1.6 Exercises

Exercise 1 *Use Tseitin's transformation to convert* $x + (y \cdot (\overline{z} \oplus x))$ *into CNF.*

Exercise 2 *Follow the scheme in Table 1.2 in Section 1.2.1 to derive the Tseitin clauses that characterise the* n*-ary Boolean formulas* $(y_1 + y_2 + \cdots + y_n)$ *and* $(y_1 \cdot y_2 \cdot \cdots \cdot y_n)$*.*

Exercise 3 *Which of the Boolean formulae below are satisfiable, and which ones are unsatisfiable?*

1. $x + x \cdot y$
2. $\overline{(x \cdot (x \rightarrow y)) \rightarrow y}$
3. $x \cdot \overline{((x \rightarrow y) \rightarrow y)}$

Convert the formulae that are unsatisfiable into conjunctive normal form (either using Tseitin's transformation or the propositional calculus) and construct a resolution refutation proof.

Exercise 4 *Construct a resolution refutation graph for the following unsatisfiable formula:*

$$y_1 \cdot y_2 \cdot y_3 \; \cdot \; (\overline{y}_1 + x) \; \cdot \; (\overline{y}_2 + \overline{x} + z) \; \cdot \; (\overline{y}_3 + \overline{z})$$

Exercise 5 *Apply the rules of the Davis-Putnam procedure (outlined in Section 1.3.3) to the following formula until you obtain an equi-satisfiable formula that cannot be reduced any further:*

$$y_1 \cdot y_2 \cdot (\overline{y}_1 + x + \overline{z}) \; \cdot \; (\overline{y}_2 + \overline{x} + z) \; \cdot \; (y_3 + \overline{z}) \; \cdot \; y_4$$

Exercise 6 *Apply the Davis-Putnam-Logeman-Loveland (DPLL) procedure (described in Section 1.3.4) to the following formula:*

$$y_1 \cdot y_2 \cdot (\overline{y}_1 + x + z) \; \cdot \; (\overline{y}_2 + \overline{x} + z) \; \cdot \; (y_3 + \overline{z}) \; \cdot \; (\overline{y}_3 + \overline{z})$$

Exercise 7 *Simulate the* conflict-driven clause learning *algorithm presented in Section 1.3.5 on the following formula:*

$$\overbrace{(\overline{x} + \overline{y} + \overline{z})}^{C_0} \cdot \overbrace{(\overline{x} + \overline{y} + z)}^{C_1} \cdot \overbrace{(\overline{x} + y + \overline{z})}^{C_2} \cdot \overbrace{(\overline{x} + y + z)}^{C_3} \cdot$$
$$\overbrace{(x + \overline{y} + \overline{z})}^{C_4} \cdot \overbrace{(x + \overline{y} + z)}^{C_5} \cdot \overbrace{(x + y + \overline{z})}^{C_6} \cdot \overbrace{(x + y + z)}^{C_7}$$

Exercise 8 *Use the approach described in Section 1.3.6 to construct a resolution refutation proof for the formula presented in Exercise 7.*

Exercise 9 *Find an unsatisfiable core of the formula*

$$(y) \cdot (x + \overline{y} + z) \cdot (\overline{x} + z) \cdot (\overline{x} + \overline{y}) \cdot (\overline{z} + \overline{y})\,.$$

(You are not allowed to provide the set of all clauses as a solution.)
Is your solution minimal?

Exercise 10 *Simplify the following formula using the substitution approach described in Section 1.3.10:*

$$\begin{aligned} w \cdot (\overline{q} + z) \cdot (\overline{q} + x) \cdot (\overline{z} + \overline{x} + q) \cdot (\overline{p} + \overline{z}) \cdot (\overline{p} + \overline{x}) \cdot (z + x + p) \cdot \\ (\overline{p} + u) \cdot (\overline{q} + u) \cdot (\overline{u} + p + q) \cdot (\overline{y} + v) \cdot (\overline{u} + v) \cdot (\overline{v} + y + u) \cdot \\ (\overline{x} + w) \cdot (\overline{v} + w) \cdot (\overline{w} + x + v) \end{aligned}$$

Exercise 11 *Use the core-guided algorithm presented in Section 1.4.3 to determine the solution of the partial MAX-SAT problem*

$$(\overline{x} + \overline{y}) \cdot (\overline{x} + z) \cdot (\overline{x} + \overline{z}) \cdot (\overline{y} + u) \cdot (\overline{y} + \overline{u}) \cdot (x) \cdot (y)\,,$$

where only the clauses (x) *and* (y) *may be dropped.*

Exercise 12 *Use the algorithm presented in Section 1.4.4 to derive all minimal correction sets for the unsatisfiable formula*

$$\overbrace{(x)}^{C_1} \cdot \overbrace{(\overline{x})}^{C_2} \cdot \overbrace{(\overline{x} + y)}^{C_3} \cdot \overbrace{(\overline{y})}^{C_4} \cdot \overbrace{(\overline{x} + z)}^{C_5} \cdot \overbrace{(\overline{z})}^{C_6}\,.$$

Exercise 13 *Derive all minimal unsatisfiable cores for the formula presented in Exercise 12.*