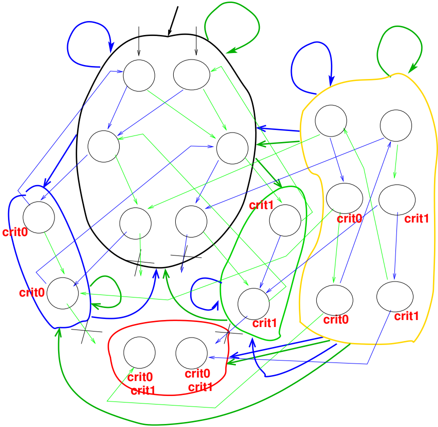

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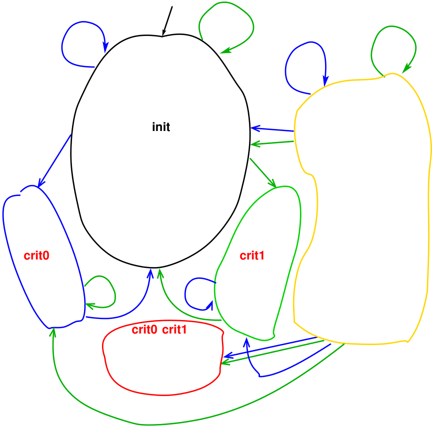

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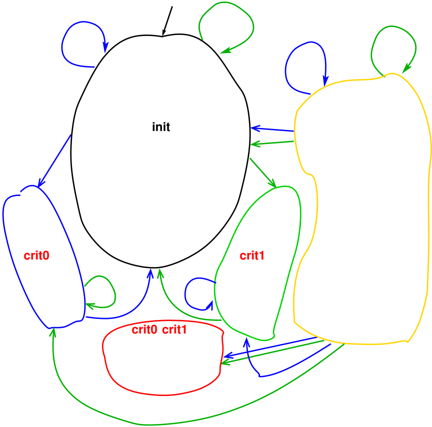

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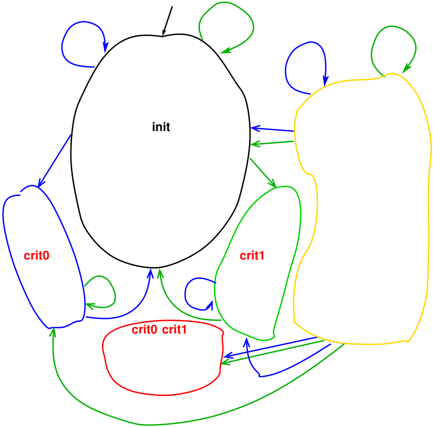

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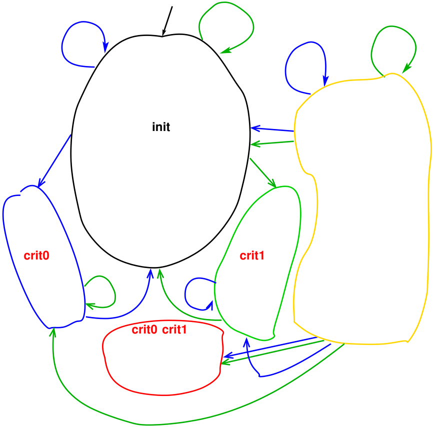

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The main motivation: avoid *state explosion*

Is our abstraction for Peterson useful?: not really,

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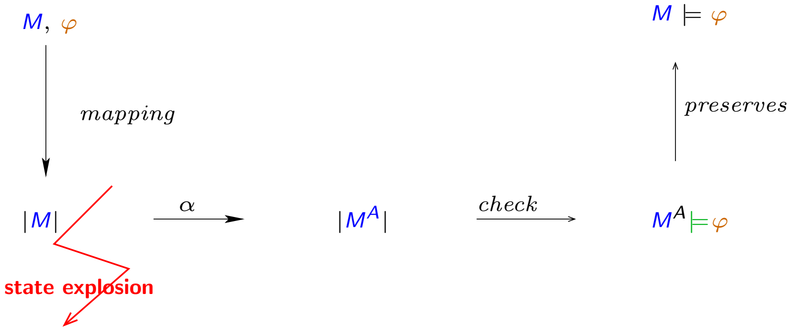

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What is particular to system verification?

We are interested in closed systems (the system + some more or less specific environment).

A system is composed of a (large) number of components (in parallel, $\|$) and there are requirements related to desired and undesired global (*emergent*) properties.

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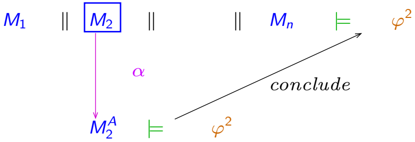

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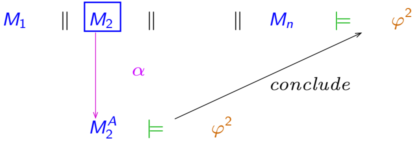

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Summary: problems we do not address

- Adequate *languages* for expressing models and requirements.
- Appropriate *composition* frameworks
- Appropriate satisfaction relations $\models$
- Algorithms $\models$ for solving verification problems $M \models \varphi$ for a given framework.
- Abstraction refinement: when both $M^A \models \varphi$ and $M^A \not\models \varphi$ fail (CEGAR approaches).